Incorporating travel time reliability in the estimation of assignment models

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Abbreviations and acronyms

AADT	annual average daily traffic
CI_t	congestion index
CV_t	coefficient of variation of travel time
%RMSE	root mean square error expressed as a percentage
GEH	a modified chi-squared statistic used in traffic model validation
HCV	heavy commercial vehicle
NB	northbound
NZTA	New Zealand Transport Agency
R^2	regression coefficient squared
$RMGEH^2$	root mean square GEH – a non-standard validation statistic
SB	southbound
SH	state highway
V/C	traffic volume to capacity ratio
WTM	Wellington Traffic Model
WTSM	Wellington Transport Strategic Model

Contents

Executive summary

This research investigated the incorporation of travel time reliability in traffic assignment models. In most traffic assignment models, route choice is determined by some function of the mean travel time and distance on the alternative routes available. Such models do not normally consider travel time reliability as a determinant of route choice. However, there is increasing evidence that travel time reliability is a significant variable that road users consider in route choice.

International literature was reviewed to identify if there were any instances where travel time reliability had been incorporated into transportation models and what approaches had been used.

Travel time reliability did not appear to have received as much attention in transport planning theory and analysis as travel time or distance in travel choice making. Nevertheless some preliminary work in the area has occurred in many countries around the world. Literature on the subject has been published in the UK and Europe, North America, Asia, Australia and New Zealand. The causes of travel time variability fall into two areas. These are changes in demand and changes in capacity. Changes in demand can depend on the day of the week, special events, or general randomness in demand. Changes in capacity include:

- incidents
- weather
- breakdowns
- debris on the route
- work zones including construction and maintenance
- traffic control devices.

It appears from the literature the incorporation of travel time reliability in modelling has not been well developed. Little experience was found of including travel time reliability in assignment models. The modelling of travel time reliability is still at a research or exploratory level.

A variety of travel time reliability measures have been used including standard statistical measures such as the standard deviation, variance and others, as well a variety of indexes such as the buffer time index. The time road users allow for a journey is a measure of their risk aversion, which is a function of trip purpose.

There is little guidance in the literature on how travel time variability should be included in deterministic models. However, relationships have been found between the standard deviation and the mean of link travel time, which should enable a utility formulation to be promising for travel time variability.

There is little statistical evidence to demonstrate that models incorporating travel time variability have greater explanatory power than those that do not. However, this may reflect that travel time reliability is at an early stage of development in the transport sector.

There is evidence that road users value travel time reliability. The limited available evidence suggests the value of reducing travel time variability is greater than that of reducing mean travel time. Anecdotal evidence indicates bus users value punctuality more than journey time reductions which may suggest vehicle users value a minute of unexpected time more than a minute of in-vehicle time.

Travel time data from 11 routes in Wellington was analysed. This data included separate surveys for each direction of travel and for the weekday morning, inter and evening peak periods.

The usefulness of differing measures of travel time variability was investigated in the context of developing a function to forecast travel time variability. Measures of variability included the standard deviation, variance, the 95th percentile value and the coefficient of variation. These measures were applied to the travel time data set from the 11 routes in Wellington.

This travel time data provided a useful basis to determine whether there were statistically based relationships between the coefficient of variation and the mean travel time and the route length as the power function proposed by Hyder Consulting (2007). The statistical validity of a variety of models was examined and a model for forecasting travel time variability based on mean travel time has been recommended.

The analysis of the data by different models indicated the relationship between the coefficient of variation of travel time (CV_t) and the congestion index (CI_t) had two distinct phases. The first phase had CV_t continuously increasing with CI_t. The second phase had CV_t continuously decreasing with CI_t. The various models had a reasonably consistent breakpoint, ψ, where ψ was approximately 1.4.

Simple relationships between travel time variability and route travel time provided a mathematically simple approach to estimating travel time variability.

The breakpoint between the two phases in this relationship represented by ψ suggested the behaviour of CV_t might undergo some change with higher levels of congestion. This could be associated with some kind of change in the traffic flow regime and should be examined further.

A simple four-node, five-link synthetic network was examined. The travel time variability function derived from the one calibrated for the Wellington network was used for a set of experiments. These experiments on the synthetic network showed increasing travel time variability cost had a greater influence on traffic route choices over a network if the trip matrix had high heterogeneity. A high heterogenic matrix is dominated by a small set of origin-destination pairs with the potential to cause congestion where a change in the validation function may lead to a rapid change in the validation. However, such responses are location specific and dependent on model formulation. In trip matrices with low heterogeneity the assignment of traffic over a network did not appear to be greatly influenced by travel time variability cost.

The validation measures changed slowly with travel time variability costs, where these costs were low and the trip matrix had high heterogeneity. Trip matrices with high heterogeneity showed rapid change in the validation measures near the optimum value of travel time variability costs.

Trip matrices with low heterogeneity showed rapid response in the validation measures with small travel time variability costs. Trip matrices with low heterogeneity showed change in the validation measures near the optimum value of travel time reliability costs. All trip matrices demonstrated slow response in the validation measures with large travel time reliability costs.

The difference between the validation measure at the optimum value of travel time variability cost and the validation measure with zero travel time variability cost was found to be statistically significant at both the 95% and 99% levels of significance. This was found to be true for all validation measures and suggested there should be a value of the travel time variability cost that clearly improved all validation measures where travel time reliability was not incorporated into the model.

The findings related to travel time reliability analysis were applied to a traffic assignment model of a real network. Models such as those using the SATURN software platform where the generalised cost function is hard wired and does not include travel time reliability were considered. Methodologies were developed to formulate the generalised cost function so it would mimic the inclusion of travel time reliability.

These methodologies were applied to a SATURN model in a real application. In particular, the impact on network-wide validation statistics and the effect on screenline GEH values were investigated.

The formulation of the standard deviation of travel time developed in this project was found to be convenient for analysing the impact of travel time reliability in traffic assignment models.

An iterative approach with these models, outside of the SATURN modelling framework, was required to make them amenable to having their generalised cost function mimic travel time variability. This iterative approach appeared to be numerically stable and converged quickly. This enabled the effects of travel time variability to be modelled with a reasonable level of convenience and without requiring undue time.

Simple numerical techniques were used to estimate optimal travel time variability costs from a range of travel time variability costs.

The incorporation of travel time variability into an assignment model did not improve network wide validation statistics. However, in the case study undertaken the model used already had a high degree of model validation. Local optimisation of network-wide validation statistics versus travel time variability cost showed the travel time variability cost was in the range of 4.9 to 5.1 times the travel time cost.

The incorporation of travel time variability into an assignment model did improve the GEH values of some selected key screenlines compared with when the travel time variability cost was zero. On some screenlines it was possible to achieve a GEH value of zero in a particular direction which represented a perfect match of modelled and observed traffic volume. However, this local improvement in GEH values resulted in an overall deterioration of the network wide validation statistics meaning other screenline GEHs deteriorated significantly.

It was possible to achieve an improvement in the GEH value simultaneously in both directions of a screenline. This was achieved on all screenlines and produced a non-zero GEH value in each direction whose value was an improvement on the GEH value when the travel time variability cost was zero. Generally the split of traffic volumes at the crossing points on the screenline remained realistic.

A travel time variability cost in the range of 3.7 to 3.8 times the travel time cost was found to improve the GEH values of all three selected screenlines in both directions simultaneously. However, the network wide validation statistics were worse than those for a travel time variability cost of zero.

A travel time variability cost in the range of 4.9 to 5.2 was found to improve the GEH values for a key screenline for both directions simultaneously and this coincided with the local optima of the network validation statistics. This represented a region of improved matching of traffic volumes at the screenline and where the deterioration in the network wide validation statistics was less significant.

The examination of route travel times in response to travel time variability cost appeared to demonstrate the correct attributes using the modelling approach advocated in this report. This requires that routes with low volume to capacity ratios have travel times that are not sensitive to travel time variability cost whereas routes with higher volume to capacity ratios have travel times that are sensitive to travel time variability cost. In simple terms, a more congested road exhibits greater travel time variability effects.

Abstract

Route choice is determined by some function of mean travel time and distance on the routes available in most traffic assignment models. Increasing traffic volumes on a route increases delay, making a particular route less desirable.

The NZTA (2010) *Economic evaluation manual* allows the benefits of improved network reliability to be monetised. However, our network models are unable to provide a convenient means of calculating the road user responses to travel time variability.

Route choice is a more complex issue than a comparison of relative travel times and distance. It appears that road users are also considering travel time variability in their route choice. Variability may occur as a result of congestion in cities or on any network as a result of road geometry, a high volume of heavy vehicles on narrow steep roads, or other reasons.

This research was carried out during 2008 to 2011 using Wellington data and sought to identify a methodology that best incorporated travel time variability into route choice models. The research determined the most useful formulation for use in models and the appropriate measure of travel time variability.

1 Introduction

Most car assignment models determine route choice by some function of the mean travel time and distance on the alternative routes available. There are a small number of purpose-built models where travel time reliability is explicitly incorporated in the route choice function. Most models respond to increasing delay on a given route by selecting other routes that have less delay for a given route length.

Even though the NZ Transport Agency's (NZTA) (2010) *Economic evaluation manual* allows the benefits of improved network reliability to be monetised, most network models are unable to provide a convenient means of calculating road user responses to travel time variability.

It has become apparent that route choice is a more complex issue than a comparison of relative travel times and distance. It appears road users are taking into account the variability of their travel time as well as mean travel time in their route choice. The variability may occur as a result of congestion in larger cities or on any network as a result of road geometry, a high volume of heavy vehicles on narrow steep roads, or for some other reason.

The purpose of this research was to establish whether a convenient and robust methodology was available for the incorporation of travel time reliability in route choice algorithms of conventional assignment models. To achieve this purpose a number of objectives needed to be realised.

The first objective of this research was to survey the international literature on the subject of travel time reliability to:

- attain an understanding of how travel time reliability influenced drivers' route choice
- document international experience involving travel time reliability
- determine a convenient means to mathematically describe travel time reliability
- ascertain any known functional relationships with other variables.

A second objective of this study was to analyse a New Zealand travel time survey data set to:

- examine the usefulness of various mathematical descriptions of travel time reliability
- test the validity of the functional relationships found in the literature review with New Zealand data
- determine a convenient formulation of travel time reliability for inclusion in assignment models.

A third objective was to gain insights into the effect of travel time reliability by application to a simple synthetic network.

The final objective of this study was to apply the methodology for incorporating travel time reliability to a real world conventional assignment model to:

- show incorporating reliability is feasible
- show the methodology is not unduly onerous and is numerically stable
- examine the impact on network wide validation statistics
- examine the impact on modelled versus observed traffic volumes on selected screenlines

- examine the impact on selected route travel times.

To achieve these objectives, this study analysed Wellington region travel time data. This study was undertaken between 2008 and 2011.

2 Literature review

2.1 Preamble

This chapter summarises the findings of a literature review on the subject of incorporating travel time reliability into the estimation of assignment models. In most car assignment models, route choice is determined by some function of the mean travel time and distance on available alternative routes. However, there is increasing evidence that travel time reliability is a significant variable that road users take into account when choosing a route.

International literature was reviewed to identify if there were any instances where travel time reliability had been incorporated into transportation models and what approaches had been taken. The literature review investigated the following and the results are discussed in this report:

- where and why accounting for travel time reliability was used
- what was the cause of travel time variability (congestion/geometric/other)
- in what kinds of models was travel time reliability incorporated (strategic multimodal/traffic assignment)
- what kind of formulation was used to incorporate travel time reliability (linear weighted objective function/generalised cost function/other)
- what was the measure of travel time reliability used (standard deviation of journey time/range of journey time/other)
- the statistical evidence to support the inclusion of travel time reliability
- the relative importance of travel time reliability versus measures such as mean travel time and distance.

2.2 Where and why accounting for travel time reliability has been used

A review of international literature showed the importance of accounting for travel time reliability has been recognised throughout the world. Literature on the subject has been published in the UK, Europe, North America, Asia, Australia and New Zealand. It would appear the issues surrounding travel time reliability are acknowledged around the globe.

In New Zealand, the *Economic evaluation manual* (NZTA 2010) provides procedures for the calculation of economic benefits associated with proposals that improve travel time reliability.

2.2.1 Economic consequences

Sumalee and Watling (2003) in the UK commented that variability in travel times could have a significant impact on people and the economic system in a community. Redundancy must be built into planning a journey which, in the case of commercial organisations, is reflected in the cost of commodities. This, they argued, would lead to a reduction in welfare for society.

Goodchild et al (2007) said travel time variability in the USA could impact significantly on supply chains. Extra time must be built into journey planning which could lead to the under-utilisation of equipment, but if this was not done, there was a risk of missing delivery windows, which could result in the loss of business.

2.2.2 Transport policy and infrastructure planning

Qiang (2007) said, in Japan, presenting travel time reliability information in ways that were meaningful for the public had outcomes such as reducing the amount of road building required, as the road network was used more efficiently by users who were able to make more informed travel choices. Ruimin (2004), in Australia, discussed the importance of travel time variability and information systems to optimise network performance.

Brownstone et al (2002) reported on a congestion pricing scheme on the San Diego I-15. From this work they were able to deduce that the relative importance of travel time reliability was greater than that of mean travel time. This was in the context of willingness to pay in a congestion pricing scheme.

Chen et al (2003) discussed travel time reliability as a measure of network level of service. They concluded travel time reliability was a useful measure of level of service that complemented other measures. This discussion was within the context of the road network in the vicinity of Los Angeles.

2.2.3 Transport modelling

A number of authors considered including travel time variability as a means of improving network models. In the USA this included the Texas Department of Transport. In the UK, the Department for Transport and the Institute of Transport Studies at Leeds University (Hollander and Liu 2008) undertook research work in this area. Similarly, research was undertaken on this subject at Monash University in Australia (Ruimin 2004). No clear conclusions were reached.

2.2.4 Public transport

Vincent (2008) undertook research that investigated the relative importance in economic terms of travel time reliability. Vincent concluded travel time reliability was an important determinant of whether a person used public transport. Although this study was primarily interested in traffic assignment it appeared travel time variability might also influence mode choice. This economic valuation considered travel time reliability in the New Zealand context.

2.3 Causes of travel time variability

Variability in travel time along any given route can have a variety of causes. It appeared from the literature that road users considered travel time variability was an important issue. Margiotta (2006) reported that in the USA road users were willing to pay for reductions in travel time variability between two to six times the rate for mean travel time reductions.

Travel time variability generally has two classes of causes. These classes relate first to changes in demand and second to those things that change route supply. In simple terms, the volume-delay curve provides the relationship between travel time and the demand divided by the capacity of a route. These two classes of events refer separately to factors that change the two components of the volume to capacity ratio. The result of change in these components is a change to the travel time.

These changes in demand and route capacity may be recurrent or isolated and they may be short term or long term (Sumalee and Watling 2003). The attributes of the causes of travel time variability are discussed in more detail below.

2.3.1 Changes In demand

Among factors that influence demand, Lomax et al (2003) listed special events and random fluctuations. Special events are clearly a generator of additional travel although it should be recognised that major events, in particular, might have special event management plans in place that could result in route capacity changes. Special events that occur more than once are recurrent sources of travel time variability and the impact will last as long as the event does.

Traffic demand is understood to exhibit a degree of randomness that leads to fluctuations in flows and therefore variability in travel time. Due to the shape of volume-delay curves, a route with inadequate base capacity will show larger variability in travel time than routes with spare base capacity. These fluctuations are recurrent but can be of short or long duration. In New Zealand, because of lower levels of daily congestion compared with some European or North American cities, these fluctuations are usually shorter.

Both Ruimin (2004) and Margiotta (2002) identified the day of the week as a source of travel time variability that produced changes in demand. This reflects business or recreational cycles where activities that occur on one day of the week might be different from another. Examples of this phenomenon include 'late night' shopping on certain days of the week or different travel behaviours associated with the work commute that might occur the day before or the day after a weekend. This type of travel time variability is usually recurrent and typically of shorter duration.

Cheu et al (2007) reported this might be more than simply a change in the volume of traffic as not only could the volume of traffic change as a result of week-day cycles but also the proportion of heavy vehicles. Ruimin reported the impact of the day of the week on travel time variability could be a particularly significant variable compared with others, depending on the overall levels of heavy vehicles.

2.3.2 Changes in capacity

These same authors listed a range of events that influence travel time by impacting on the capacity of the route. These include:

- incidents
- weather
- breakdowns
- debris on the route
- work zones including construction and maintenance
- traffic control devices.

The most common example of an incident is a vehicle crash. This can be minor or serious and is not usually recurrent. Crashes can lead to a partial or complete closure of a road. Incidents include crashes occurring in the opposite flow direction that can act as a distraction to drivers.

Weather-related events include slips, flooding, snow, ice and also sun strike. They are not usually recurrent and, depending on their severity, can have short-term or long-term consequences. Major weather interventions can completely close a road but events like sun strike can reduce effective capacity.

Breakdowns reduce effective capacity by occupying a lane and by providing a distraction to other road users. These are not usually recurrent events and are usually short term. Debris reduces the capacity of one or more lanes by requiring vehicles to slow down or avoid them. These are not usually recurrent events and are short lived.

Construction and maintenance activities are both a distraction to road users and can reduce the effective capacity of a road. Construction and maintenance are non-recurrent activities. Construction can be long term whereas maintenance is usually shorter in duration. However, both construction and maintenance activities can be planned to reduce the impact on traffic flow.

Poorly timed traffic signals and at-grade railroad crossings are examples of traffic control devices that cause a reduction in effective road capacity. In the case of traffic signals the effective road capacity can vary with the arrival rate of trains, and at which part of the signal phasing the trains arrive. This may influence the variability of delay at the crossing.

Margiotta (2002) provided data from the USA that indicated the additional delay and economic cost caused by incidents appeared to be greater than the outcome of other events that reduced effective road capacity.

2.4 Models that incorporate travel time reliability

Few examples were found of mainstream network traffic models that had incorporated travel time reliability, except for some in the context of specialised research.

Reckner et al (2005) undertook research that considered the differing responses of risk-averse and risk-taking road users in terms of trip departure and route choices. They assessed that most route choice models worked on the assumption road users had perfect knowledge of travel time on route options. In reality, most road users do not have perfect knowledge of travel time on route options and the amount of risk a user is prepared to accept may impact on route choice.

Reckner et al examined a variety of methods to build in responses to variability in travel time in assignment models, including both deterministic and stochastic approaches. Among the deterministic approaches were models that used a travel time disutility function that was a weighted function of mean travel time and the standard deviation of travel time. Reckner et al also considered a mixed logit route choice formulation.

Qiang (2007) and Hollander and Liu (2008) developed micro-simulation models incorporating travel time variability. Qiang's model required the maximum and minimum travel times on a route to be specified and used an assumed distribution of travel times within that range. Hollander and Liu separately incorporated spatial, temporal and stochastic variability into travel times.

Cheu et al (2007) developed an assignment model where the link travel time was replaced by a link disutility function. The link disutility function is a weighted function of mean travel time and travel time variance. This approach requires the calibration of the link disutility function.

In summary, there are few examples of network models that have incorporated travel time reliability. Where deterministic network models have been developed they generally use a disutility function which is a weighted sum of mean travel time and a measure of travel time variability. Both the standard deviation and the variance of travel time have been used as this measure of travel time variability.

The articles identified in the literature appeared to suggest the use of travel time reliability in modelling was not well developed. Few lessons have been learned. The modelling of travel time reliability is still very much at a research or exploratory level with no clear conclusions on its benefits.

2.5 Measures of travel time reliability

The concept of travel time reliability is not as well developed as other areas of transportation research. Accordingly the thinking around appropriate parameters for measuring travel time reliability is still in its infancy. Tseng (2005) commented there was no generally accepted measure of travel time reliability.

A variety of measures of travel time reliability were found in the literature. Lomax et al (2003) and Margiotta (2002) discussed the use of standard statistical measures that included mean travel time, the standard deviation, variance and various percentile travel times. Their work represents some of the founding thinking in the area of travel time reliability measurement.

Reckner et al (2005) undertook investigations of risk taking in travel decisions in California that found the standard deviation of travel time was strongly correlated with mean travel time for a given length of route. This was within the context of recurrent travel time variability. Chen et al (2003) supported this finding. A correlation between mean travel time and the standard deviation of travel time is expected because as the traffic volumes increase on a route, the road becomes more congested. Increased levels of congestion lead to greater mean travel times and greater variability in travel times.

Brownstone et al (2002) and Chen et al (2003) reported on research where 90th and 95th percentile travel times were used in conjunction with the mean travel time as a measure of travel time variability. Brownstone et al's work related to a willingness to pay for travel time savings and improvement in travel time reliability. The difference between the 90th or 95th percentile travel time and the mean travel time was used as a measure of travel time variability. Goodchild et al (2007) and Lomax et al (2003) introduced the buffer index that normalised this measure by dividing the difference by the mean travel time and expressing it as a percentage.

Lomax et al (2003) introduced the 'tardy trip indicator'. This is a measure of how often a traveller is unacceptably late. This indicator is the converse of the 90th or 95th percentile travel time considered by Brownstone et al (2002) and Chen et al (2003). Lomax et al discussed the 'variability index' which is the ratio of the mean peak period variability in minutes to the off-peak variability.

Tseng (2005) built upon the above work by considering the behaviour of road users who responded to travel time variability on a route. Tseng used the equation

$$T = E(t) + \lambda\sigma(t) \qquad \text{(Equation 2.1)}$$

where T is the time a road user allows to undertake a journey

$E(t)$ is the expected travel time for that journey

$\sigma(t)$ is the standard deviation of the travel time on that journey.

Tseng argued that λ was a measure of how risk averse the road user was in the context of a particular trip. A low value of λ indicated the road user was less risk averse and a high value of λ indicated the road user was highly risk averse. It was expected λ would vary according to trip purpose. For example, a journey to the airport to catch a plane or a journey to deliver a commodity that had an absolute deadline for delivery could be expected to have high values of λ.

In summary, the literature indicated a variety of travel time reliability measures had been used but the development of thinking in this area was still in its infancy. Standard statistical measures such as the standard deviation, variance and various percentile values have been used as well a variety of indexes such as the buffer time index. The time road users allow for a journey is a measure of their risk aversion which is a function of trip purpose.

2.6 The formulation of travel time reliability for incorporation in models

The modelling of recurrent travel time variability requires some functional formulation. Cheu (2007) proposed a link disutility function that incorporated both mean travel time and the standard deviation of travel time. The functional form of this is

$$U = \alpha E(t) + \beta\sigma(t) \qquad \text{(Equation 2.2)}$$

where α and β are constants that would need to be calibrated and U is the link disutility function that would replace travel mean travel time.

This kind of formulation is dependent on being able to estimate both $E(t)$ and $\sigma(t)$ as functions of demand volume.

The behaviour of $E(t)$ as a function of volume is a well understood relationship and incorporated in models using volume delay curves. The usefulness of equation 2.2 in a modelling context is dependent on being able to determine $\sigma(t)$ for differing link volumes.

As discussed above, Reckner et al (2005) found there was a strong correlation between mean travel time and the standard deviation of travel time. Arup (2004) investigated the relationship between the standard deviation of travel time and mean travel time. Both found that for traffic volumes below the capacity of the link there existed a linear relationship between the standard deviation and mean travel time.

$$\sigma(t) = \gamma E(t) + \delta \qquad \text{for } v < C \qquad \text{(Equation 2.3)}$$

where γ and δ are constants, v is the link volume and C is the link capacity.

Arup (2004) commented that, for volumes above the link capacity, the standard deviation was no longer a linear function of mean travel time. The relationship was more complex and needed to take into account the profile of demand including before the period being modelled. This was because whenever volume exceeded capacity a queue would be generated. Arup advocated making some simple assumptions about the shape of the demand profile which would enable a relationship to be found between the mean travel time and its standard deviation. However, many of these modelling approaches were developed for congested networks.

Hyder Consulting (2007) found a relationship for urban areas in the UK of the form

$$CV_t = \alpha CI_t^{\beta} d^{\delta} \qquad \text{(Equation 2.4)}$$

where CV_t is the coefficient of variation = $\sigma(t)/E(t)$

where CI_t is the congestion index = $E(t)/T_0$

where d is route length in km

where T_0 is the free-flow travel time for the route

where α, β and δ are constants that require calibrating.

When Hyder Consulting estimated this model, they obtained average values for α to be 0.16, β to be 1.02 and δ equal to –0.39. This result appears to be consistent with the findings of Reckner et al (2005) but, unlike Arup, would suggest the standard deviation of route travel time is not a linear function of mean route travel time but is approximately related to the square of the mean route travel time.

Hyder Consulting (2008) applied these relationships to estimate the economic benefits of several road schemes that had an impact on both mean travel time and travel time variability. They found road schemes that reduced congestion produced increased economic benefits.

The literature review showed there was little guidance on how travel time variability should be included in deterministic models. However, relationships did exist between the standard deviation and the mean of link travel time, which would enable a utility formulation to be promising for modelling recurrent travel time variability.

2.7 Statistical evidence for incorporating travel time reliability

This literature review did not find any statistical evidence to demonstrate models that incorporated travel time variability had greater explanatory power than those that did not. This is because there is little analysis of this issue available and reflects the early stage of development that currently exists in the transport sector in allowing for travel time reliability.

2.8 The relative importance of travel time reliability

Sumalee and Watling (2003) commented that travel time variability could have significant impacts on people and the economy. Travel time variability could add significant stress to drivers particularly if their trip purpose had high value or there was some absolute deadline to meet.

Travel time variability means additional time has to be allowed if goods need to be delivered by a specified time. This leads to additional cost for goods and inefficient use of resources. Goodchild et al (2007) commented that travel time variability was a particularly significant issue for perishable goods.

There have been a number of studies that have attempted to monetise travel time variability. Tseng reported that his analysis produced a wide range of monetary values for travel time reliability. Brownstone et al (2002) found, in a congestion pricing context, a willingness to pay more for increased travel time reliability than a mean travel time reduction. This was in the context of recurring congestion.

Margiotta (2006) undertook work that compared the willingness to pay for reduced travel time variability and mean travel time. He found the willingness to pay for reduced travel time variability was two to six times that of a reduced mean travel time.

While there is limited information available on the value of travel time reliability it would appear road users value travel time reliability. The limited available evidence suggests the value of reducing travel time variability is greater than that of reducing mean travel time.

2.9 Summary

Although the analysis of travel time reliability appears not to be well developed some preliminary work in the area has occurred in many countries around the world. Literature on the subject has been found in the UK and Europe, North America, Asia, Australia and New Zealand. The various authors have discussed travel time reliability from a variety of perspectives including its economic consequences, its modelling, transport policy and infrastructure planning and public transport.

The causes of travel time variability fall into two areas: changes in demand and changes in capacity.

A variety of travel time reliability measures have been used including standard statistical measures such as the standard deviation, variance and various percentile values, as well indexes such as the buffer time index. The time road users allow for a journey is a measure of their risk aversion, which is a function of trip purpose.

There is little guidance in the literature on how travel time variability should be included in deterministic models. However, relationships do exist between the standard deviation and the mean of link travel time, which should enable a utility formulation for recurrent travel time variability.

There is little statistical evidence in the literature to demonstrate that models incorporating travel time variability have greater explanatory power than those that do not. This reflects the early stage of development that currently exists in the transport sector in allowing for travel time reliability.

It would appear that road users value travel time reliability. The limited available evidence suggests the value of reducing travel time variability is greater than that of reducing mean travel time.

3 Data analysis

3.1 Preamble

This section documents the investigations into the development of a travel time variability function and analyses data from travel time surveys on 11 routes in Wellington. The data includes separate surveys for each direction of travel and each of the weekday morning, inter and evening peak periods. The effect of the travel time variability function on route choice was examined on a simple synthetic model and used as the basis of a methodology for incorporation in an assignment model of a real network.

The research considered the usefulness of differing measures of travel time variability in the context of developing a function to forecast travel time variability. Measures of variability included the standard deviation, variance, the 95th percentile value and the coefficient of variation. These measures were applied to the travel time data set involving the 11 routes in Wellington.

The travel time data was analysed to understand whether there were statistically based relationships between the coefficient of variation and the mean travel time and the route length. The statistical validity of a variety of models was examined and a model for forecasting travel time variability has been recommended.

3.2 Description of the travel time data set

Travel time surveys were undertaken on 11 routes in the Wellington region during August 2007:

- SH1 – Waikanae to Whitford-Brown
- SH1 – Whitford Brown to Ghuznee Street
- SH1 – Ghuznee Street to Airport
- Fergusson Drive (Upper Hutt town centre) to Silverstream to Dowse Drive via SH2
- Dowse Drive to Ghuznee Street via SH2 and SH1
- Porirua (Mungavin) on SH1 to Seaview via SH58, SH2, Naenae to Seaview
- Wellington Railway Station to Island Bay via Waterloo Quay via Kent/Cambridge Terrace, Adelaide Road, John Street and The Parade
- SH2 – Featherston to Upper Hutt
- city centre at Bunny Street to Karori at Makara Road via Tinakori Road
- Woburn to Wellington Railway Station via Whites Line, The Esplanade, SH2 and Aotea Quay
- Seaview to Petone Railway Station via Petone Esplanade.

Where routes, such as SH1 above, have sections of different standard, these were split up to reflect sections of road with consistent standards. A travel time survey was undertaken on each route for three periods: morning, inter and evening peak, in both directions. Morning peak was defined as 7am to 9am, inter peak 11am to 3pm and evening peak 4pm to 6pm.

The surveys used a floating car approach and were undertaken by transportation consultants for the Greater Wellington Regional Council. In each survey, the timings were taken on five separate runs for each of the above 66 permutations of route, time of day and direction. Care was taken to ensure representative conditions associated with the particular time of day prevailed and that events, such as road works that produce atypical conditions, did not occur.

These surveys provided data for a mix of multi-lane motorway, highway and local roads and included a blend of short and long routes. They are summarised in appendix A.

3.3 Data normalisation

The 11 routes surveyed had different lengths and different route characteristics such as geometry and capacity. Direct analysis of the data from the surveys is unlikely to show meaningful relationships for travel time variability independent of these route characteristics. Further, unless a non-dimensional analysis is undertaken the coefficients produced by statistical models have dimensions associated with them and vary depending on the units that are used to measure the variables in the statistical models.

In order to remove the effect of these route characteristics it is necessary to normalise the data and present it in a dimensionless form. The analytical work undertaken by Hyder Consulting (2007) used convenient dimensionless forms that are reproduced here. These are:

$$CV_t \text{ is the coefficient of variation} = \sigma(t)/E(t) \qquad \text{(Equation 3.1)}$$

$$CI_t \text{ is the congestion index} = E(t)/T_0 \qquad \text{(Equation 3.2)}$$

where σ(t) is the standard deviation of the travel time on a given route

where E(t) is the mean travel time on a given route at a particular time, and

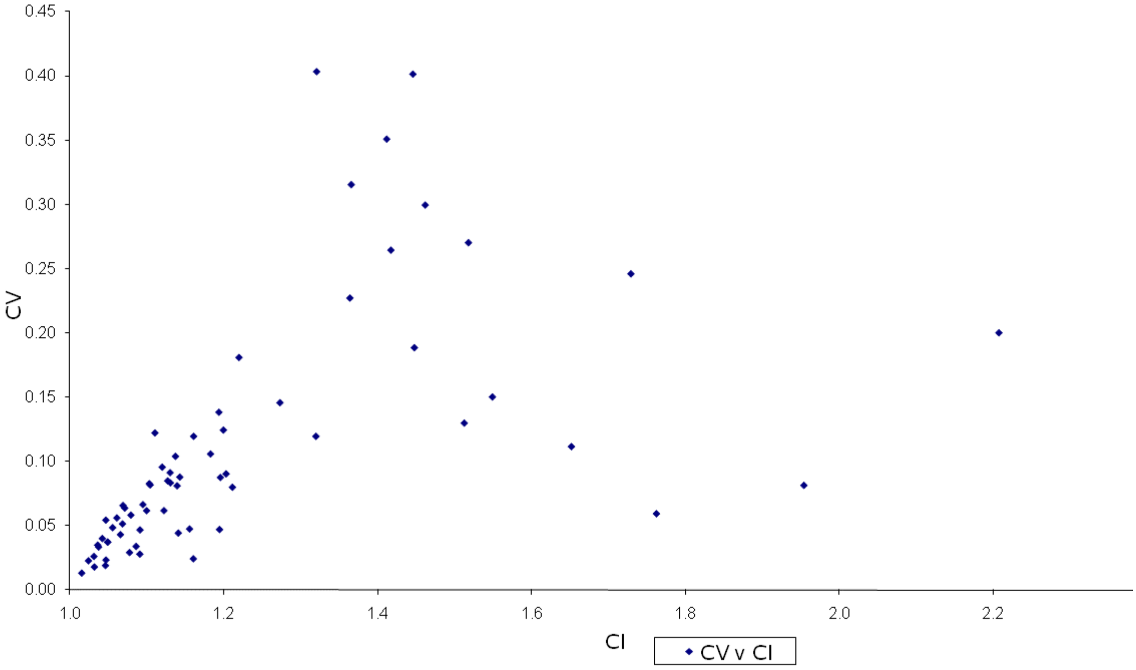
T_0 is the free-flow travel time observed on the route. T_0 was calculated by considering the time taken at the legal speed limit over link length with an adjustment made for average delay at intersections.

In effect the coefficient of variation, CV_t, is a dimensionless measure of travel time variability and the congestion index, CI_t, is a dimensionless measure of travel time that recognises the prevailing level of congestion on the route of interest.

3.4 General travel time variability patterns

CV_t is plotted against CI_t to consider whether possible relationships exist between the variables. This plot is shown in figure 3.1. Examination of figure 3.1 shows a general trend of CV_t increasing with CI_t for values of CI_t from 1.0 up to approximately 1.5. For CI_t greater than 1.5, continuing increases in CV_t do not appear to be maintained. The correlation coefficient of CV_t on CI_t has an R^2 of 0.30. This demonstrates a weak positive correlation between CV_t and CI_t so that CV_t is generally greater when CI_t increases.

Figure 3.1 CV vs CI



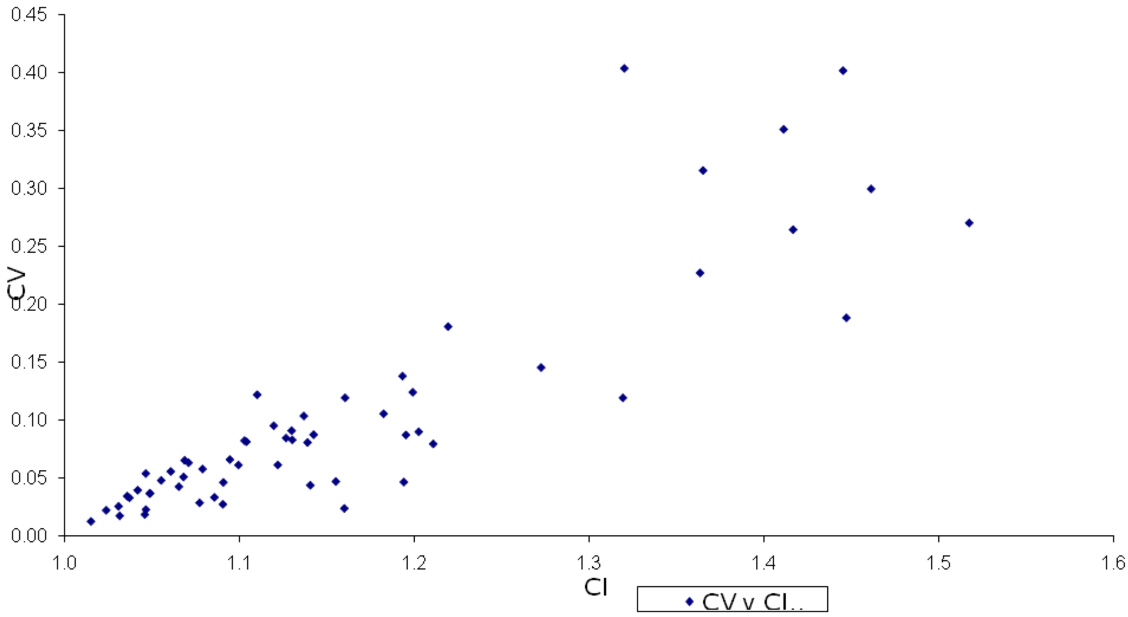
If the points with CI_t greater than 1.5 are treated as 'outliers' and removed then the distribution of points shown in figure 3.2 is obtained. The value of CI_t equal to 1.5 has been chosen from inspection of figure 3.1 but has been generalised in the analyses in later sections of this report.

Generally, these outliers relate to routes in or approaching central Wellington. The outliers occur in pairs – one outlier at a location and traffic direction in the morning peak is matched by another at the same location in the evening peak but in the opposite traffic direction.

Figure 3.2 shows a strong correlation between CV_t and CI_t and the correlation coefficient R^2 has a value of 0.77. It would appear there is some evidence of a correlation between CV_t and CI_t which is strengthened if the points with CI_t greater than 1.5 are removed. As discussed in later sections this result is pointing to two distinct flow phases where the phase transition occurs near CI_t equal to 1.5. At lower values of CI_t flow becomes more variable as CI_t increases and at higher values of CI_t flow breaks down and travel times become more certain but longer as CI_t increases.

Another explanation may be contained in the findings of Arup (2004) who argued that in highly congested situations the mean and standard deviation of travel time was dependent on the demand profile, including that before the modelled period. When CI_t is greater than 1.5 this means actual travel times are more than 50% greater than free-flow travel times which would suggest there is significant congestion.

Figure 3.2 CV vs CI outliers removed



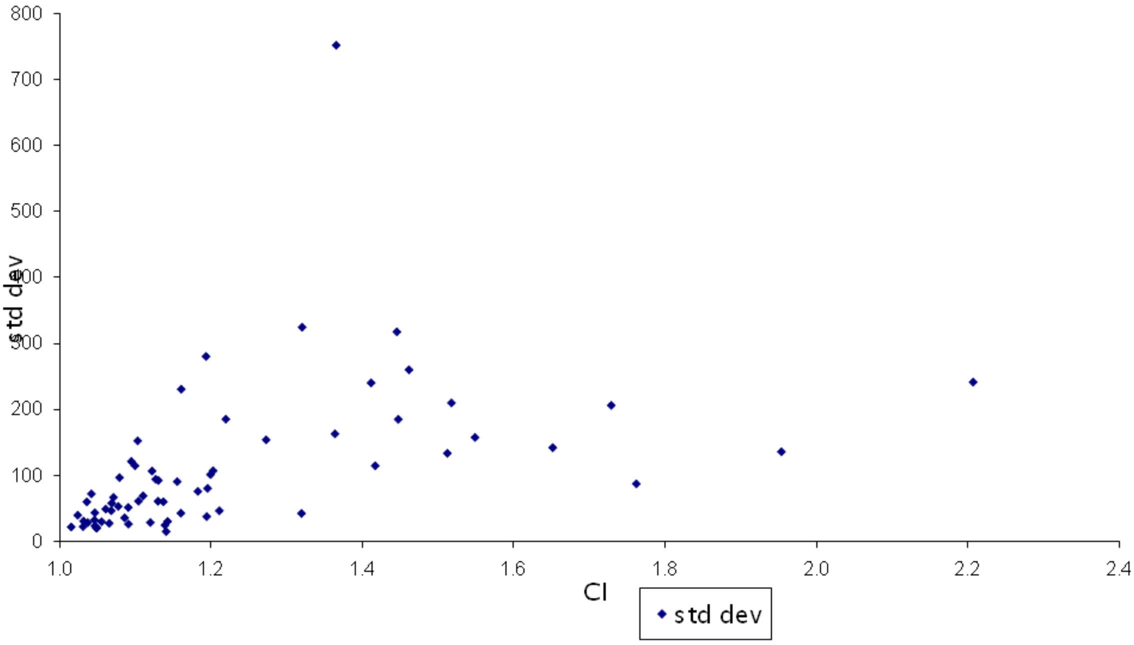
3.5 Alternative measures of travel time reliability

Even though the concept of travel time reliability has not been as well developed as other areas of transportation research, there was some mention of it in the literature. Lomax et al (2003) and Margiotta (2002) discussed the use of standard statistical measures including mean travel time, the standard deviation, variance and various percentile travel times. Their work represents some of the founding thinking in the area of travel time reliability measurement.

The standard statistical measures of standard deviation, variance, percentile and the coefficient of variation are related. The variance is the square of the standard deviation and, in the case of normally distributed travel times, the 95th percentile travel time is approximately the mean travel time plus two standard deviations. The coefficient of variation has been used here as a normalised form of the standard deviation and is equal to the standard deviation divided by the mean travel time. These measures are mathematically interchangeable.

Figures 3.3 and 3.4 show the standard deviation of travel time plotted against CI_t and the variance plotted against CI_t. Both figures 3.3 and 3.4 have a significant outlier when CI_t is approximately 1.37. The respective standard deviation and variance is significantly greater than those for other values of CI_t. These outliers represent a measurement of travel time variability at the same location, time period and traffic direction.

Figure 3.3 Standard deviation vs CI



The correlation coefficient R^2 for the standard deviation plotted against CI_t is 0.22 and the R^2 for the variance plotted against CI_t is 0.06. These R^2 values are inferior to those obtained for CV_t plotted against CI_t and do not provide a compelling reason to adopt either the standard deviation or variance as the measure of travel time variability.

When figures 3.3 and 3.4 are compared with figures 3.1 and 3.2 there are no obvious advantages in directly using the standard deviation or the variance as the basis of the analysis. Further, the measures in figures 3.3 and 3.4 are easily manipulated into the form used in figures 3.1 and 3.2. For these reasons it is recommended that the dimensionless normalised form be used as the basis of further analysis.

3.6 Development of a travel time variability function using the full data set

The following analysis uses the full data set presented in figure 3.1. In the next section an analysis is undertaken where outliers are removed, as shown in figure 3.2.

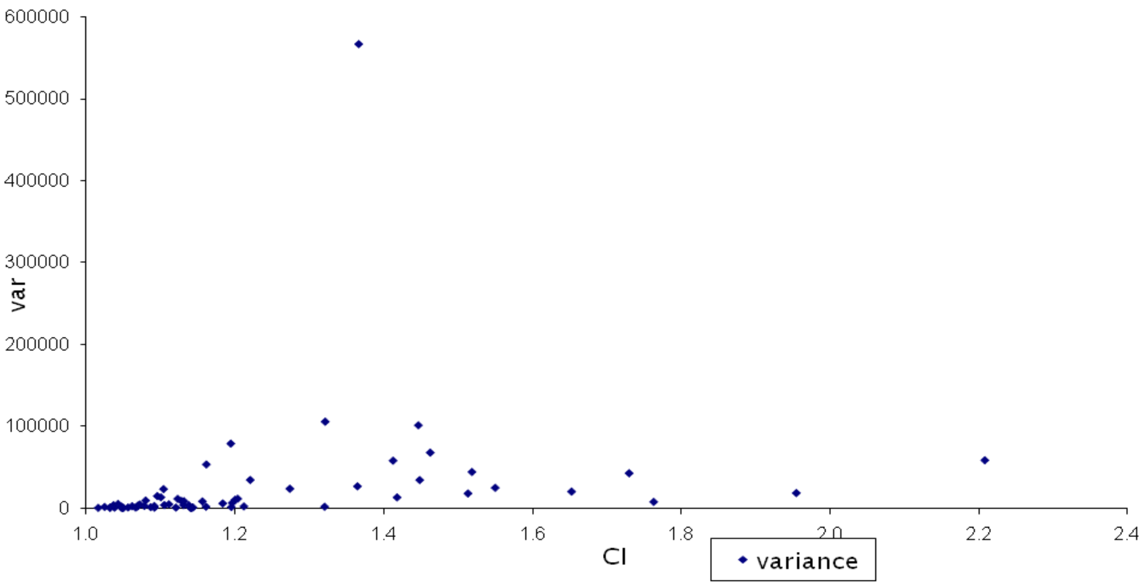
A variety of models are investigated. These look at a wide range of potential functional relationships between CV_t and CI_t. Of particular interest is the model of Hyder Consulting (2007) identified in the literature review. This is of the form:

$$CV_t = \alpha CI_t^{\beta} d^{\delta} \qquad \text{(Equation 3.3)}$$

where CV_t is the coefficient of variation = $\sigma(t)/E(t)$

where CI_t is the congestion index = $E(t)/T_0$

Figure 3.4 Variance vs CI



where d is route length in km

where T_0 is the free-flow travel time for the route

and α, β and δ are constants that require calibrating

Equation 3.3 is mathematically equivalent to

$$\xi = \theta + \beta\zeta + \delta\eta \qquad \text{(Equation 3.4)}$$

where $\xi = \log_{10}(CV_t)$

where $\zeta = \log_{10}(CI_t)$

where $\eta = \log_{10}(d)$

and $\theta = \log_{10}(\alpha)$

which is linear in form.

This means a functional form can be investigated where functions of $\log_{10}(CI_t)$ are analysed against $\log_{10}(CV_t)$. There is no requirement to use logarithms to base 10 other than for convenience, as any logarithmic base can be used. This includes the linearised form as in equation 3.4 above and more complex alternatives.

The following models are investigated:

- $CV_t = \theta + \beta CI_t$ (A)
- $\xi = \theta + \beta\zeta$ (B)
- $\xi = \theta + \beta\zeta + \delta\eta$ (C)
- $\xi = \theta + \beta\zeta + \delta\zeta^2$ (D)

- $\xi = \theta + \beta\zeta + \delta\zeta^2 + \kappa\eta$ (E)
- $\xi = \theta + \beta\zeta + \delta\zeta^2 + \kappa\eta + \gamma\eta^2$ (F)
- $\xi = \theta + \beta\zeta + \delta\eta + \kappa\zeta\eta$ (G)
- $\xi = \theta + \beta\log_{10}(\zeta + \delta) + \kappa\eta$ (H)

where $\xi = \log_{10}(CV_t)$ which is a simple transform of CV_t

where $\zeta = \log_{10}(CI_t)$

and $\eta = \log_{10}(d)$.

Model A is mathematically equivalent to finding a straight line through figure 3.1. Terms in ζ^2, η^2 and $\zeta\eta$ that appear in models D, E, F and G allow non-linear curve fits to be investigated.

The coefficients calculated from the data are shown in table 3.1 and the statistical measures of the validity of the models are shown in table 3.2.

Table 3.1 Calculated coefficients of the models

Model	θ	β	δ	κ	γ
A	-0.1595	0.2174			
B	-1.3718	3.1727			
C	-1.1424	2.9239	-0.1759		
D	-1.6403	10.2561	-25.8131		
E	-1.6229	10.2089	-25.7059	-0.01254	
F	-2.3871	10.1754	-25.7357	1.4271	-0.6355
G	-0.6809	-2.4652	-0.5623	4.8888	
H	-0.2094	0.6545	-0.00368	-0.0455	

Table 3.2 Statistical measures of significance

Model	R_{adj}^2	F	t_β	t_δ	t_κ	t_γ
A	0.299	27.3	0.921			
B	0.429	48.4	43.4			
C	0.436	25.2	40.0	-0.650		
D	0.672	66.0	140.2	-1217		
E	0.667	43.3	139.6	-1212	-0.0463	
F	0.670	33.2	139.1	-1214	5.27	-1.03
G	0.487	20.9	-33.7	-2.07	63.4	
H	0.650	40.3	1.52	-0.00852	-0.168	

The coefficients in table 3.1 show a certain amount of stability as the models become more complex. This is demonstrated by the reasonably constant value of θ for models B to E, and the reasonably constant values of β and δ for models D to F.

These coefficients suggest CV_t will generally increase with increasing CI_t, but with the models that contain quadratic terms, large values of CI_t will allow a decline in CV_t. It should be noted that a decrease in CV_t does not necessarily mean a decrease in the travel time variability, as measured by the standard deviation of travel time, as CV_t is the coefficient of variation equal to $\sigma(t)/E(t)$. The coefficients suggest CV_t will generally decrease with increasing d. The above conclusions need to be qualified by the possibility there may be some correlation between CI_t and d.

In table 3.2 the R_{adj}^2 term, R^2 adjusted for differing degrees of freedom, indicates models need to be fairly complex before a reasonable fit is obtained as in models D, E, F and H. The F-statistics calculated for each of the models A to H are statistically significant at both the 95% and 99% levels indicating all the models have significant explanatory power.

The t-statistics for the various coefficients provide some useful insights. In the case of model A the coefficient of CI_t is not statistically significant at the 95% level. In models B to G the coefficients of ζ ($\zeta = \log_{10}(CI_t)$) are statistically significant at both the 95% and 99% levels. In models D, E and F, which contain terms in ζ^2, the coefficients are found to be significant at the 95% and 99% levels.

The t-statistics for coefficients of d and η ($\eta = \log_{10}(d)$) are not statistically significant at the 95% level except in model F where the coefficient of η is statistically significant at both the 95% and 99% levels if a quadratic term in η is included in the model.

These results suggest models D, E and F are the most promising.

3.7 Development of a travel time variability function with outliers removed

The analysis in section 3.6 is repeated. This time it is assumed a number of points in figure 3.1 are outliers and are distorting the analysis performed in section 3.6. This is because figure 3.1 appears to show consistent trends for lower values of CI_t but not for larger values. It may be that some other relationships exist above a particular threshold in CI_t. This is explained by road users responding to more severe congestion in a way that is different from lower levels of traffic.

Such a possibility will be explored in this section. The outlier points are removed as in figure 3.2 and the analysis presented in section 3.6 is repeated.

The results of this analysis are presented in tables 3.3 and 3.4.

Table 3.3 Calculated coefficients of the models with outliers removed

Model	θ	β	δ	κ	γ
A	-0.6594	0.6574			
B	-1.5619	6.8285			
C	-1.4756	6.6809	-0.06563		
D	-1.6964	11.6663	-28.0507		
E	-1.6426	11.4396	-27.2312	-0.03796	
F	-1.9757	11.4557	-27.5567	0.6470	-0.3179
G	-1.0760	0.9671	-0.4007	5.1055	
H	0.6045	1.5783	0.0302	-0.02982	

Table 3.4 Statistical measures of significance with outliers removed

Model	R_{adj}^2	F	t_β	t_δ	t_κ	t_γ
A	0.772	193.2	5.11			
B	0.748	169.6	149.8			
C	0.747	84.7	146.6	-0.206		
D	0.771	96.2	255.9	-3450		
E	0.768	63.3	2509	-3349	-0.119	
F	0.767	47.5	251.3	-3389	2.030	-0.460
G	0.781	68.5	21.2	-1.26	101.3	
H	0.773	65.2	7.92	0.152	-0.0935	

As in table 3.1, the coefficients shown in table 3.3 show some stability as the models become more complex. This is demonstrated by the reasonably constant value of θ for models B to F, and the reasonably constant values of β and δ for models D to F. These results show there is more stability when the outliers are removed.

These coefficients suggest CV_t will generally increase with increasing CI_t, but with the models that contain quadratic terms, larger values of CI_t, will allow CV_t to plateau and decrease. The coefficients suggest CV_t will generally decrease with increasing d. The above conclusions need to be qualified by the possibility there may be some correlation between CI_t and d.

In table 3.4, the R_{adj}^2 term indicates all the models show good fit. In particular the simple model A: $CV_t = \theta + \beta CI_t$, provides a good fit with only two coefficients. It is interesting to note that the two coefficients for model A are closely related, with one being approximately the negative of the other.

This result suggests a model of the form $CV_t = \beta(CI_t - 1)$ should be considered with β being approximately 0.658. Such a model has practical and theoretical appeal as it is simple in form, with just a single coefficient, and easy to use for calculations. Further, such a model would suggest that under free-flow conditions, where $CI_t = 1$, the travel time variability would be zero. This property is consistent with theoretical expectations.

If a relationship can be found between CV_t and CI_t using measured values of CV_t and CI_t, then this relationship can be used to forecast unknown values of CV_t if traffic conditions change. An example of this application includes measuring current patterns of CV_t and CI_t and asking what these might be in 20 years time if traffic continues to grow with no changes to the network, or alternatively what CV_t and CI_t might be if a road improvement or other measure is put in place. The F-statistics calculated for each of the models A to H are statistically significant at both the 95% and 99% levels indicating the models have significant explanatory power.

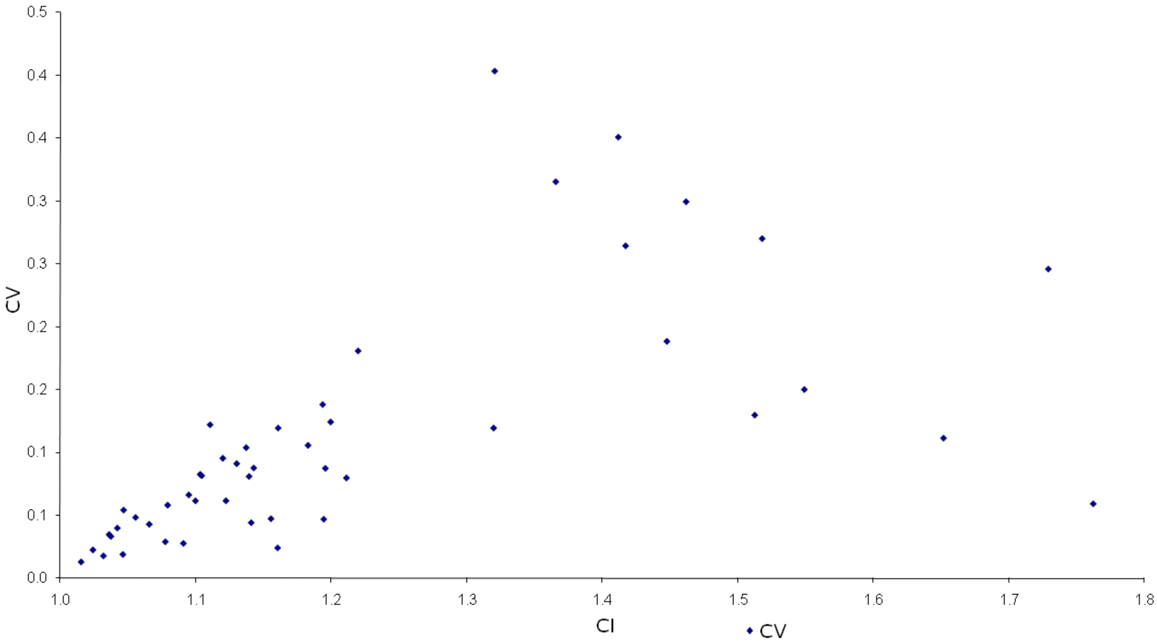
The t-statistics for the various coefficients provide some useful insights. In the case of model A the coefficient of CI_t is statistically significant at the 95% and 99% levels. In models B to G the coefficient of ζ ($\zeta = \log_{10}(CI_t)$) is statistically significant at both the 95% and 99% levels. In model H the coefficient of $\log_{10}(\zeta)$ is statistically significant at both the 95% and 99% levels. In models D, E and F, which contain terms in ζ^2, the coefficients are found to be significant at the 95% and 99% levels.

The t-statistics for coefficients of d and η ($\eta = \log_{10}(d)$) are not found to be statistically significant at the 95% level except in model F where the coefficient of η is statistically significant at the 95% but not at the 99% level if a quadratic term in η is included in the model.

A comparison of table 3.2 with table 3.4 indicates that the removal of the outlier points generally produces higher values of R_{adj}^2. The values of the F-statistics in table 3.4 are generally larger than in table 3.2 indicating a higher level of statistical significance in the model. The t-statistics for the coefficients of ζ, ζ^2 and $\log_{10}(\zeta)$ are generally larger in magnitude in table 3.4 than in table 3.2 indicating greater statistical validity for the inclusion of these terms.

Neither table 3.2 nor table 3.4 provide statistical evidence for the validity of terms in d, η ($\eta = \log_{10}(d)$) and η^2. This result shows there is little statistical evidence to support models of the form of equation 3.3 developed by Hyder Consulting (2007) using data from the UK. There does not appear to be a clear reason for this.

Figure 3.5 Non expressway CV vs CI



3.8 Development of a travel time variability function with allowance for different types of roads

The question is posed as to whether the type of road may have an impact on travel time reliability. Essentially the data set contains two types of roads: urban roads, and motorways and expressways. The motorways and expressways are typically higher standard roads and are multi-laned with higher posted speeds. They have few but well designed junctions. Urban roads frequently have single lane sections, lower speed regimes, and a greater frequency and variety of junctions.

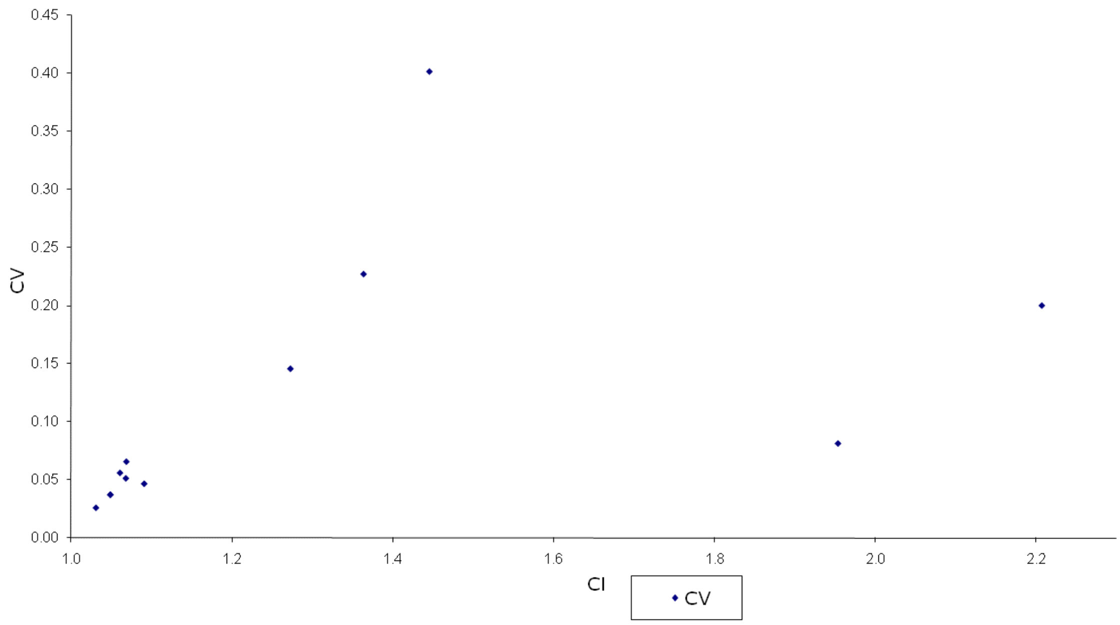
The analysis in section 3.6 is repeated, including the outlier points. This time the points in figure 3.1 are separated into these two groups of roads and analysed separately. These data sets are presented in figures 3.5 and 3.6.

The results of the analysis of urban roads that are not motorways or expressways are presented in tables 3.5 and 3.6.

Table 3.5 Calculated coefficients of the models for urban roads

Model	θ	β	δ	κ	γ
A	-0.2510	0.2965			
B	-1.4155	3.8938			
C	-1.3095	3.7126	-0.07880		
D	-1.7717	14.1670	-44.8777		
E	-1.7007	14.0155	-44.7374	-0.05191	
F	-1.3204	14.3629	-46.2103	-0.7948	0.3261
G	-0.5698	-6.5224	-0.7631	10.4308	
H	-0.1898	0.7136	-0.00247	-0.00622	

Figure 3.6 Expressway CV vs CI



As in tables 3.1 and 3.3, the coefficients in table 3.5 show some stability as the models become more complex. This is demonstrated by the reasonably constant value of θ for models B to F, and the reasonably constant values of β and δ for models D to F.

These coefficients suggest CV_t will generally increase with increasing CI_t, but with the models that contain quadratic terms, larger values of CI_t will allow CV_t to plateau. The coefficients suggest CV_t will generally decrease with increasing d. The above conclusions need to be qualified by the possibility there may be some correlation between CV_t and d.

Table 3.6 Statistical measures of significance for urban roads

Model	R_{adj}^2	F	t_β	t_δ	t_κ	t_γ
A	0.373	27.4	1.33			
B	0.470	40.9	60.8			
C	0.462	20.3	58.0	-0.257		

Model	R_{adj}^2	F	t_β	t_δ	t_κ	t_γ
D	0.709	56.5	221.2	-2953		
E	0.704	37.1	218.9	-2944	-0.169	
F	0.699	27.4	224.3	-3041	2.590	0.465
G	0.689	34.6	-101.8	-2.49	178.5	
H	0.634	27.3	1.74	-0.00603	-0.0203	

In table 3.6 the R_{adj}^2 term, indicates that models D to H show good fit and models A to C have a modest fit. Generally the fits are better than those in table 3.2 where all the data is used but not as good as in table 3.4 where the outliers have been removed.

The F-statistics calculated for each of the models A to H are statistically significant at both the 95% and 99% levels indicating the models have significant explanatory power.

The t-statistics for the various coefficients provide some useful insights. In the case of model A the coefficient of CI_t is not statistically significant at the 95% level. In models B to G the coefficient of ζ (ζ = $\log_{10}(CI_t)$) is statistically significant at both the 95% and 99% levels. In model H the coefficient of $\log_{10}(\zeta)$ is not statistically significant at the 95% level. In models D, E and F which contains terms in ζ^2 the coefficients are found to be significant at the 95% and 99% levels.

The t-statistics for coefficients of d and η (η = $\log_{10}(d)$) are not found to be statistically significant at the 95% level except in model F where the coefficient of η is statistically significant at the 95% but not at the 99% level if a quadratic term in η is included in the model.

The t-statistics for the coefficients of ζ, ζ^2 and $\log_{10}(\zeta)$ are generally larger in magnitude in table 3.6 than in table 3.2 but not as large as those in table 3.4.

Table 3.6, as with tables 3.2 and 3.4, does not provide statistical evidence for the validity of terms in d, η (η = $\log_{10}(d)$) and η^2. This result shows there is little statistical evidence to support models of the form of equation 3.3 developed by Hyder Consulting (2007) using data from the UK.

The results of the analysis of motorways and expressways are presented in tables 3.7 and 3.8.

Table 3.7 Calculated coefficients of the models for motorways and expressways

Model	θ	β	δ	κ	γ
A	-0.05687	0.1313			
B	-1.3132	2.1325			
C	-0.5378	2.0182	-0.6219		
D	-1.5852	9.8814	-23.3098		
E	-0.9933	9.6389	-22.8406	-0.4704	
F	-1.1910	9.6389	-22.8406	-0.1456	-0.1322
G	-1.3596	10.2548	0.04499	-6.8026	
H	0.06557	0.4768	-0.00984	-0.3992	

Table 3.8 Statistical measures of significance for motorways and expressways

Model	R_{adj}^2	F	t_β	t_δ	t_κ	t_γ
A	0.209	2.64	0.336			
B	0.422	7.30	18.9			
C	0.407	3.84	17.9	-5.21		
D	0.763	16.4	87.5	-601.6		
E	0.764	11.1	85.3	-589.5	-3.94	
F	0.734	7.31	85.3	-589.5	-1.22	-0.451
G	0.406	2.82	90.8	0.377	-50.1	
H	0.669	7.12	0.752	-0.0155	-3.35	

Comparison of table 3.7 with table 3.1 shows similar patterns in the stability of the coefficients. The coefficients of ζ and ζ^2 in table 3.7 are smaller in magnitude than those in table 3.5. This suggests motorways and expressways have flatter curves when CV_t is plotted against CI_t.

Again table 3.7 suggests CV_t increases with CI_t although the coefficient of quadratic terms suggests this may plateau or decrease with large values of CI_t. The coefficients of terms involving distance indicate CV_t may decrease with increasing values of d.

Table 3.8 indicates the model fits are not particularly good for expressways and motorways. The values of R_{adj}^2 are generally modest except with models D, E and F where terms in ζ^2 are used. The F-statistics generally do not indicate a level of statistical significance in any of the models except for models E and F. This is partly due to a small number of data points being available for expressways and motorways. In models E and F, the F-statistics are significant at the 95% level but not at the 99% level.

The t-statistics of terms in ζ and ζ^2 are statistically significant at the 95% and 99% levels. Other terms do not generally have statistically significant t-statistics.

3.9 Extension of a simple linear travel time variability function

In this section the simple linear model A of the form

$$CV_t = \vartheta + \beta CI_t \qquad \text{(Equation 3.5)}$$

and the simpler theoretically more satisfying model

$$CV_t = \beta(CI_t - 1) \qquad \text{(Equation 3.6)}$$

are further developed.

Equation 3.5 and its special case represented by equation 3.6 have been determined by a least squares regression fit where points from the data set where CI_t is greater than 1.5 are removed as in figure 3.2. The value of CI_t where points with larger values are excluded is generalised to ψ.

Equation 3.5 is now used for all values of CI_t and the difference between observed values of CV_t and those estimated by equation 3.5 are found. This difference is $\Delta(CV_t)$. That is:

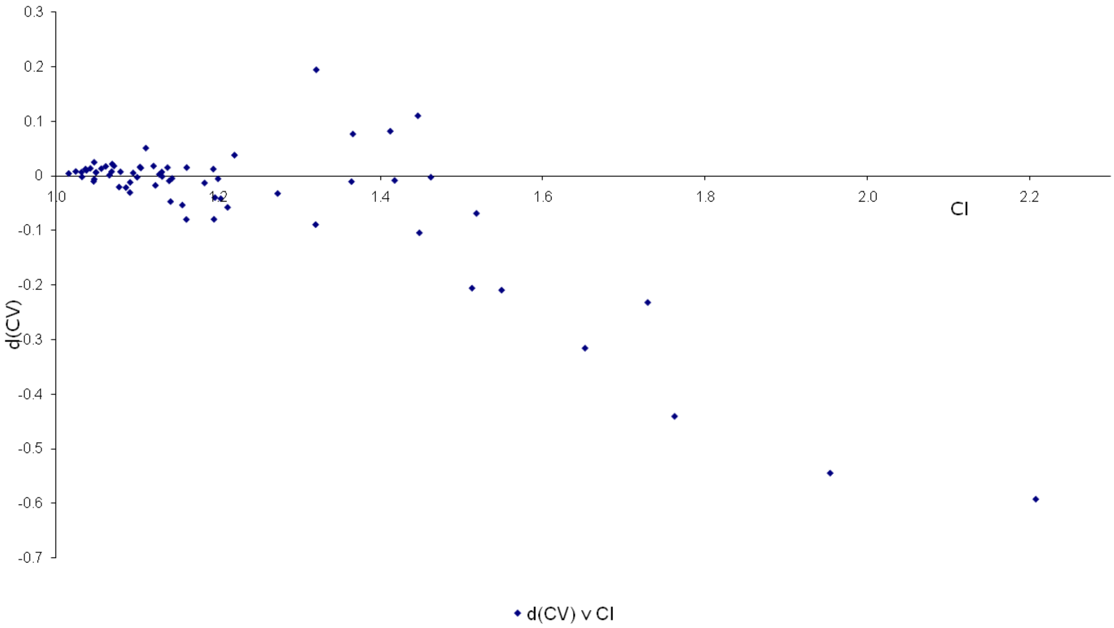
$$\Delta(CV_t) = CV_t^o - (\vartheta + \beta CI_t)$$ (Equation 3.7)

where CV_t^o is the observed value of CV_t.

$\Delta(CV_t)$ is plotted against CI_t in figure 3.7 and d in figure 3.8 for $\psi = 1.5$.

Figure 3.7 shows the expected collection of points with $\Delta(CV_t)$ close to zero for CI_t from 1.0 to 1.5. For CI_t greater than 1.5 there appears to be a monotonic decline in $\Delta(CV_t)$ with increasing CI_t. Figure 3.8 shows no obvious relationship between $\Delta(CV_t)$ and d.

Figure 3.7 Δ(CV) v CI



If ψ is allowed to vary away from 1.5 it is possible to find a value of ψ and models for CV_t and $\Delta(CV_t)$ that give the best fit of the data. Such an analysis has been undertaken and the results for an optimal data fit obtained using R_{adj}^2:

$\psi = 1.395$

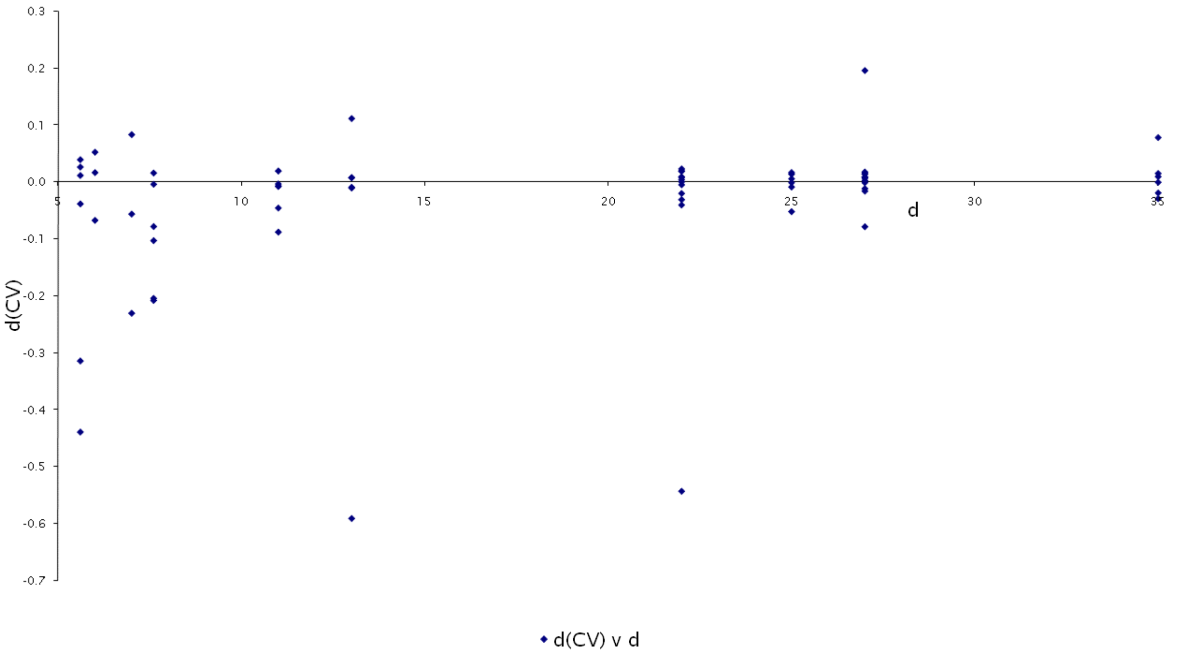
$$CV_t = -0.6714 + 0.6677CI_t + \Delta(CV_t)$$ (Equation 3.8)

where

$$\Delta(CV_t) = 0 \qquad CI_t \leq 1.395$$ (Equation3.9)

$$= 1.2306 - 0.8822CI_t \qquad CI_t > 1.395$$

Figure3.8 Δ(CV) vs distance d



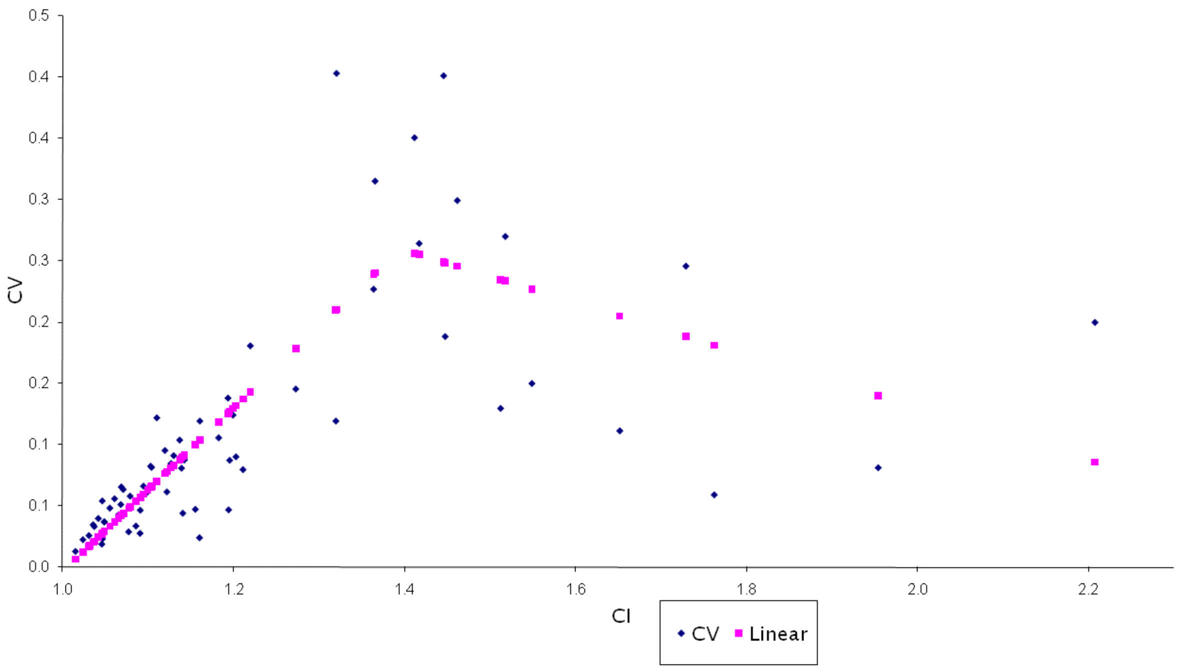
The statistical measure of validity for these models is R_{adj}^2 =0.786 with the F-statistic being 118. This measure indicates a good model fit for the complete data set, comparable with the results achieved for the truncated data set used to produce table 3.4.

The t-statistic for the coefficient of CI_t in equation 3.8 is 7.73, which is significant at both the 95% and 99% levels. The t-statistic for the coefficient of CI_t in equation 3.9 is -3.69, which is significant at both the 95% and 99% levels.

The coefficients of equation 3.8 are again closely related and are approximately the form of equation 3.6 with β = 0.670.

The model for a travel time variability function represented by equations 3.8 and 3.9 appears to fit the data well. The statistical measures of goodness of fit appear to support this model, as do the t-statistics of the individual coefficients.

Figure 3.9 CV vs CI



The behaviour of this model for values of CI_t greater than1.395 is consistent with figure 3.7. Equations 3.8 and 3.9 and figure 3.7, if accepted, would suggest CV_t increases as traffic volumes increase above free-flow conditions, but beyond a point (represented by CI_t = 1.395) CV_t would then decrease as traffic volumes continue to increase.

In figure 3.9 this linear model is superimposed on top of the observed data. The model shows CV_t increasing up to the breakpoint as CI_t increases and after the breakpoint CV_t decreases as CI_t continues to increase.

A second refinement is to investigate a linear functional relationship before the breakpoint with a rectangular hyperbolic functional relationship following the breakpoint. The rectangular hyperbolic relationship will prove mathematically convenient in the analysis later in this study. This is outlined in equation 3.10.

$$CV_t = \vartheta + \beta CI_t \qquad \text{for } CI_t \leq \psi \qquad \text{(Equation 3.10)}$$
$$= \delta + \kappa / CI_t \qquad \text{for } CI_t \geq \psi$$

The coefficients θ, β, δ and κ are constrained so that CV_t is continuous at CI_t equal to ψ.

The optimum combination of these coefficients is given by:

θ = -0.7094, β = 0.7023, δ = -0.3105, κ =0.8465 with ψ = 1.418.

A relationship of the form of equation (3.6) where the constant θ is zero is given by equation 3.11.

$$CV_t = \beta(CI_t\text{-}1) \qquad \text{for } CI_t \leq \psi \qquad \text{(Equation 3.11)}$$
$$= \delta + \kappa / CI_t \qquad \text{for } CI_t \geq \psi$$

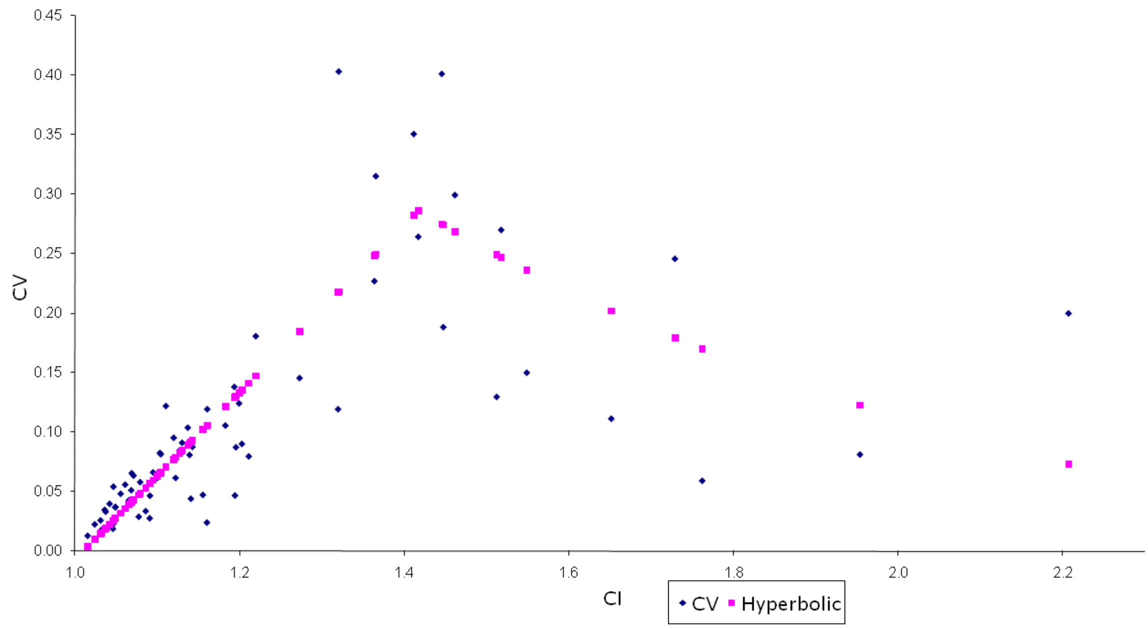
The coefficients β, δ and κ are constrained so that CV_t is continuous at CI_t equal to ψ.

The optimum combination of these coefficients is given by:

β = 0.7058, δ = -0.3105, κ =0.8465 with ψ = 1.411.

The hyperbolic part of equation 3.11 cuts the horizontal axis at CI_t equal to 2.726.

Figure 3.10 CV vs CI



R_{adj}^2 = 0.599, the F-statistic for the fit is 32.5 which is statistically significant at both the 95% and 99% levels. The t-statistic for the coefficient β is 6.97 and κ is 10.8. These are both statistically significant at the 95% and 99% levels. Again as with many of the earlier analyses θ approximately equals - β.

In figure 3.10 this linear-hyperbolic model is superimposed on top of the observed data. The model shows CV_t increasing linearly up to the breakpoint as CI_t increases and after the breakpoint CV_t decreases, this time with a hyperbolic curve, as CI_t continues to decrease.

A further refinement is to directly fit equation 3.6 and the more complex equation 3.12 to the data.

$$CV_t = \beta(CI_t - 1) + \gamma(CI_t - 1)^2 \quad \text{(Equation 3.12)}$$

Equation 3.12 is an extension of equation 3.6 and like equation 3.6 has the property that when CI_t equals 1, CV_t is zero. However, the additional term, $\gamma(CI_t - 1)^2$, allows a non linear curve fit and an extra degree of freedom which may produce a better fit to the data.

The results of this refinement are provided in tables 3.9 and 3.10 for the optimum cut-off point for CI_t of 1.49.

Figure 3.11 CV vs CI – 1

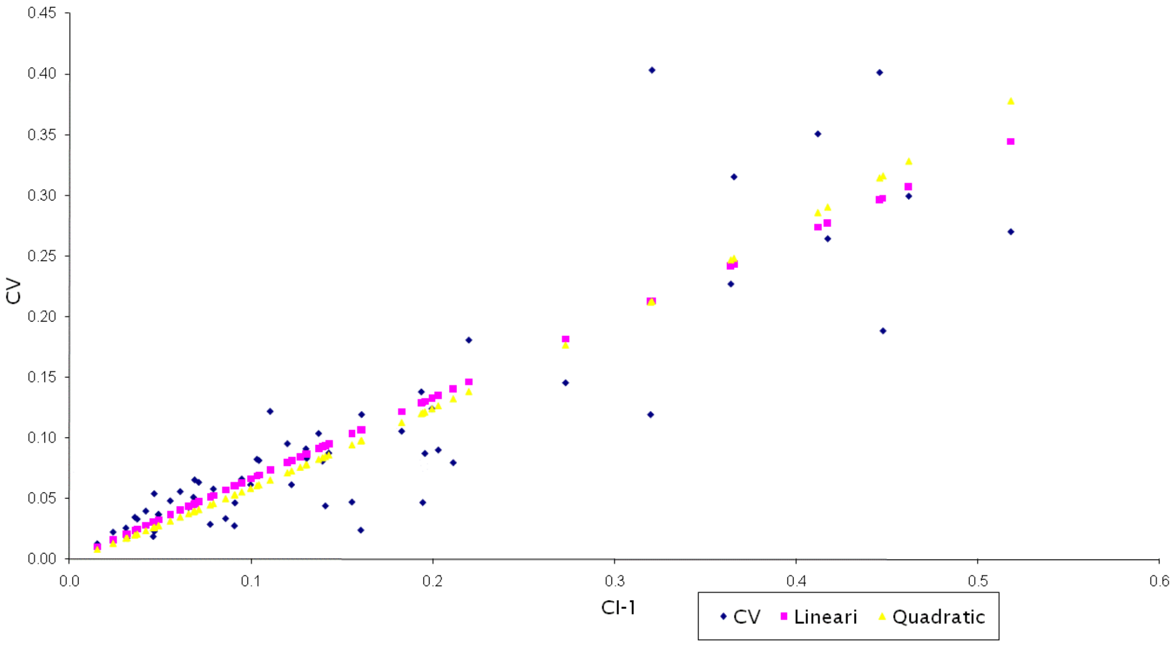


Table 3.9 Calculated coefficients of the models

Model	β	γ
$CV_t = \beta(CI_t - 1)$	0.6650	
$CV_t = \beta(CI_t - 1) + \gamma(CI_t - 1)^2$	0.5572	0.3323

Table 3.10 Statistical measures of significance

Model	R_{adj}^2	F	t_β	t_γ
$CV_t = \beta(CI_t - 1)$	0.769	189.6	5.51	
$CV_t = \beta(CI_t - 1) + \gamma(CI_t - 1)^2$	0.772	97.1	4.17	5.70

Figure 3.11 shows observed values of CV_t plotted against CI_t-1 with the linear and quadratic models from tables 3.9 and 3.10 superimposed.

Both the linear ($CV_t = \beta(CI_t - 1)$) and quadratic ($CV_t = \beta (CI_t - 1) + \gamma(CI_t - 1)^2$) models suggest travel time variability will increase as route travel time and route congestion increase. The quadratic model suggests the rate of increase of travel time variability will also increase with route travel time and route congestion as the coefficient, γ, is positive.

The calculated value of R_{adj}^2 shows a good model fit for both the linear and the quadratic model. The R_{adj}^2 indicates the data fit for the quadratic model is better than the linear model but the amount of improvement is only marginal. The F-statistics indicate the model fits are statistically significant at both the 95% and 99% levels for both the linear and quadratic models. The t-statistics of all the coefficients of both the linear and quadratic models are statistically significant at both the 95% and 99% levels. Examination of figure 3.11, would suggest the values of CV_t obtained from equations 3.6 and 3.12 are not greatly different. This indicates that little benefit is achieved by adding the extra complexity implicit in equation 3.12.

A useful result where a breakpoint, ψ, is identified in the behaviour of CV_t causing it to monotonically increase for values of CI_t up to and including ψ and then monotonically decline for values of CI_t greater than ψ, is that the maximum value of CV_t is calculated by:

$$maximum\ CV_t = CV_t(\psi) = \beta(\psi - 1) \quad \text{(Equation 3.13)}$$

using the linear model and

$$maximum\ CV_t = CV_t(\psi) = \beta(\psi - 1) + \gamma(\psi - 1)^2 \quad \text{(Equation 3.14)}$$

using the quadratic model.

3.10 Implications of the linear, quadratic and hyperbolic travel time variability functions

This section considers the mathematical implications of equations 3.6 and 3.12, and then equations 3.9 and 3.11. It looks in particular at the impact on travel time variability if the mean route travel time changes. As discussed earlier, the mean route travel time may change for a variety of reasons, such as ongoing traffic growth or the implementation of a road improvement.

3.10.1 Case 1: Both mean route travel times occur before the breakpoint

σ_1 represents the travel time variability when the mean travel time is t_1 and σ_2 represents the travel time variability when the mean travel time is t_2. Equations 3.1, 3.2 and 3.6 imply that:

$$(\sigma_1 / t_1)/(t_1/t_0 - 1) = \beta = (\sigma_2 / t_2)/(t_2/t_0 - 1) \quad \text{(Equation 3.15)}$$

where both t_1/t_0 and t_2/t_0 are less than or equal to ψ which can be rewritten as both t_1 and t_2 are less than or equal to ψt_0; and β is the model coefficient, and in the case of the Wellington data used, is found to be 0.6650 and t_0 is the route free-flow travel time.

Equation 3.15 can be rearranged to give:

$$\sigma_2 / \sigma_1 = (t_2 / t_1).\{(t_2 - t_0)/(t_1 - t_0)\} \quad \text{(Equation 3.16)}$$

Equation 3.16 enables the calculation of a new travel time variability directly from an old travel time variability if the old, new mean and free-flow route travel times are known. This calculation does not require the knowledge of the coefficient β in equation 3.6.

Equations 3.1, 3.2 and 3.12 can be rearranged to give:

$$\sigma_2 / \sigma_1 = (t_2 / t_1).\{[\beta t_0(t_2 - t_0)+\gamma(t_2 - t_0)^2]/[\beta t_0(t_1 - t_0)+\gamma(t_1 - t_0)^2]\} \quad \text{(Equation 3.17)}$$

As with equation 3.16, equation 3.17 allows the calculation of a new travel time variability from an old travel time variability with the knowledge of the old, new and free-flow route travel times. Equation 3.17 requires the value of the coefficients β and γ to be known. This means, when using the quadratic model, the new travel time reliability cannot be calculated from an old travel time reliability without knowledge of β and γ. However, if using the linear model, knowledge of β is not required. Equations 3.16 and 3.17 are valid, provided there is no break point ψ between the two values of CI_t associated with t_1 and t_2.

3.10.2 Case 2: Both mean route travel times occur after the breakpoint

σ_1 represents the travel time variability when the mean travel time is t_1, and σ_2 represents the travel time variability when the mean travel time is t_2. Using the linear model for values of CI_t greater than ψ, equations 3.8 and 3.9 can be written as:

$$CV_t = \lambda + \omega CI_t \qquad \text{(Equation 3.18)}$$

Using algebraic manipulation equation 3.19 is obtained:

$$\sigma_2 / \sigma_1 = (\lambda t_0 t_2 + \omega t_2^2)/(\lambda t_0 t_1 + \omega t_1^2) \qquad \text{(Equation 3.19)}$$

where both t_1/t_0 and t_2/t_0 are greater than or equal to ψ which can be rewritten as both t_1 and t_2 are greater than or equal to ψt_0.

If the hyperbolic model is used, as in equation 3.11, algebraic manipulation produces:

$$\sigma_2 / \sigma_1 = (\delta t_2 + \kappa t_0)/(\delta t_1 + \kappa t_0) \qquad \text{(Equation 3.20)}$$

where, again, both t_1/t_0 and t_2/t_0 are greater than or equal to ψ which can be rewritten as both t_1 and t_2 are greater than or equal to ψt_0.

3.10.3 Case 3: The mean route travel times occur either side of the breakpoint

In this case the two mean travel times straddle the breakpoint. Using a linear model both before and after the break point, algebraic manipulation produces:

$$\sigma_2 / \sigma_1 = t_2(\lambda t_0 + \omega t_2)/(\beta t_1 (t_1 - t_0)^2) \qquad \text{(Equation 3.21)}$$

If the linear model is used before the breakpoint and the hyperbolic model is used after the breakpoint, algebraic manipulation produces:

$$\sigma_2 / \sigma_1 = t_2(\delta t_0 + \kappa t_2)/(\beta t_1 (t_1 - t_0)^2) \qquad \text{(Equation 3.22)}$$

where t_1/t_0 is less than or equal to ψ and t_2/t_0 is greater than or equal to ψ. This can be rewritten as t_1 being less than or equal to ψ t_0 and t_2 is greater than or equal to ψ t_0.

For completeness, using the quadratic model before the breakpoint and the linear model after the breakpoint produces equation 3.23, and using the quadratic model before the breakpoint and the hyperbolic model after the breakpoint produces equation 3.24.

$$\sigma_2 / \sigma_1 = t_0 t_2(\lambda t_0 + \omega t_2)/(t_1(t_1 - t_0)(\beta t_0 + \gamma(t_1 - t_0))) \qquad \text{(Equation 3.23)}$$

$$\sigma_{2/} \sigma_1 = t_0^2(\delta t_2 + \kappa t_0)/(t_1(t_1 - t_0)(\beta t_0 + \gamma(t_1 - t_0))) \qquad \text{(Equation 3.24)}$$

where, again, t_1/t_0 is less than or equal to ψ and t_2/t_0 is greater than or equal to ψ. This can be rewritten as t_1 being less than or equal to ψt_0 and t_2 is greater than or equal to ψt_0.

The models developed here are based on Wellington data. It is expected the coefficients obtained in other places may be different. Further, the break points described by the parameter ψ required by equations 3.8 and 3.9 may also vary from location to location. If this is the case then these parameters will need to be determined at each location. The advantage of equation 3.16 is that it is not reliant on determining these coefficients and, as discussed in section 3.9, equation 3.17 is unlikely to provide a significantly different estimate of σ_2/σ_1 from the estimate obtained in equation 3.16. In the cases of equations 3.20 and 3.24 knowledge of these local coefficients is required.

3.11 Summary

Data from the 2007 travel time surveys in Wellington was useful for investigating travel time variability. The data provided a mix of shorter and longer routes, local roads, highways and motorways, and two-lane and multiple-lane roads.

The methodology of Hyder Consulting (2007) provided a convenient approach for data normalisation, enabling travel time data to be converted into dimensionless variables.

Analysis of the travel time variability data appeared to show a relationship between travel time variability and route mean travel time and therefore the congestion on the route. There did not appear to be a relationship between travel time variability and route distance.

A variety of models were investigated to estimate travel time variability. When the total data set was used the data fits were not good unless complex models were used. These models indicated relationships existed between travel time variability and normalised route travel time but did not support a relationship with route length. The coefficients of the normalised route travel time variables were generally found to be statistically significant.

If data, where the route travel time was approximately 50% or greater than the free-flow route travel time, was excluded then good data fits could be achieved with simple models of travel time variability. These models did not support the inclusion of route travel distance. Simple refinements of these models provided good data fit.

The analysis of the data by different models gave evidence that the relationship between CV_t and CI_t had two distinct phases. In the first phase CV_t monotonically increased with CI_t and in the second phase CV_t monotonically decreased with CI_t. The various models had a reasonably consistent breakpoint, ψ, where ψ was approximately 1.4. It appeared this breakpoint indicated a point where traffic flow became so congested the flow broke down. Increasing congestion beyond this breakpoint led to increasing delay but the amount of delay was less uncertain.

Linear, quadratic and hyperbolic relationships between travel time variability and normalised route travel time provided a mathematically simple approach to estimating travel time variability.

Further research would be helpful to demonstrate, or otherwise, the transferability of the relationships obtained to other locations. In particular, it would be interesting to observe how the coefficients of the models obtained in the above investigations changed in locations outside Wellington. This would include identifying the value of the breakpoint represented by ψ in equations 3.8 and 3.9.

The breakpoint represented by ψ in equations 3.8 and 3.9 suggested the behaviour of CV_t might undergo some change with higher levels of congestion, possibly associated with some kind of change in the traffic flow regime. This phenomenon should be examined further.

4 Application to a simple network

4.1 Preamble

The next stage of the project built upon the literature review and data analysis to consider how travel time reliability might be incorporated into an assignment model and to test whether the effects of doing so were statistically significant.

A simple hypothetical network created for this investigation was examined. However, the road link attributes such as capacity, free-flow travel time and others were based on characteristics measured in the model calibration phase of this study in Wellington. The travel time reliability function was derived from the one calibrated for the Wellington network in the previous phase of this project.

Two assignment techniques were investigated. These were the deterministic 'all or nothing' and the logit formulation. Each was examined for its suitability to incorporate travel time reliability and the effects of doing so were identified. Techniques were identified to incorporate travel time reliability and recommendations have been made.

4.2 Description of the network and trip matrices

The simple network developed had link characteristics similar to those found for the Wellington network in the previous phase of this project.

The network is shown in figure 4.1 and has four nodes connected by five links. Each of the links is a two lane two-way road. The characteristics of the road links are shown in table 4.1.

For the purposes of the investigation equation 4.1 is assumed to be the relationship between the coefficient of variation of travel time and the congestion index. This equation has been derived from the work in section 3.9 based on the analysis of data in Wellington.

$$CV_t = \sigma/t = 0.7058(CI-1) \qquad CI_t \le 1.4104 \qquad \text{(Equation 4.1)}$$

$$= -0.3105+0.8465/CI_t \qquad CI_t \ge 1.4104 \text{ and } \le 2.7262$$

$$= 0 \qquad CI_t \ge 2.7262$$

where $CI_t = t/t_0$ the congestion index

CV_t is the coefficient of variation of link travel time

σ is the standard deviation of link travel time

t_0 is the free-flow link travel time

t is the link travel time.

Figure 4.1 Simple network outline

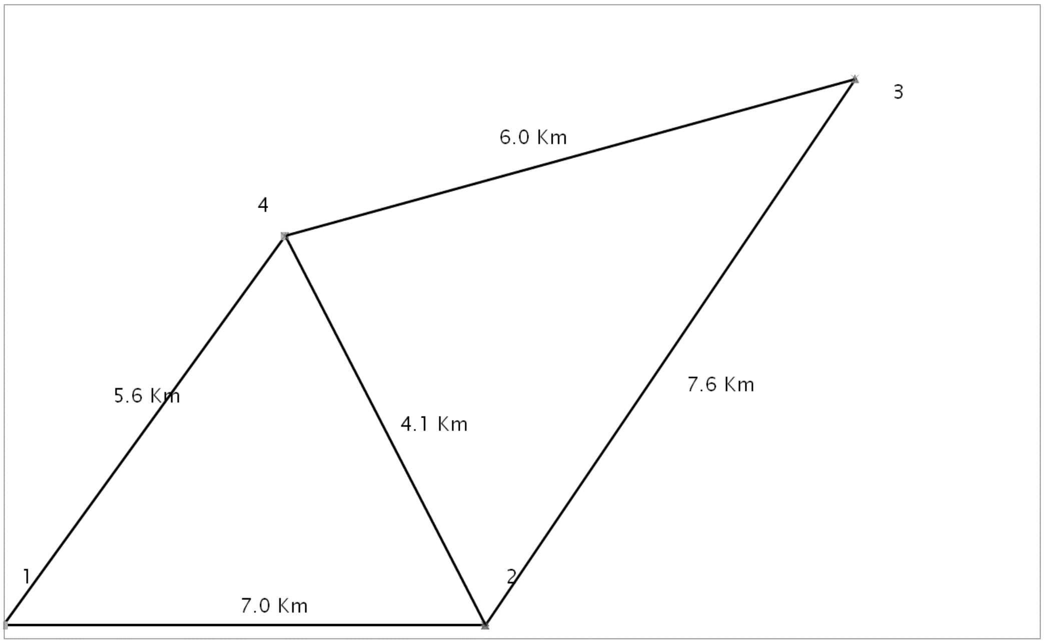


Table 4.1 Network link characteristics

Link	1–2	1–4	2–1	2–3	2–4	3–2	3–4	4–1	4–2	4–3
Length (km)	7.0	5.6	7.0	7.6	4.1	7.6	6.0	5.6	4.1	6.0
Free-flow travel time (secs)	486	775	486	680	272	683	513	842	307	513
Capacity (veh/h)	2000	1400	2000	2050	1700	1900	2050	2000	1800	1900

Equation 4.1 can be manipulated to see more directly how the standard deviation of link travel time varies with the congestion index as in equation 4.2. This gives a more direct representation of travel time reliability and its functional form is initially a quadratic function of link travel time up to a break point followed by a linear function of link travel time.

$$CV_t' = \sigma/t_0 = 0.7058CI_t(CI_t-1) \qquad CI_t \leq 1.4104 \qquad \text{(Equation 4.2)}$$

$$= -0.3105CI_t+0.8465 \qquad CI_t \geq 1.4104 \text{ and } \leq 2.7262$$

$$= 0 \qquad CI_t \geq 2.7262$$

or

$$CV_t' = CI_tCV_t \qquad \text{(Equation 4.3)}$$

These relationships can be seen in figure 4.2 which shows CV_t is a linear function of CI_t monotonically increasing for values of CI_t up to 1.4104. For CI_t greater than 1.4104, CV_t declines hyperbolically to zero at CI_t equal to 2.7262. Essentially this means the standard deviation of travel time, which is directly proportional to CV_t', is a monotonically increasing quadratic function of CI_t up to CI_t equal to 1.4104, and then declines linearly to zero at CI_t equal to 2.7262.

Two test trip matrices were developed so that assignment with travel time reliability could be investigated. They are shown below in node-to-node form.

Table 4.2 Trip matrix T_1 (vehicles/h)

Origin/destination	1	2	3	4	Total
1	0	1550	250	400	2200
2	1500	0	450	4000	5950
3	300	450	0	375	1125
4	400	4000	375	0	4775
Total	2200	6000	1075	4775	14,050

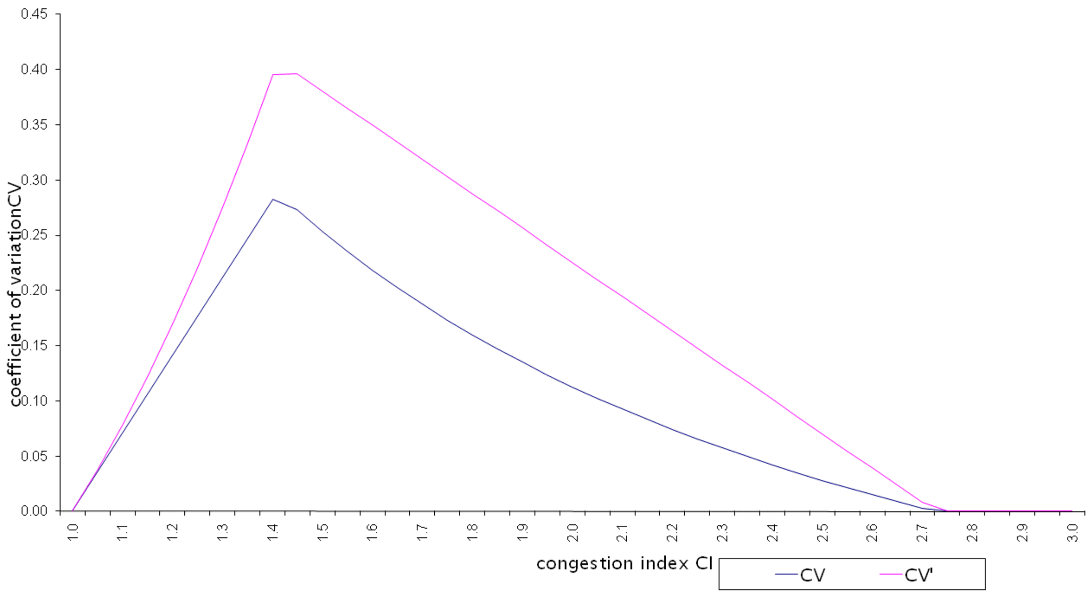
Table 4.3 Trip matrix T_2 (vehicles/h)

Origin/destination	1	2	3	4	Total
1	0	1550	1375	1250	4175
2	1500	0	1675	2500	5675
3	1420	1625	0	1280	4325
4	1250	2500	1175	0	4925
Total	4170	5675	4225	5030	19,100

T_1 and T_2 were chosen to provide a contrast in trip patterns. T_1 has heavy demands in a selected number of origins and destinations with the rest of the origins and destinations having light demands. T_2 has less contrast between the sizes of the origin destination pairs.

The leading diagonal of both T_1 and T_2 are all zeros. This is because we are only concerned with the assignment of trips from zone to zone and the values of intra-zonal trips have no relevance.

Figure 4.2 CV vs CI



4.3 The effect of travel time reliability on link volumes

Trip matrices T_1 and T_2 can be assigned to the network shown in figure 4.1. Two assignment methods are used. These are the 'all or nothing' and the logit formulation.

The trip matrix is assigned in increments. The all or nothing routine assigns all the trips for each origin-destination pair in each increment to the lowest travel cost route. After each increment of the matrix has been assigned the travel costs on any particular link are updated for the whole network and the next increment of the trip matrix is assigned. This is continued until the whole trip matrix is assigned.

In the logit assignment process a similar approach is followed except that instead of all the trips of a particular origin-destination pair being assigned to the lowest cost route, trips are assigned to alternative route options using the logit formulation shown in equation 4.4.

$$T^k_{ij} = T_{ij}e^{\theta c_k}/(\Sigma_g T_{ij}e^{\theta c_g}) \qquad \text{(Equation 4.4)}$$

where T_{ij} are the number of trips travelling between origin i and destination j

T^k_{ij} are the number of trips travelling between origin i and destination j using route k

θ is a factor used to convert travel costs to dis-utilities and

c_k is the travel cost of using route k.

Applying either an all or nothing or a logit assignment routine is dependent, among other things, on the determination of the route travel cost. A simple cost function is postulated as shown in equation 4.5 and is of the form discussed in section 2.6.

$$c_k = \delta d_k + \tau t_k + \varsigma\sigma_k \qquad \text{(Equation 4.5)}$$

where d_k is the length of route k

t_k is the travel time of route k

σ_k is the standard deviation of travel time on route k

δ,τ and ς are perceived costs per unit distance, time and travel time reliability respectively.

Equation 4.5 is a simple generalisation of a perceived cost function used in many assignment models that contains only travel distance and travel time. It is important to recognise that equation 4.5 represents perceived costs and not actual costs. For example, road users do not understand the full costs associated with distance. Typically wear- and tear-related costs are not consciously recognised. For the purposes of the investigations that follow, typical values of δ and τ have been adopted. These are $0.0875/km and $16.27/hr respectively. In the work that follows ς will be varied to understand its effect with other values held constant.

A simple spreadsheet model is used to do these assignments for both the all or nothing and logit formulations. Speed flow curves are developed for each of the links as a function of traffic volume and link capacity. The speed flow curves assume link travel time increases with the fourth power of the traffic volume to capacity ratio and are calibrated to broadly approximate Wellington conditions. These speed flow curves enable a value of t_k to be calculated for each link at each incremental assignment.

In practice a route will be the sum of several road links, each with their own individual costs. This means that d_k, and t_k are the resulting values from summing the respective values on each constituent road link. However, σ_k cannot be found by summing the values on the constituent road links. As this network is simple it is not too onerous to calculate σ_k over each route.

Figures 4.3 to 4.6 show the maximum, mean and minimum link flow rates per hour on the network shown in figure 4.1and how these vary with travel time reliability cost. Figure 4.3 is an all or nothing assignment using trip matrix T_1. Figure 4.4 is a logit assignment using trip matrix T_1. Figure 4.5 is an all or nothing assignment using trip matrix T_2. Figure 4.6 is a logit assignment using trip matrix T_2.

Examination of figures 4.3 and 4.4 and then 4.5 and 4.6 shows that the use of a different assignment process, whether all or nothing or a logit assignment, has negligible impact on these measures of distribution of traffic on the various road links. Irrespective of the travel time reliability cost, the mean, maximum and minimum traffic volumes assigned to the network do not appear to differ significantly if an all or nothing or logit assignment is used.

Figure 4.3 Flow parameters – matrix T_1 – all or nothing

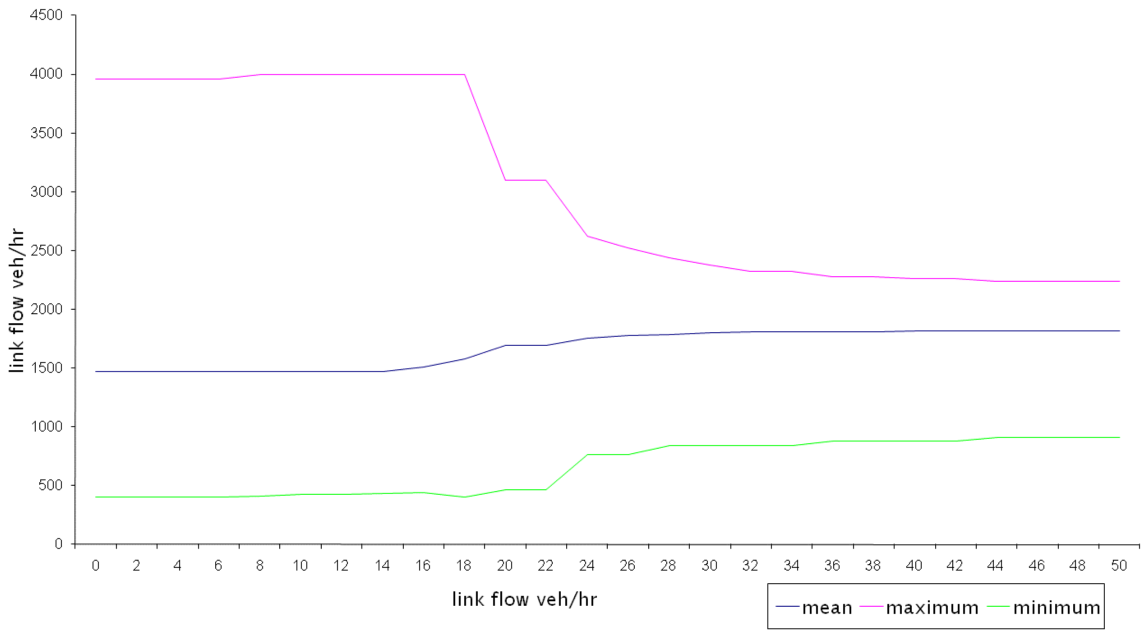


Figure 4.4 Flow parameters – matrix T_1 – logit

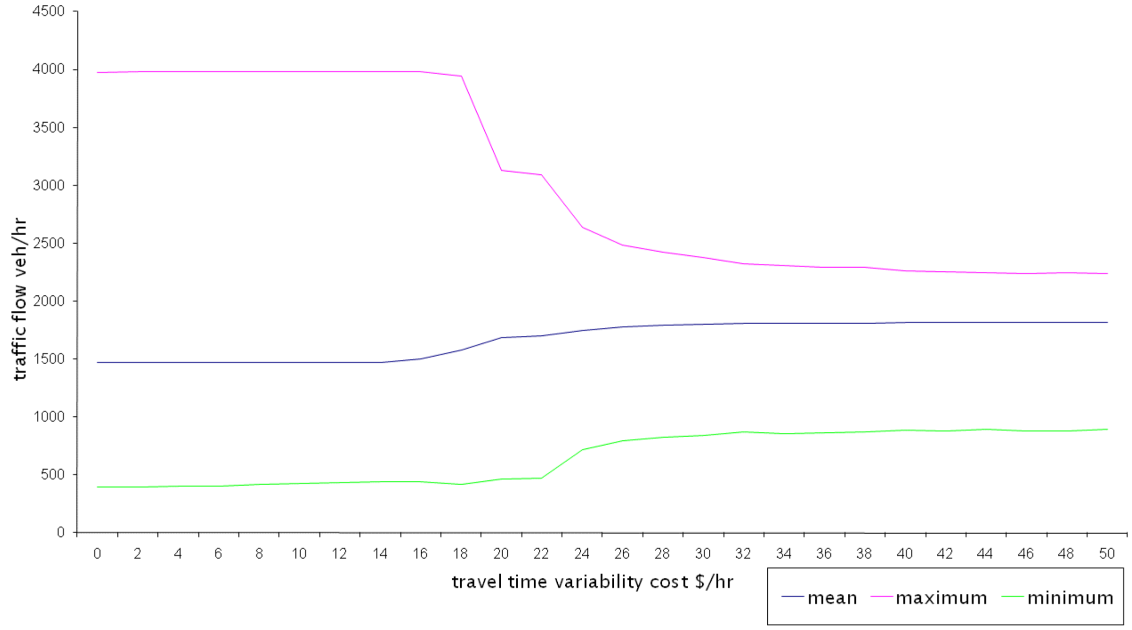
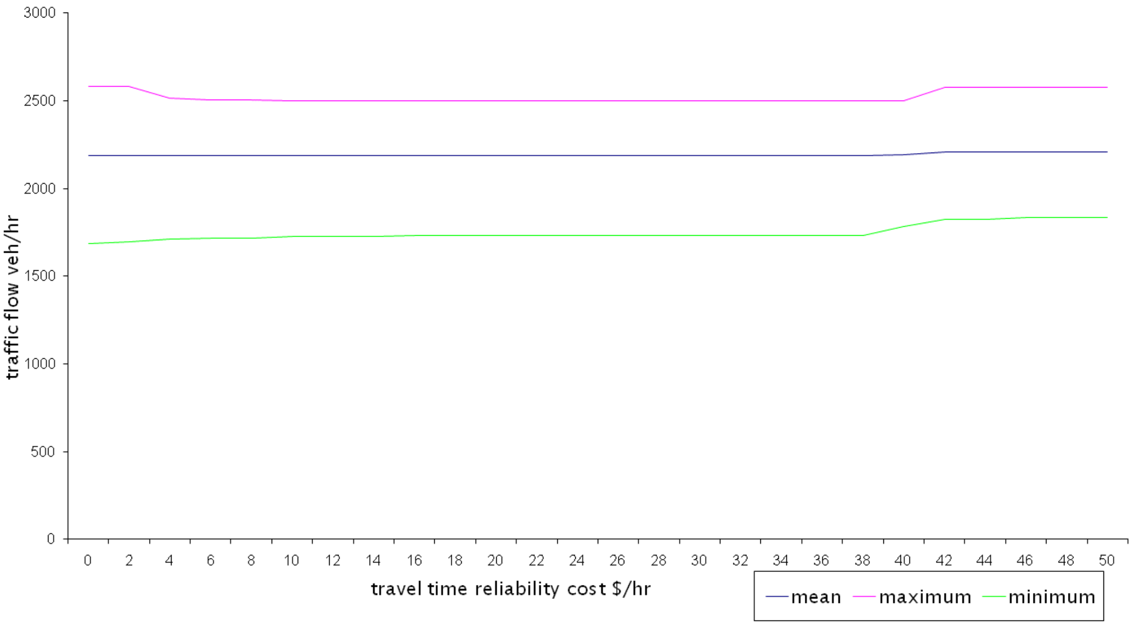


Figure 4.5 Flow parameters – matrix T_2 – all or nothing



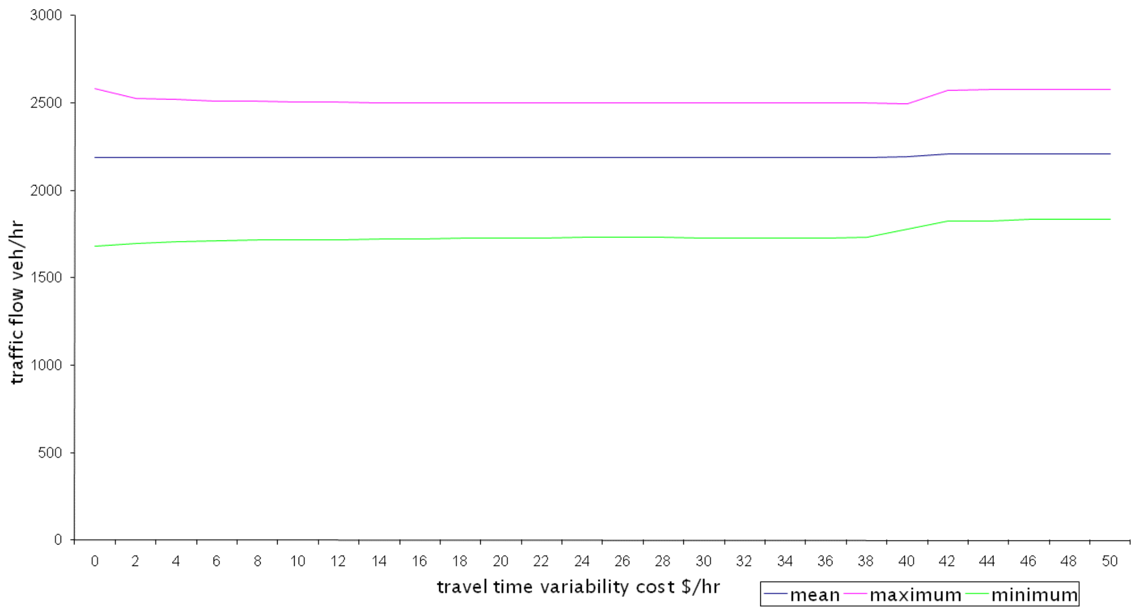
If figures 4.3 and 4.4 are compared to figures 4.5 and 4.6 it can be seen the former show a greater response to increasing travel time reliability cost than figures 4.5 and 4.6. It would appear that trip matrices where there is a dominance of certain movements over the remainder of the trip movements, subject to the overall volume of trips, would be more responsive to increasing travel time reliability cost.

In figures 4.3 to 4.6 there is little change in the maximum, mean and minimum link flow values for low values of travel time reliability cost. However, in figures 4.3 and 4.4 once the travel time reliability cost exceeds approximately $20/hr, the maximum, mean and minimum link flows change quickly and begin to converge whereas in figures 4.5 and 4.6 they still remain relatively constant.

It appears the pattern and volume of the trip matrix may have an impact on the response of link flows to travel time reliability cost. This is likely to reflect travel time reliability becoming a more significant issue as the volume to capacity ratio on a link increases.

To examine this effect, a measure of the extent to which a trip matrix is dominated by certain elements is required. Let T_{ij} be the elements of a trip matrix assigned to a network. This is the measure of the amount the matrix is dominated by certain elements and can be defined by considering the standard deviation of the off diagonal elements of the trip matrix, Π.

Figure 4.6 Flow parameters – matrix T_2 – logit



$$\Pi = \sqrt{(\Sigma\Sigma (T_{ij} - \Gamma_{ij})^2)/m} \qquad \text{(Equation 4.6)}$$

where $\Gamma_{ij} = 0 \qquad i = j$

$= \Gamma = \Sigma\Sigma T_{ij}/m \qquad i \neq j$

the mean value of the off-diagonal trip matrix elements where m is the number of off-diagonal elements.

Let the measure of trip matrix heterogeneity Λ be defined by:

$$\Lambda = \Pi/\Gamma \qquad \text{(Equation 4.7)}$$

This is the standard deviation of the off-diagonal elements divided by the mean of the off-diagonal elements of the trip matrix. The standard deviation of the off-diagonal elements is divided by the mean of the off-diagonal elements so Λ is not dependent on the total number of trips in the trip matrix. A matrix where all the off-diagonal elements are identical will have a Λ equal to zero.

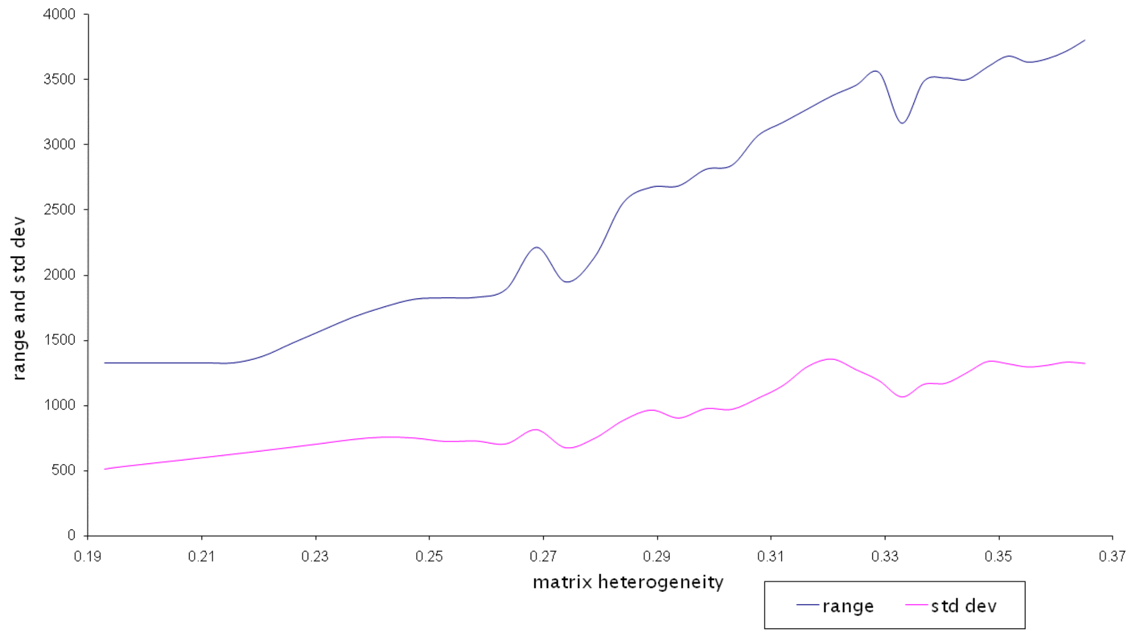
This Λ is also known as the coefficient of variation of the off-diagonal elements of the trip matrix, which is a normalised standard deviation. T_1, defined in table 4.2, has a Λ of 0.329 whereas T_2, defined in table 4.3, has a Λ of 0.258.

As there are 12 off-diagonal elements in T_1 and T_2 there is a large number of ways the elements of these matrices can be varied each with a consequential value of Λ. To get an understanding of the impact of matrix heterogeneity just the movements between nodes 2 and 4 are varied so a consistent trend can be developed. These have been selected as they are the clearest examples of movements whose magnitudes are at a scale different from the remainder of the trip movements.

Figure 4.7 shows the spread of traffic assigned to the various links of the network expressed as a range and standard deviation of link traffic volumes versus trip matrix heterogeneity for a travel time reliability cost of $16.27/hr.

It can be seen from figure 4.7 that the range and standard deviation of link traffic volumes are highly non-linear with respect to matrix heterogeneity. The spread of traffic assigned across the network is generally greater with more heterogeneous trip matrices.

Figure 4.7 Range and standard deviation of link traffic volumes vs matrix heterogeneity



However, this spread of traffic across the network does not monotonically increase with trip matrix heterogeneity. This is because the travel costs of alternative routes may become more attractive with higher volumes of traffic. It might be expected with more complex networks that the effects of any particular alternative route becoming more attractive in terms of travel cost will be less significant and so the range and standard deviation of traffic assigned to the links of the network might appear to increase more smoothly.

The range and standard deviation of traffic volumes assigned across the network is illustrated further by examination of figures 4.8 and 4.9.

Figure 4.8 Variance of link volumes vs travel time reliability cost – matrix T_1

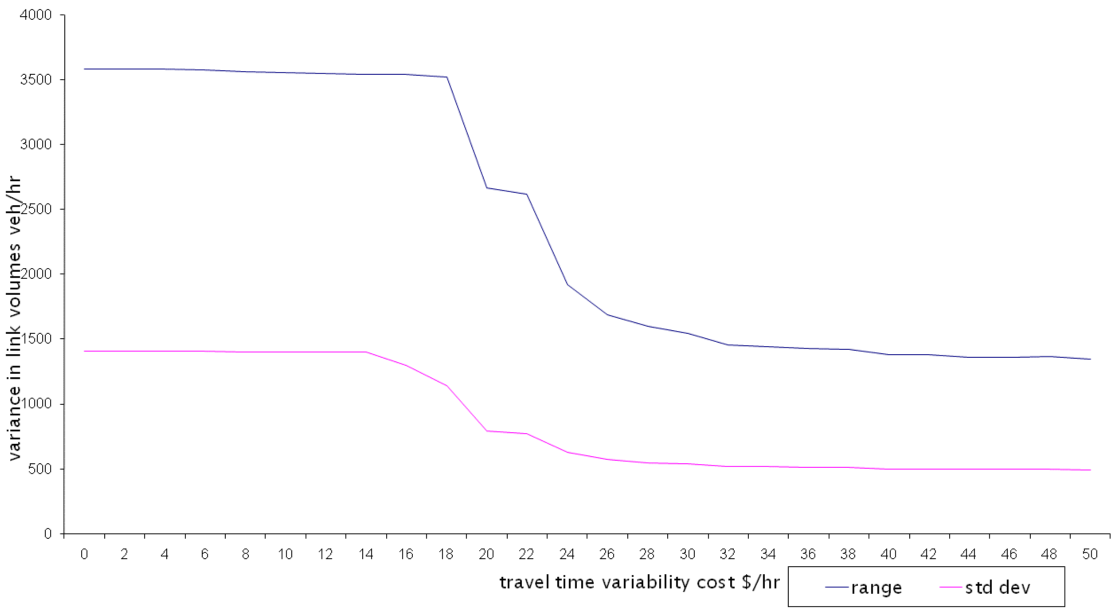


Figure 4.9 Variance of link volumes vs travel time reliability cost – matrix T_2

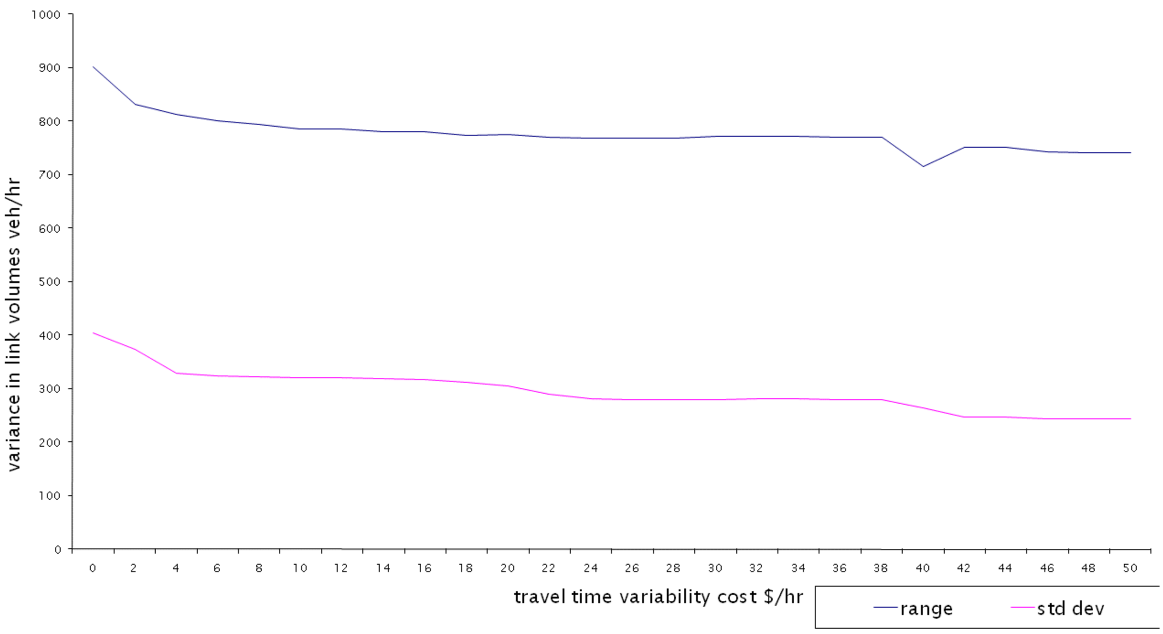


Figure 4.8 shows the range and standard deviation of traffic distributed across the network links for matrix T_1, which is a trip matrix with heterogeneity of 0.329 (showing high dominance by certain trip movements). Figure 4.9 shows the same plot but this time for trip matrix T_2, which is a trip matrix of heterogeneity 0.258 (showing little dominance by any trip movements).

It can be seen from these plots that large values of travel time reliability cost have a significant impact on heterogeneous trip matrices that reduces the range and standard deviation of traffic assigned to the network. This makes the distribution of traffic across the various links more uniform. With fewer heterogeneous trip matrices the effect is less pronounced. In simple terms, as people value travel time reliability more they are more inclined to seek alternative routes where reliability may be greater. However, a less heterogeneous trip matrix means traffic volumes are typically more evenly spread across the network reducing the likelihood of any one route being much more congested than another. This reduces the incentive to use alternative routes.

4.4 Measures of assignment validation

Equation 4.7 provides a means of incorporating travel time reliability into the assignment of traffic onto a network. To do this there needs to be a way of determining the travel time reliability cost parameter ς.

One approach is to use a value of ς determined from willingness to pay surveys. For example Margiotta (2006) undertook investigations that indicated the willingness to pay for reduced travel time variability was two to six times that of willingness to accept a reduced mean travel time. However, it has been found in assignment models that road users respond to a perceived travel distance cost rather than an actual travel distance cost in their route choice decisions.

An approach that recognises this is to examine route choice decisions and select a value of ς that optimises the assignment model fit. In order to do this, measures of model validation need to be considered.

Three measures of model validation are in common use. The first examines the correlation coefficient R^2 for link volumes from the model with observed traffic volumes. The second uses the root-mean-square-error measure and the final measure involves GEH statistics.

The correlation coefficient R^2 compares the model assigned link volumes with actual observed volumes. The NZTA (2010) *Economic evaluation manual* requires the correlation coefficient R^2 to be at least 0.85 overall and 0.95 in the vicinity of any scheme being investigated for acceptable validation. Hence, the optimal choice of ς is one that maximises R^2.

The root-mean-square-error measure calculates the statistic:

$$\%RMSE = 100 \sqrt{\Sigma((v_{obs} - v_{mod})^2 / (n-1))} / (\Sigma v_{mod}/n) \qquad \text{(Equation 4.8)}$$

where v_{obs} is the observed link volume

v_{mod} is the modelled link volume

n is the number of road links

and the summation is carried out over all n road links. For acceptable validation the *Economic evaluation manual* requires the %RMSE to be 30 or less. Hence, the optimal choice of ς is one that minimises %RMSE. In essence the root-mean-square-error measure is a modified form of a chi-squared test.

GEH statistics are calculated for individual road links in accordance to

$$GEH = \sqrt{(2(v_{mod} - v_{obs})^2/(v_{mod} + v_{obs}))} \qquad \text{(Equation 4.9)}$$

The *Economic evaluation manual* (NZTA 2010) requires that, for an acceptable level of validation:

60% of the links in the network have GEH < 5.0

95% of GEH < 10.0

100% of GEH < 12.0.

As this link-by-link approach is not convenient for the problem of choosing a ς that optimises validation it is proposed to define a modified GEH that examines the whole network at once. This statistic is the root-mean-squared-GEH defined by:

$$RMGEH^2 = \sqrt{(\Sigma GEH_i^2)} \quad \text{(Equation 4.10)}$$

where GEH_i is the GEH from equation 4.9 calculated for the ith road link. In essence $RMGEH^2$ is the magnitude of the n dimensional vector whose ith element is the GEH statistic for the ith road link and n is the number of road links. Hence, the optimal choice of ς is one that minimises $RMGEH^2$. Again, $RMGEH^2$ is a modified chi-squared test.

In the investigations that followed, each of these measures was examined to see whether they were useful in determining whether a validation was improved by incorporating travel time reliability. It may be that a validation is so flawed that incorporating travel time reliability shows an improvement in the validation.

4.5 Validation tests

A series of experiments assigned matrices T_1 and T_2 to the network shown in figure 4.1. A selection of observed traffic volumes was assumed for the tests, which included all or nothing and logit based assignments. These observed flows were chosen to ensure a range of ς results that optimised the validations. This was done to get a greater understanding of the influence of ς. The inputs into these tests are summarised in appendix B.

In each test ς was allowed to vary and the resulting R^2, %RMSE and $RMGEH^2$ were calculated. This allowed the identification of ς that approximately optimised each of R^2, %RMSE and $RMGEH^2$ for each test. There was no reason to believe that each of R^2, %RMSE and $RMGEH^2$ would have the same value of ς for an optimum value.

The attributes of the tests are shown in table 4.4.

Table 4.4 Experiment attributes

	E1	E2	E3	E4	E5	E6	E7	E8
Matrix	T_1	T_1	T_2	T_2	T_1	T_1	T_2	T_2
Assignment	all or nothing	all or nothing	all or nothing	all or nothing	logit	logit	logit	logit

The experiments with the attributes summarised in table 4.4 occurred in pairs. This allowed two sets of observed link flows to be tested for each matrix and assignment process.

4.5.1 Experiment E1

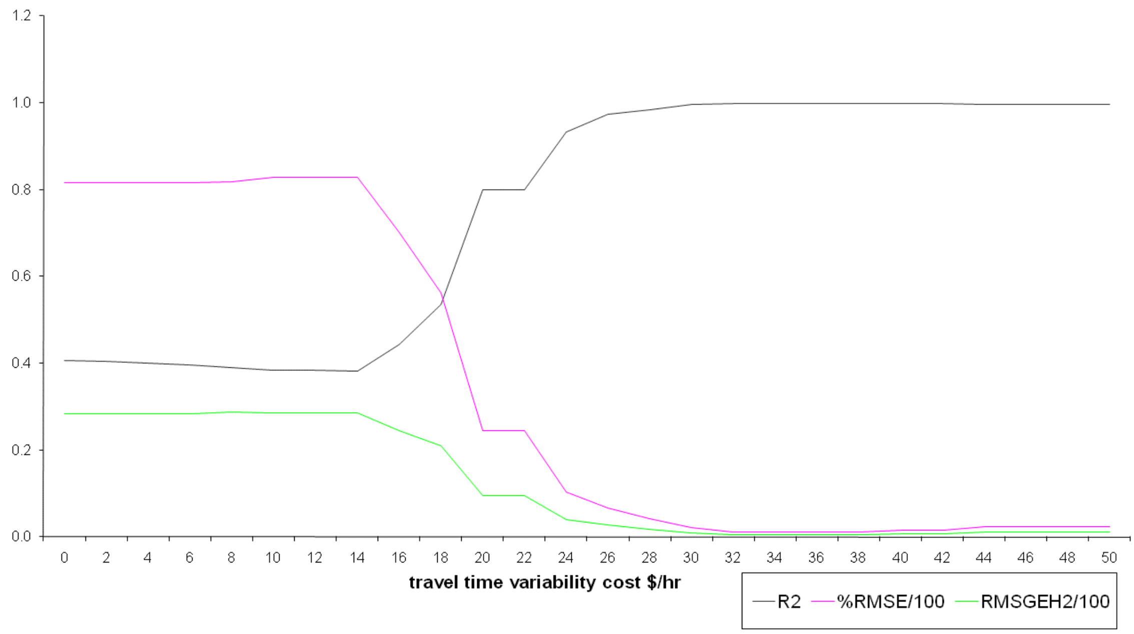
A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.10 for experiment E1. This is the assignment of a trip matrix with high heterogeneity, T_1, using an all or nothing assignment.

From figure 4.10 it can be seen that initially increasing travel time reliability cost ς does not improve validation significantly. It is not until the travel time reliability cost exceeds $14/hr that the three measures of validation show significant improvement. After the point where ς optimises R^2, %RMSE and $RMGEH^2$, increasing the values of ς sees a slow deterioration in R^2, %RMSE and $RMGEH^2$.

Figure 4.10 illustrates there are ranges where R^2, %RMSE and $RMGEH^2$ remain constant with increasing travel time reliability cost and this includes a range where R^2, %RMSE and $RMGEH^2$ appear to have optimum values.

In appendix C a numerical optimisation method is provided that iteratively allows better values of ς to be estimated. However, the lack of a single point where ς optimises R^2, %RMSE and $RMGEH^2$ limits the effectiveness of this optimisation method.

Figure 4.10 Validation measures vs travel time reliability cost matrix T_1 – all or nothing



The range where the three measures of R^2, %RMSE and $RMGEH^2$ are optimised appears to be coincident and in the case of figure 4.10 appears to be for values of ς between $35/hr and $38/hr. Near the optimum value of travel time reliability cost the validation measures do not change quickly.

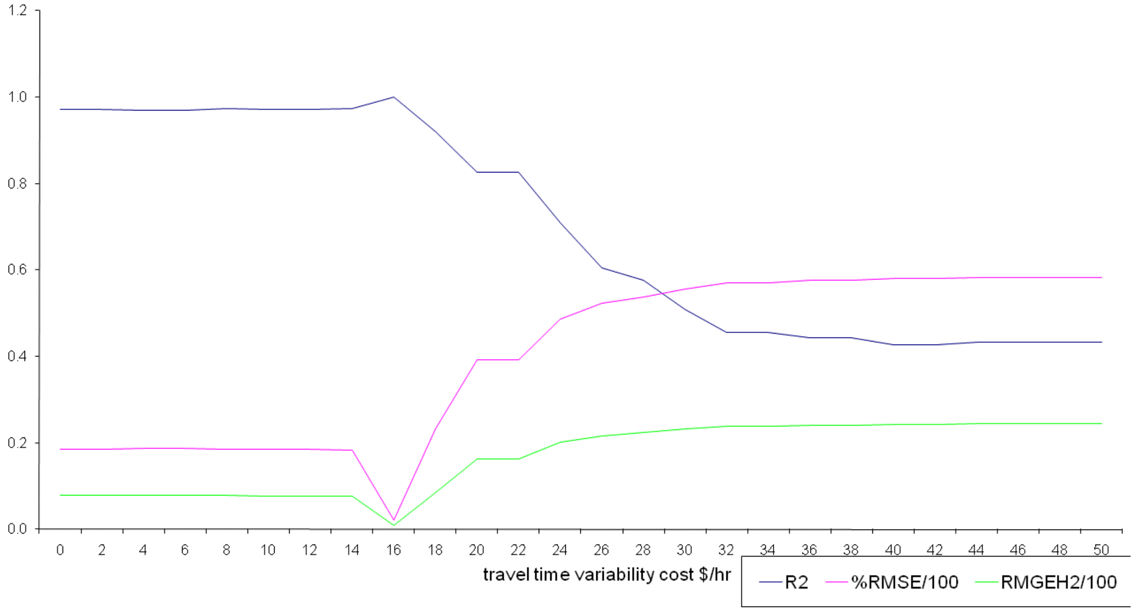
4.5.2 Experiment E2

A graph of R^2, %RMSE and RMGEH2 with travel time reliability cost ς is shown in figure 4.11 for experiment E2. This is the assignment of a trip matrix with high heterogeneity using an all or nothing assignment.

From figure 4.11 it can be seen that initially increasing travel time reliability cost ς does marginally improve. It is not until the travel time reliability cost exceeds $14/hr that the three measures of validation show a more noticeable improvement. After the point where ς optimises R^2, %RMSE and $RMGEH^2$, increasing values of ς sees rapid changes in R^2, %RMSE and $RMGEH^2$ but these eventually reduce and changes in R^2, %RMSE and $RMGEH^2$ become small.

Figure 4.11 shows ranges where R^2, %RMSE and $RMGEH^2$ remain constant with increasing travel time reliability cost. This includes a range where R^2, %RMSE and $RMGEH^2$ appear to have optimum values although this does not seem to be the case in figure 4.11 due to insufficient definition in the graph.

Figure 4.11 Validation measures vs travel time reliability cost – matrix T_1 – all or nothing



The lack of a single point where ς optimises R^2, %RMSE and $RMGEH^2$ limits the effectiveness of the optimisation method provided in appendix C.

The range where the three measures of R^2, %RMSE and $RMGEH^2$ are optimised appears to be coincident and in the case of figure 4.11 appears to be for values of ς in the vicinity of $16/hr. Near the optimum value of travel time reliability cost the validation measures change quickly.

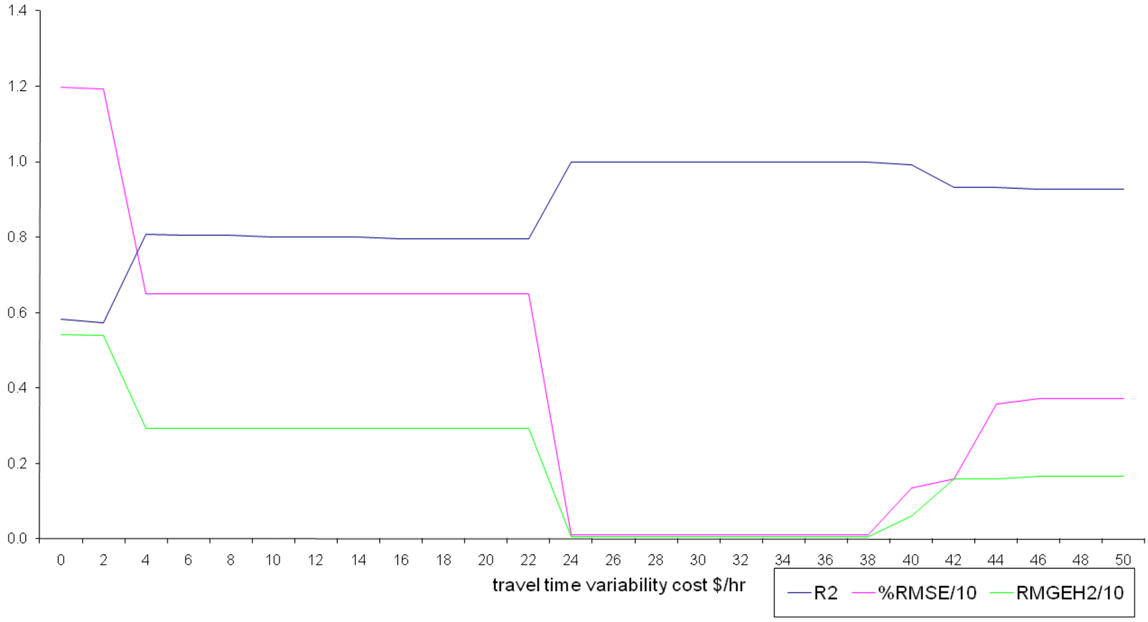
4.5.3 Experiment E3

A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.12 for experiment E3. This is the assignment of a trip matrix with low heterogeneity using an all or nothing assignment.

From figure 4.12 it can be seen that initially increasing travel time reliability cost ς does not improve validation significantly. It is not until the travel time reliability cost exceeds $3/hr, which is a low value of travel time reliability cost, that the three measures of validation show significant improvement. Again R^2, %RMSE and $RMGEH^2$ remain constant until the travel time reliability cost reaches approximately $24/hr where a second improvement in R^2, %RMSE and $RMGEH^2$ occurs. After the point where ς optimises R^2, %RMSE and $RMGEH^2$, increasing values of ς sees steady deterioration in R^2, %RMSE and $RMGEH^2$, which occurs through a series of steps.

Figure 4.12 also shows ranges where R^2, %RMSE and $RMGEH^2$ remain constant with increasing travel time reliability cost. This includes a range where R^2, %RMSE and $RMGEH^2$ appear to have optimum values.

Figure 4.12 Validation measures vs travel time reliability cost – matrix T_2 – all or nothing



The lack of a single point where ς optimises R^2, %RMSE and $RMGEH^2$ limits the effectiveness of the optimisation method provided in appendix C.

The range where the three measures of R^2, %RMSE and $RMGEH^2$ are optimised appears to be coincident and in the case of figure 4.12 appears to be for values of ς in the range of $23/hr to $39/hr. Near the optimum value of travel time reliability cost the validation measures change slowly.

4.5.4 Experiment E4

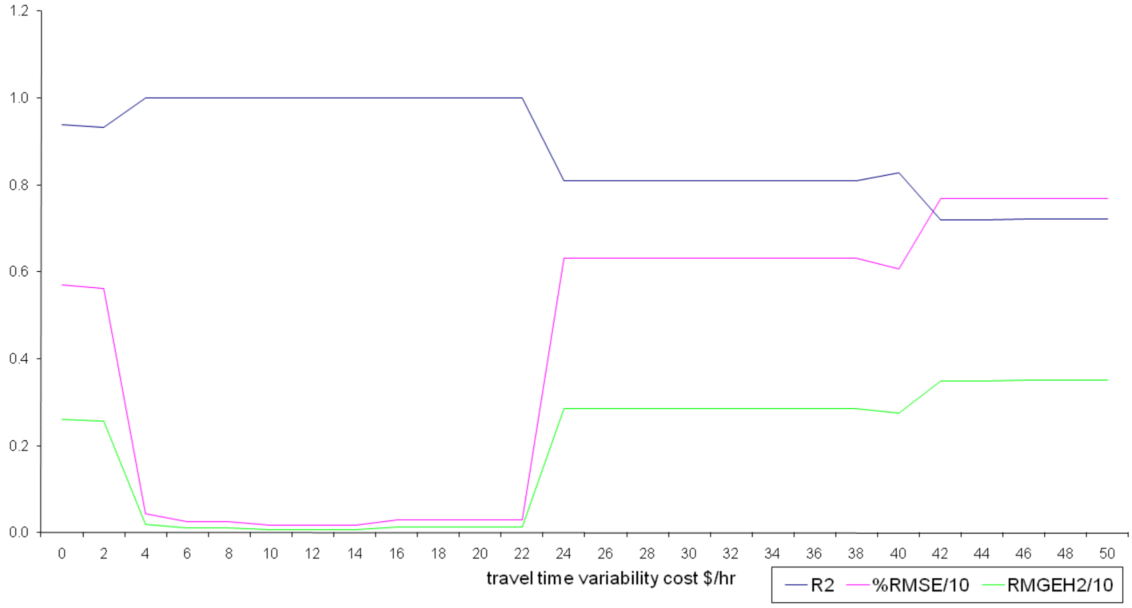
A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.13 for experiment E4. This is the assignment of a trip matrix with low heterogeneity using an all or nothing assignment.

From figure 4.13 it can be seen that initially increasing travel time reliability cost ς does not improve validation significantly. It is not until the travel time reliability cost exceeds $3/hr that the three measures of validation show significant improvement. It would appear this is a threshold at which one or more trip movements switch route using the all or nothing assignment, and significantly improves validation. After the point where ς optimises R^2, %RMSE and $RMGEH^2$ increasing values of ς sees deterioration in R^2, %RMSE and $RMGEH^2$ which occur through a series of steps.

Figure 4.13 also shows ranges where R^2, %RMSE and $RMGEH^2$ remain constant with increasing travel time reliability cost. This includes a range where R^2, %RMSE and $RMGEH^2$ appear to have optimum values.

The lack of a single point where ς optimises R^2, %RMSE and $RMGEH^2$ limits the effectiveness of the optimisation method provided in appendix C.

Figure 4.13 Validation measures vs travel time reliability cost – matrix T_2 – all or nothing



The range where the three measures of R^2, %RMSE and $RMGEH^2$ are optimised appears to be coincident and in the case of figure 4.13 appears to be for values of ς in the range of \$9/hr to \$15/hr. Near the optimum value of travel time reliability cost the validation measures change slowly.

4.5.5 Experiment E5

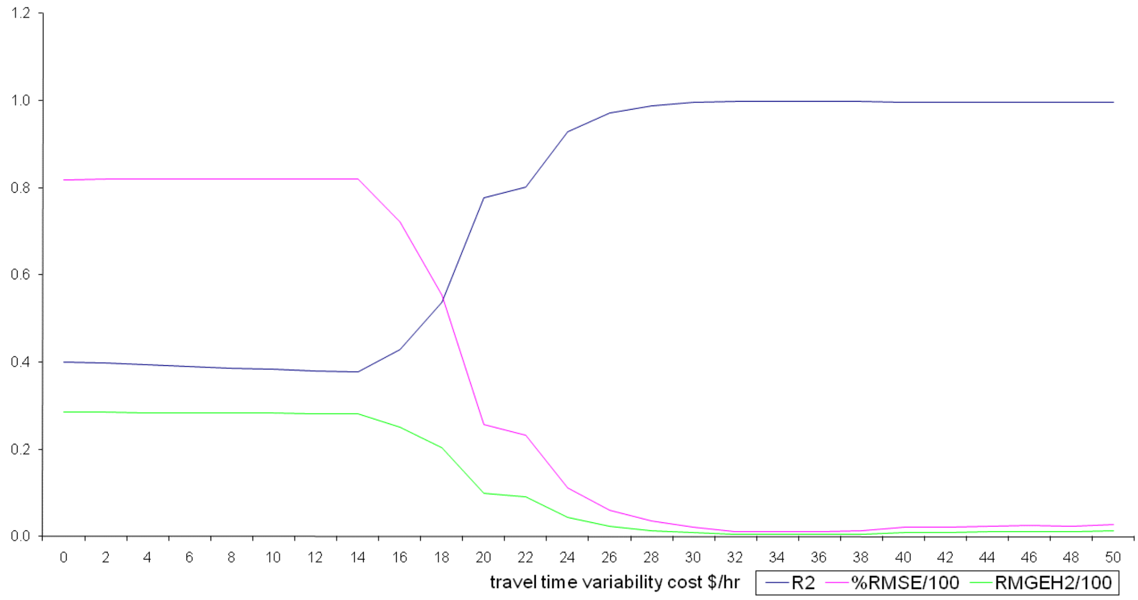
A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.14 for experiment E5. This is the assignment of a trip matrix with high heterogeneity using a logit assignment.

From figure 4.14 it can be seen that initially increasing travel time reliability cost ς does not improve validation. It is not until the travel time reliability cost exceeds \$14/hr that the three measures of validation show significant improvement. After the point where ς optimises R^2, %RMSE and $RMGEH^2$ increasing values of ς sees a slow deterioration in R^2, %RMSE and $RMGEH^2$.

Figure 4.14 illustrates that R^2, %RMSE and $RMGEH^2$ continue to vary, albeit slowly in some situations, with increasing travel time reliability cost. This behaviour is different from that encountered with the all or nothing assignment where in some ranges R^2, %RMSE and $RMGEH^2$ remain constant.

In appendix C a numerical optimisation method is provided that iteratively allows better values of ς to be estimated. Because the logit assignment provides a smooth curve this numerical method can be used to calculate point where ς optimises R^2, %RMSE and $RMGEH^2$. However, figure 4.14 shows local optima exist that are not the global optimum and so this method must be applied carefully to ensure the procedure converges to the global optimum and not a local optimum.

Figure 4.14 Validation measures vs travel time reliability cost – matrix T_1 – logit



The three measures of R^2, %RMSE and $RMGEH^2$ are optimised at a value of ς which is approximately the same for each of R^2, %RMSE and $RMGEH^2$. However, the optimum values of ς are not exactly coincident. This experiment shows the curvature in %RMSE is the greatest of the three measures near the optimum point and is the easiest in which to calculate the optimum point. The optimum point for ς for %RMSE is $33.95/hr. Near the optimum value of travel time reliability cost the validation measures do not change quickly.

4.5.6 Experiment E6

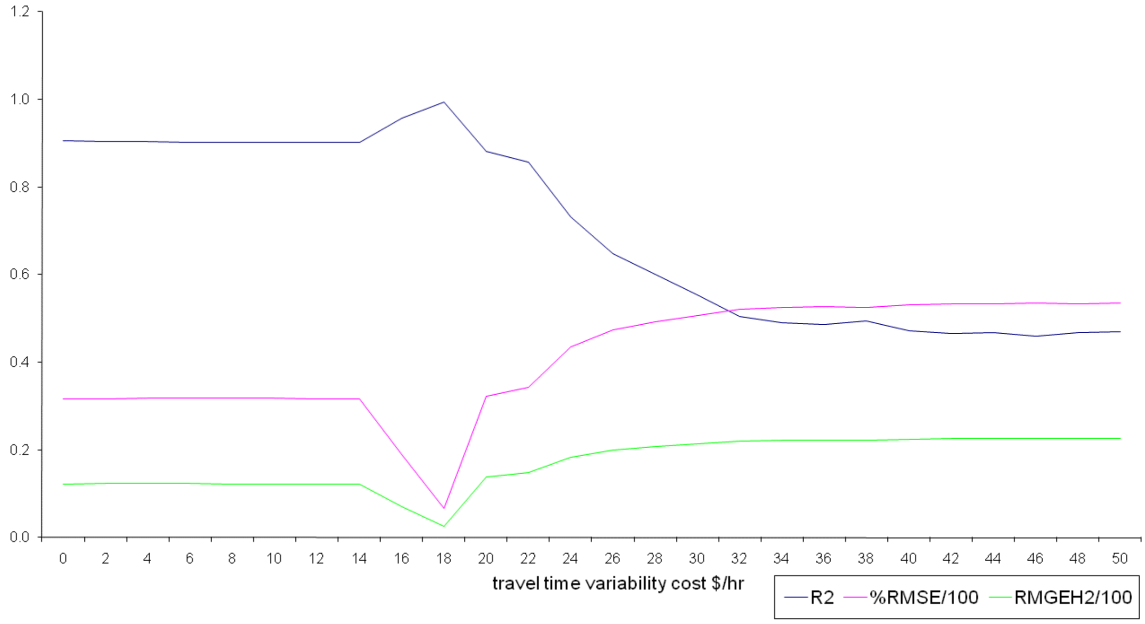
A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.15 for experiment E6. This is the assignment of a trip matrix with high heterogeneity using a logit assignment.

From figure 4.15 it can be seen that initially increasing travel time reliability cost ς does not improve validation until a travel time reliability cost of $14/hr is reached. After the point where ς optimises R^2, %RMSE and $RMGEH^2$ increasing values of ς sees steady deterioration in R^2, %RMSE and $RMGEH^2$ but this eventually plateaus.

Figure 4.15 illustrates that R^2, %RMSE and $RMGEH^2$ continues to vary, albeit slowly in some situations, with increasing travel time reliability cost. This behaviour is different from that encountered with the all or nothing assignment where ranges were encountered where R^2, %RMSE and $RMGEH^2$ remained constant.

In appendix C a numerical optimisation method is provided that iteratively allows better values of ς to be estimated. Because the logit assignment provides a smooth curve this numerical method can be used to calculate the point where ς optimises R^2, %RMSE and $RMGEH^2$. However, figure 4.15 shows local optima exist which are not the global optimum and so this method must be applied carefully to ensure the procedure converges to the global optimum and not a local optimum.

Figure 5.15 Validation measures vs travel time reliability cost – matrix T_1 - logit



The three measures of R^2, %RMSE and $RMGEH^2$ are optimised at a value of ς that is approximately the same for each. However, the optimum values of ς are not exactly coincident. This experiment shows the curvature in %RMSE is the greatest of the three measures near the optimum point and is the easiest in which to calculate the optimum point. The optimum point for ς for %RMSE is $17.05/hr. Near the optimum value of travel time reliability cost the validation measures change quickly.

4.5.7 Experiment E7

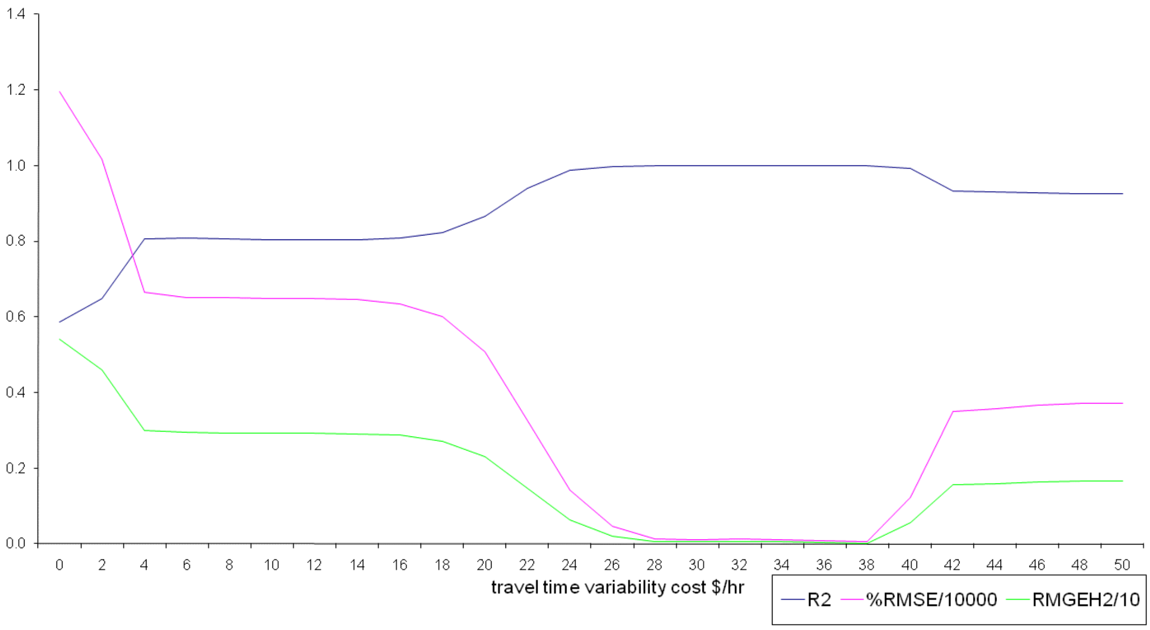
A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.16 for experiment E7. This is the assignment of a trip matrix with low heterogeneity using a logit assignment.

From figure 4.16 it can be seen that initially increasing travel time reliability cost ς does improve validation. After the point where ς optimises R^2, %RMSE and $RMGEH^2$, increasing values of ς sees steady deterioration in R^2, %RMSE and $RMGEH^2$ but this eventually plateaus.

Figure 4.16 illustrates that R^2, %RMSE and $RMGEH^2$ continue to vary, albeit slowly in some situations, with increasing travel time reliability cost. This behaviour is different from that encountered with the all or nothing assignment where ranges were encountered where R^2, %RMSE and $RMGEH^2$ remained constant.

In appendix C a numerical optimisation method is provided that iteratively allows better values of ς to be estimated. Because the logit assignment provides a smooth curve this numerical method can be used to calculate the point where ς optimises R^2, %RMSE and $RMGEH^2$. However, figure 4.16 shows local optima exist which are not the global optimum and so this method must be applied carefully to ensure the procedure converges to the global optimum and not a local optimum.

Figure 5.16 Validation measures vs travel time reliability cost – matrix T_2 – logit



The three measures of R^2, %RMSE and $RMGEH^2$ are optimised at a value of ς that is approximately the same for each of R^2, %RMSE and $RMGEH^2$. However, the optimum values of ς are not exactly coincident. This experiment shows the curvature in %RMSE is the greatest of the three measures near the optimum point and is the easiest in which to calculate the optimum point. The optimum point for ς for %RMSE is $38.29/hr. Near the optimum value of travel time reliability cost the validation measures change slowly.

4.5.8 Experiment E8

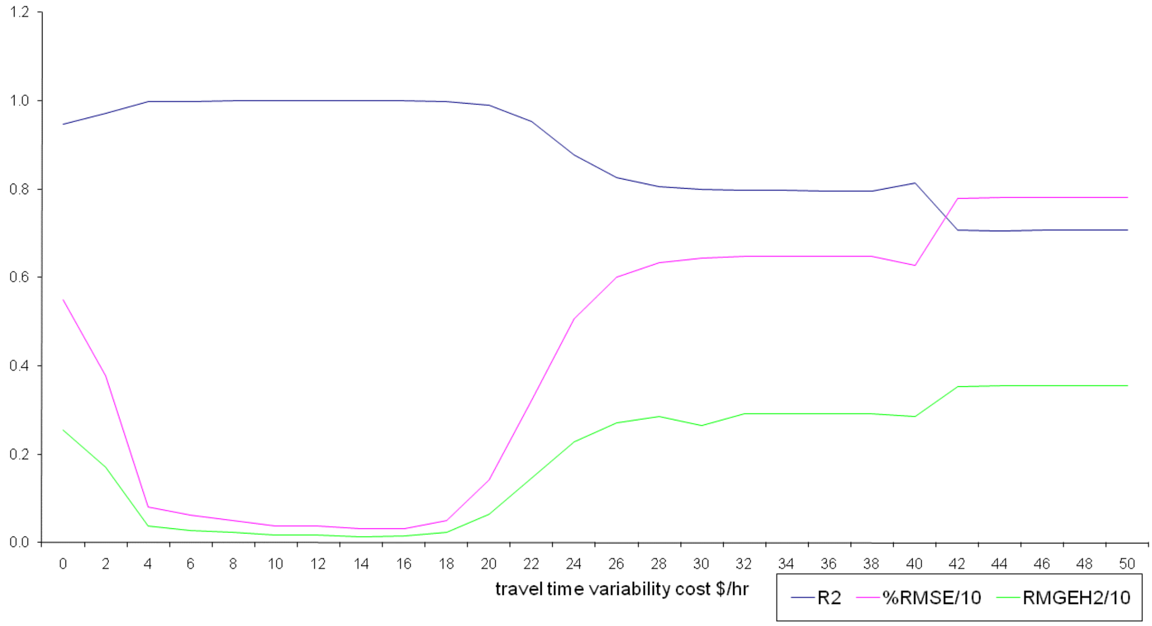
A graph of R^2, %RMSE and $RMGEH^2$ with travel time reliability cost ς is shown in figure 4.17 for experiment E8. This is the assignment of a trip matrix with low heterogeneity using a logit assignment.

From figure 4.17 it can be seen that initially increasing travel time reliability cost ς does improve validation. After the point where ς optimises R^2, %RMSE and $RMGEH^2$, increasing values of ς sees steady deterioration in R^2, %RMSE and $RMGEH^2$ but this eventually plateaus.

Figure 4.17 illustrates that R^2, %RMSE and $RMGEH^2$ continue to vary, albeit slowly in some situations, with increasing travel time reliability cost. This behaviour is different from that encountered with the all or nothing assignment where ranges were encountered where R^2, %RMSE and $RMGEH^2$ remained constant.

In appendix C a numerical optimisation method is provided that iteratively allows better values of ς to be estimated. Because the logit assignment provides a smooth curve this numerical method can be used to calculate the point where ς optimises R^2, %RMSE and $RMGEH^2$. However, figure 4.17 shows local optima exist which are not the global optimum and so this method must be applied carefully to ensure the procedure converges to the global optimum and not a local optimum.

Figure 4.17 Validation measures vs travel time reliability cost – matrix T_2 – logit



The three measures of R^2, %RMSE and $RMGEH^2$ are optimised at a value of ς that is approximately the same for each of R^2, %RMSE and $RMGEH^2$. However, the optimum values of ς are not exactly coincident. This experiment shows the curvature in %RMSE is the greatest of the three measures near the optimum point and is the easiest in which to calculate the optimum point. The optimum point for ς for %RMSE is $13.09/hr. Near the optimum value of travel time reliability cost the validation measures change slowly meaning nearby values of the validation statistics are only marginally worse.

4.5.9 Results summary

Experiments E1 to E8 had a broad range of characteristics. Four of the experiments used an all or nothing assignment routine and the other four used a logit assignment. Two different trip matrices were used with one having low heterogeneity and the other having high heterogeneity. Eight different 'observed' link flows were used which broadly corresponded to a selection of travel time reliability costs that were small and a selection that was large.

From the eight experiments above the following results were observed:

- the all or nothing assignment produced a set of validation measures that were not a smooth function of travel time reliability cost but contained a series of steps
- the all or nothing assignment might not produce a discrete value of travel time reliability cost to optimise the validation measures
- the optimum value of travel time reliability cost for each validation measured R^2, %RMSE and $RMGEH^2$ with an all or nothing assignment appearing to be in the same range
- the logit assignment produced a set of validation measures that were a smooth function of travel time reliability cost

- the logit assignment produced similar results for the travel time reliability cost that optimised the validation measures
- the optimum value for each validation measure for logit assignment was approximately coincident
- the validation measures each had more than one local point with a zero gradient so care had to be taken to ensure the correct value of travel time reliability cost was chosen for optimising the various validation measures
- trip matrices with high heterogeneity showed slow response in the validation measures with small travel time reliability costs
- trip matrices with low heterogeneity showed rapid response in the validation measures with small travel time reliability costs
- all trip matrices showed slow response in the validation measures with large travel time reliability costs
- trip matrices with high heterogeneity showed slow change in the validation measures near the optimum value of travel time reliability costs
- trip matrices with low heterogeneity rapidly changed in the validation measures near the optimum value of travel time reliability cost
- of the three validation measures R^2, %RMSE and $RMGEH^2$, the measure %RMSE gave the sharpest definition of the optimum value of travel time reliability cost with logit assignments.

These results are summarised in table 4.5 below.

Table 4.5 Experiment results

	E1	E2	E3	E4	E5	E6	E7	E8
Matrix	T_1	T_1	T_2	T_2	T_1	T_1	T_2	T_2
Assignment	all or nothing	all or nothing	all or nothing	all or nothing	logit	logit	logit	logit
Validation measure form	not smooth	not smooth	not smooth	not smooth	smooth	smooth	smooth	smooth
Response to small travel time cost	slow	slow	rapid	rapid	slow	slow	rapid	rapid
Response to large travel time cost	slow	slow	slow	slow	slow	slow	slow	slow
Response near optimum value	slow	slow	rapid	rapid	slow	slow	rapid	rapid

4.6 Statistical significance of improved validation by including travel time reliability

In this section the statistical significance of improved validation of including travel time reliability is tested using the results of the experiments in the previous section. This is done by testing the statistical significance of the difference in the validation measure when the travel time reliability cost is zero and optimum.

4.6.1 The correlation coefficient R^2

Elementary college statistical theory teaches that the difference in the transformation, ξ, of the correlation coefficient R^2 given by equation 4.11 is approximately normally distributed.

$$\xi = 0.5\log_e((1 + R)/(1 - R)) \qquad \text{(Equation 4.11)}$$

Examination of table 4.6 shows the value of the transformed correlation coefficient for the optimum R^2, ξ_{opt}, is significantly different from the value when travel time reliability cost is zero, at both the 95% and 99% levels. This was found to be the case for all eight experiments E1 to E8.

Table 4.6 Statistical significance of R^2

	E1	E2	E3	E4	E5	E6	E7	E8
R_0^2	0.406	0.972	0.583	0.934	0.400	0.906	0.587	0.946
R_{opt}^2	0.999	1.000	1.000	1.000	0.999	1.000	1.000	1.000
ξ_0	0.754	2.468	1.005	2.067	0.745	1.849	1.011	2.141
ξ_{opt}	4.153	4.594	5.868	5.195	4.118	5.769	6.502	4.975
ξ_{95}	3.499	3.941	5.215	4.541	3.465	5.116	5.849	4.321
ξ_{99}	3.294	3.735	5.009	4.336	3.259	4.910	5.643	4.116

4.6.2 The root-mean-square-error %RMSE

The test of statistical significance of the difference in %RMSE when %RMSE is optimised and the travel time reliability cost is zero assumes %RMSE is approximately normally distributed. A sample standard deviation of %RMSE is calculated by using all the values calculated for travel time reliability cost equal to zero to $50/hr, in integer increments, for each experiment. The results of this analysis are shown in table 4.7.

Table 4.7 Statistical significance of %RMSE

	E1	E2	E3	E4	E5	E6	E7	E8
$\%RMSE_0$	81.623	18.484	11.966	5.707	81.896	31.601	11.964	5.501
$\%RMSE_{opt}$	1.110	2.150	0.118	0.171	1.049	0.628	0.052	0.270
$\%RMSE_{95}$	15.302	9.545	1.497	1.411	15.235	5.565	1.314	1.424
$\%RMSE_{99}$	19.763	11.870	1.930	1.801	19.693	7.117	1.710	1.787

Examination of table 4.7 shows when the value of %RMSE is optimised, $\%RMSE_{opt}$, it is sufficiently different from the value when travel time reliability cost is zero to be statistically significant at both the 95% and 99% levels. This was found to be the case for all eight experiments E1 to E8.

4.6.3 The root-mean-GEH2 $RMGEH^2$

The test of statistical significance of the difference in $RMGEH^2$ when $RMGEH^2$ is optimised and the travel time reliability cost is zero assumes $RMGEH^2$ is approximately normally distributed. A sample standard deviation of

$RMGEH^2$ is calculated by using all the values calculated for travel time reliability cost equal to zero to \$50/hr, in integer increments, for each experiment. The results of this analysis are shown in table 4.8.

Examination of table 4.8 shows when the value of $RMGEH^2$ is optimised, $RMGEH^2_{opt}$, it is sufficiently different from the value when travel time reliability cost is zero to be statistically significant at both the 95% and 99% levels. This was found to be the case for all eight experiments E1 to E8.

Table 4.8 Statistical significance of $RMGEH^2$

	E1	E2	E3	E4	E5	E6	E7	E8
$RMGEH^2_0$	28.457	7.907	5.420	2.601	28.494	12.263	5.420	2.554
$RMGEH^2_{opt}$	0.507	0.859	0.054	0.077	0.440	0.307	0.023	0.124
$RMGEH^2_{95}$	5.406	3.964	0.672	0.640	5.317	2.544	0.594	0.645
$RMGEH^2_{99}$	6.960	4.940	0.866	0.817	6.849	3.246	0.774	0.808

4.7 Summary

A simple four node, five link network was used to conduct a series of experiments using travel time reliability cost to improve the validation of an assignment. An equation based on the results of chapter 3 of this report was used to provide a relationship between travel time variability and link travel time for a 'real world' Wellington network.

On the basis of the experiments conducted in this report it would appear increasing travel time reliability cost might influence the distribution of traffic over a network more if the trip matrix had high heterogeneity. In trip matrices with low heterogeneity the assignment of traffic over a network did not appear to be influenced as much by travel time reliability cost. This conclusion should be tested with a wider range of trip matrices to check validity.

Eight experiments were undertaken. Four of these experiments used an all or nothing assignment approach and the other four used a logit methodology. Two test trip matrices were used. The first trip matrix had high heterogeneity and the second matrix had low heterogeneity. Three measures of assignment validation were reported in the experiments. These were the correlation coefficient R^2, the root-mean-square-error %RMSE and a refinement of the GEH statistic – the root-mean-GEH^2 $RMGEH^2$.

These experiments allowed the following conclusions to be drawn for the networks considered:

- The all or nothing assignment produced a set of validation measures that were not a smooth function of travel time reliability cost but contained a series of steps.
- The all or nothing assignment might not produce a discrete value of travel time reliability cost that optimised the validation measures. The optimum value of travel time reliability cost for each of the validation measured R^2, %RMSE and $RMGEH^2$ with an all or nothing assignment appearing to be in the same range.
- The logit assignment produced a set of validation measures that were a smooth function of travel time reliability cost. The logit assignment produced an approximately coincident discrete value of travel time reliability cost that optimised the validation measures. The optimum value for each validation measure for logit assignment was approximately coincident. However, in the case of a trip matrix with high heterogeneity the validation measures changed slowly in the vicinity of the optimum value of

travel time variability cost. The validation measures each had local optima that were not the global optimum with logit assignment.

- On the basis of the matrices used, trip matrices with high heterogeneity showed slow response in the validation measures with small travel time reliability costs. Trip matrices with high heterogeneity showed rapid change in the validation measures near the optimum value of travel time reliability costs.
- Trip matrices with low heterogeneity showed rapid response in the validation measures with small travel time reliability costs. Trip matrices with low heterogeneity showed change in the validation measures near the optimum value of travel time reliability costs. All trip matrices showed slow response in the validation measures with large travel time reliability costs.
- Of the three validation measures R^2, %RMSE and $RMGEH^2$, the measure %RMSE gave the sharpest definition of the optimum value of travel time reliability cost with logit assignments.
- The difference between the validation measure at the optimum value of travel time reliability cost and the validation measure with zero travel time reliability cost was found to be statistically significant at both the 95% and 99% levels of significance. This was found to be true for all validation measures.

The analysis undertaken in this phase of the project appeared to provide a useful basis to investigate data taken from a real network. It is recommended that the estimation of an appropriate travel time reliability cost be undertaken by optimising the %RMSE as the measure of validation. Logit assignment provides the sharpest definition of travel time reliability.

These results were taken forward to the next phase of this project where real trip matrices, link traffic volumes and networks were considered. It was expected that on networks like Wellington where peak traffic flows showed a definite dominance of trips to and from the Wellington central business district the results for trip matrices with high heterogeneity might be useful. Trip matrices for other urban areas or time periods might not have such a dominance of origin destination pairs and the findings for trip matrices with low heterogeneity might be more useful.

5 Application to a real network

5.1 Preamble

This section documents the application of the travel time reliability analysis undertaken in the earlier parts of this project to a 'real world' traffic assignment model. Models which have the capability of including travel time reliability direct through an appropriate formulation of the generalised cost function are discussed first.

Models such as those that use the SATURN software platform where the generalised cost function is hard wired and does not include travel time reliability are discussed next. Methodologies are developed to formulate the generalised cost function so it mimics the inclusion of travel time reliability.

These methodologies are applied to a SATURN model in a real application. The results of this application are reported and conclusions drawn. In particular, the impact on network wide validation statistics is reported and the effect on screenline GEH values. The relationship between the appropriate travel time variability cost and travel time cost is investigated.

5.2 Models with flexible generalised cost functions

A number of models exist in New Zealand where the formulation of the generalised cost function is flexible. This allows travel time reliability to be included directly into the traffic model and used as part of the trip assignment process. An example of such a model is the Western Bay of Plenty Traffic Model built by Beca (September 2009) and used to analyse the Tauranga Eastern Link toll project.

This model uses a generalised cost function of the form:

$$G_c = \delta d + \Tau t + \varsigma\sigma + \kappa \qquad \text{(Equation 5.1)}$$

where G_c is the generalised cost in dollars that the assignment algorithm seeks to minimise

where δ is the distance weighting in $/km

where d is travel distance in km

where $\Tau$ is the time weighting in $/min

where t is the travel time in minutes

where ς is the travel time reliability weighting in $/min

where σ is the standard deviation of travel time in minutes

where κ is a constant in $.

The constant κ can represent a fixed charge such as a toll on a link.

Such models have incorporated travel time reliability directly into the generalised cost function that determines route choice in trip assignment and require no further attention in this research.

Examples of these models are few and they are purpose built. As a result they require more effort to develop than models with standard generalised cost formulations and there is limited international experience of their use. Where such models are in place there do not appear to be any more issues in their application than for any other model.

5.3 Models with fixed generalised cost functions

Many assignment models fix the generalised cost function in the form:

$$G_c = \delta d + \tau t + \kappa \qquad \text{(Equation 5.2)}$$

where G_c is the generalised cost in dollars that the assignment algorithm seeks to minimise

where δ is the distance weighting in \$/km

where d is travel distance in km

where τ is the time weighting in \$/min

where t is the travel time in minutes

where κ is a constant in \$.

Frequently the generalised cost is expressed in equivalent time and the dimension of δ is min/km, τ is dimensionless and t and κ are in minutes. This is the case with the Wellington models.

Examples of such models with fixed generalised cost functions include traffic assignment models based on the SATURN software platform.

Incorporating travel time reliability into the fixed generalised cost function formulation requires equation 5.1 to be amended to the form shown in equation 5.2. The problem is to recast equation 5.1 into a form that is consistent with equation 5.2 so that it can be accommodated by assignment models.

From the investigations of travel time reliability using Wellington data a relationship of the form is found:

$$\begin{aligned} \sigma/t = CV_t &= \alpha(CI_t - 1) && CI_t < \psi_1 && \text{(Equation 5.3)} \\ &= \beta + \gamma/CI_t && \psi_1 \leq CI_t < \psi_2 \\ &= 0 && \Psi_2 \leq CI_t \end{aligned}$$

where σ is the standard deviation of link journey time in minutes

where CV_t is the coefficient of variation of link journey time

where CI_t is the congestion index and is equal to t/t_0.

In the case of the data from Wellington it was found that:

$\alpha = 0.7058$

$\beta = -0.3105$

$\gamma = 0.8465$

$\psi_1 = 1.4104$

$\psi_2 = 2.7262$

Equation 5.3 enables equation 5.1 to be recast into the form of equation 5.2 and can be rewritten as:

$$\sigma = \alpha(t/t_0 - 1)t \qquad CI_t < \psi_1 \qquad \text{(Equation 5.4)}$$

$$= \beta\, t + \gamma t_0 \qquad \psi_1 \le CI_t < \psi_2$$

$$= 0 \qquad \psi_2 \le CI_t$$

Equation 5.1 becomes:

$$G_c = \delta d + \tau t + \varsigma\sigma + \kappa$$

$$= \delta d + \tau t + \varsigma\alpha(t/t_0 - 1)t + \kappa \qquad CI_t < \psi_1 \qquad \text{(Equation 5.5)}$$

$$= \delta d + \tau t + \varsigma\beta\, t + \varsigma\gamma t_0 + \kappa \qquad \psi_1 \le CI_t < \psi_2$$

$$= \delta d + \tau t + \kappa \qquad \psi_2 \le CI_t$$

$$G_c = \delta d + \tau t + \varsigma\sigma + \kappa = \delta d + \tau^* t + \kappa^* \qquad \text{(Equation 5.6)}$$

where $\tau^* = \tau + \varsigma\alpha(t/t_0 - 1) \qquad \kappa^* = \kappa \qquad CI_t < \psi_1$

$$\tau^* = \tau + \varsigma\beta \qquad \kappa^* = \kappa + \varsigma\gamma t_0 \qquad \psi_1 \le CI_t < \psi_2$$

$$\tau^* = \tau \qquad \kappa^* = \kappa \qquad \psi_2 \le CI_t$$

Equation 5.6 successfully recasts equation 5.1 in the form of equation 5.2. It should be appreciated that in the condition $CI_t < \psi_1$ τ^*, the right-hand side is not a constant but a function of travel time t. This requires an iterative application of the traffic model where τ^* is updated at each successive iteration. For $CI_t \ge \psi_1$ both τ^* and κ^* are not functions of t but constants.

However, this calculation is not straightforward as the calculation of the correct values of τ^* and κ^* requires care as it may be that successive iterations cross the thresholds set by ψ_1 and ψ_2. This requires the routines that calculate τ * and κ * to perform logical tests on the value of CI_t. As τ^* and κ^* are continuous, but not smooth functions, this is not expected to affect convergence of this iterative process.

The above formulation can be used if the distance, time and travel time reliability can be calculated for the whole route from the trip origin to the destination. This is because, even though travel time and distance are additive, the standard deviation of travel time is not. This requires a whole route travel time standard deviation to be calculated.

If there are a number of links between an origin and a destination using a particular route then the total route travel time t is given by:

$$T = \Sigma\, t_i \qquad \text{(Equation 5.7)}$$

and total travel distance d is:

$$D = \Sigma\, d_i \qquad \text{(Equation 5.8)}$$

where the summation is over all the links on the route. If it is assumed the travel time variability of one link is independent of another then the variances for the whole route can be summed. This gives the total variance as:

$$\sigma^2 = \Sigma \sigma_i^2 \qquad \text{(Equation 5.9)}$$

or

$$\sigma = (\Sigma \sigma_i^2)^{1/2} \qquad \text{(Equation 5.10)}$$

This also sets up an iterative scheme for the route assignment. Each σ_i can be calculated for a particular link i at each iteration with equation 5.4 using the travel time on the link from the previous iteration. The overall route travel time standard deviation can then be calculated using equation 5.10. This is added as part of κ for the whole route. This is repeated until convergence is achieved.

A key issue is the assumption that underpins equation 5.9. This requires that the correlation between travel time variations between links is small. Equation 5.11 below gives the overall variance as a function of two distributions.

$$\sigma^2_{tot} = \sigma^2_1 + \sigma^2_2 + 2cor \qquad \text{(Equation 5.11)}$$

Where $\sigma^2_{tot\ t}$ is the total variance across two road links, σ^2_1 is the variance across link 1, σ^2_2 is the link across link 2 and 'cor' is the correlation between links 1 and 2. Equation 5.11 can be generalised to more than two road links.

The assumption that the route variance is the sum of the link variances in equation 5.9 means the correlations between links on a route sum to a small number when compared with the sum of the variances. A high volume on a link is likely to mean there will be some correlation with downstream links. However, if the number of links and origins and destinations on a network is high then the flow on link 1 may only have a small correlation with link 2 as significant portions of the traffic travel to links other than link 2. Link 2 may also receive traffic from links other than link 1 which also dilutes the correlation between link 1 and link 2.

This line of reasoning suggests that in networks with a large number of links and a large number of origins and destinations, the sum of the correlations (which can be positive and negative) between links may be sufficiently low when compared with the sum of the variances (which can only be positive). However, because of the importance of this assumption it is recommended that the size of the sum of the correlations between links on the network compared with the sum of the variances be investigated as a subsequent research project.

In the event that equation 5.10 is found to break down, which is expected to be more likely in networks with a smaller number of links and origins and destinations, then equation 5.10 cannot be used. In such situations route standard deviations will need to be found directly using equation 5.3. This task should not be overly onerous as such a network is expected to contain a smaller number of links.

5.4 Iterative process

The use of equation 5.10 in conjunction with equation 5.4 requires an iterative process. This process seeks to find the value of ς that minimises the validation statistics such as GEH, RSME and R^2. In reality there may be no one value of ς that optimises all three of these statistics simultaneously so it may be necessary to find a balance which could be a linear combination of these statistics.

The iterative process requires the calculation of these statistics for a chosen value of ς. This means equations 5.1 and 5.4 are used together in a further iterative loop. The first iteration sets ς to zero in equation 5.1 to obtain the travel times on each link after the trip matrix is assigned. This assignment is updated using equation 5.1 with ς set at its chosen value and this is multiplied into the value of time t on the link found from the previous iteration. The assignment is repeated to find a new value of each link time. This process is repeated until convergence in the value of link times is obtained as well as the validation statistics.

A new value of ς is chosen and the process is repeated to find a new set of converged link travel times and validation statistics. A further value of ς is chosen and so on. In this way validation statistics are calculated as a function of ς.

Inspection of this array of validation statistics as a function of ς hopes to reveal a value of ς where the value of the validation statistics is optimised. This requires a manual consideration of the pattern of the validation statistics with the value of ς. As discrete values of ς have been chosen to calculate the validation statistics there is a need to converge on to the optimal point.

This can be expedited by fitting a quadratic function:

$$f_i = a\,\varsigma_i^2 + b\,\varsigma_i + c \qquad \text{(Equation 5.12)}$$

through three points in the vicinity of the optimum point where f_i is the validation statistic at the ith point, ς_i is the travel time variability cost at the ith point and a, b and c are constants that enable a quadratic in ς_i to be fitted. The constants a, b and c can be determined using the formulae in appendix C. Again, the selection of the three points requires manual determination of the range of where the optimum value of ς is likely to lie with each iteration.

From inspection of the outputs of the modelling of the validation statistics the three closest values of travel time variability cost to a range that optimises the validation statistic are selected. Each of these in turn is used in equation 5.12 as the ith point. Three points are chosen as three are required to define a quadratic and a quadratic is the simplest polynomial that has an optimum point defined where its derivative is zero.

The next best estimate for ς is calculated by setting

$$df/d\varsigma = 0 = 2a\,\varsigma + b \qquad \text{(Equation 5.13)}$$

or

$$\varsigma = -b/2a \qquad \text{(Equation 5.14)}$$

Equation 5.14 is used to calculate an improved estimate of the optimal point and a new value of the validation statistic is calculated from the model. A further set of the closest three points can be chosen and this process is repeated using equations 5.12 and 5.14 to further improve the estimate of the optimum point.

5.5 Description of the case study

A case study was implemented using the 2009 Wellington Traffic Model (WTM). This model covers the area of Wellington from Johnsonville in the north, Karori in the west, Lyall Bay in the south and Miramar in the east. The model has 229 zones, 3064 nodes and 4650 links built on a SATURN software platform. This number of zones and links may assist in reducing the magnitude of the sum of correlations between links compared with the sum of the variances of travel time. The model has a generalised cost function of the form shown in equation 5.2 hardwired into the software platform. Accordingly the iterative process described in the previous section is required.

The WTM is a meso-level model which telescopes into the Wellington city part of the network modelled by the four-stage strategic model for the Wellington region known as the Wellington Transport Strategic Model (WTSM). Matrices which link origins and destinations in the coarse strategic model are exported to the WTM and these matrices undergo a partitioning process so that the trips in the coarser WTSM trip matrix cells can be allocated to the more detailed WTM cells.

The WTSM is calibrated on 2006 census data and traffic data. Trip matrices from the WTSM have to be factored up to estimate 2009 levels in the WTM.

In this case study a 2009 validated SATURN traffic model for Wellington city was used. As this model has been validated this limited the potential for improvement in the global validation statistics such as R^2 and %RMSE although some improvement might be possible. The morning one-hour peak model was used as this had the poorest validation of the three time periods.

Two changes to the base WTM were made. These were:

- The Evans Bay amenity factor was removed which ensured the correct balance in traffic between the bay route and SH1 and reflected that, despite longer distances and slower speeds, traffic used this route because of the pleasant views available. The removal of the amenity factor was necessary as this variable was required to implement the iterative process reflecting travel time variability. The removal of the amenity factor may have reduced the overall validation of the model; however, it was possible the inclusion of travel time reliability might compensate for this to some extent.
- The inclusion of travel time variability required the use of the SATURN variable FREE88 which allowed the travel time variability to be inputted in free format.

Both these changes meant the model was not directly comparable with the validated WTM. However, on reflection it was considered these changes would not be significant and would allow the investigation to proceed efficiently.

The generalised cost function in the model used a distance cost of $0.0875/km and a travel time cost of $16.27/hr and multiples of $16.27/hr for the travel time variability cost. The travel time variability cost varied from 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 times $16.27/hr to give a broad range of costs. This range more than straddled the range indentified by Margiotta (2006) who found travel time variability costs were two to six times travel time costs.

The standard deviation in link travel time calculated from the link travel time using equation 5.4 in one iteration was used in the next iteration. The iterations continued until the standard deviations in successive iterations across the whole network varied by no more than 1.5%. This process required no more than six iterations for any given multiple of $16.27/hr used as the travel time variability cost.

This result indicated a high degree of stability in the iterative process even though there was a considerable variation in link travel time standard deviation from one link to another across the network for a given travel time variability cost. In this respect, it was found the process was not overly cumbersome for a network as large as the Wellington city network. However, it did require the development of an appropriate macro to calculate link travel time standard deviations for each of the iterations.

The macro enabled the iterative process that used equation 5.6 to proceed efficiently for each value of travel time variability cost. The estimation of the travel time variability cost that optimised the validation statistics using equations 5.12 to 5.14 took place outside this process on a spreadsheet.

The following statistics were reported:

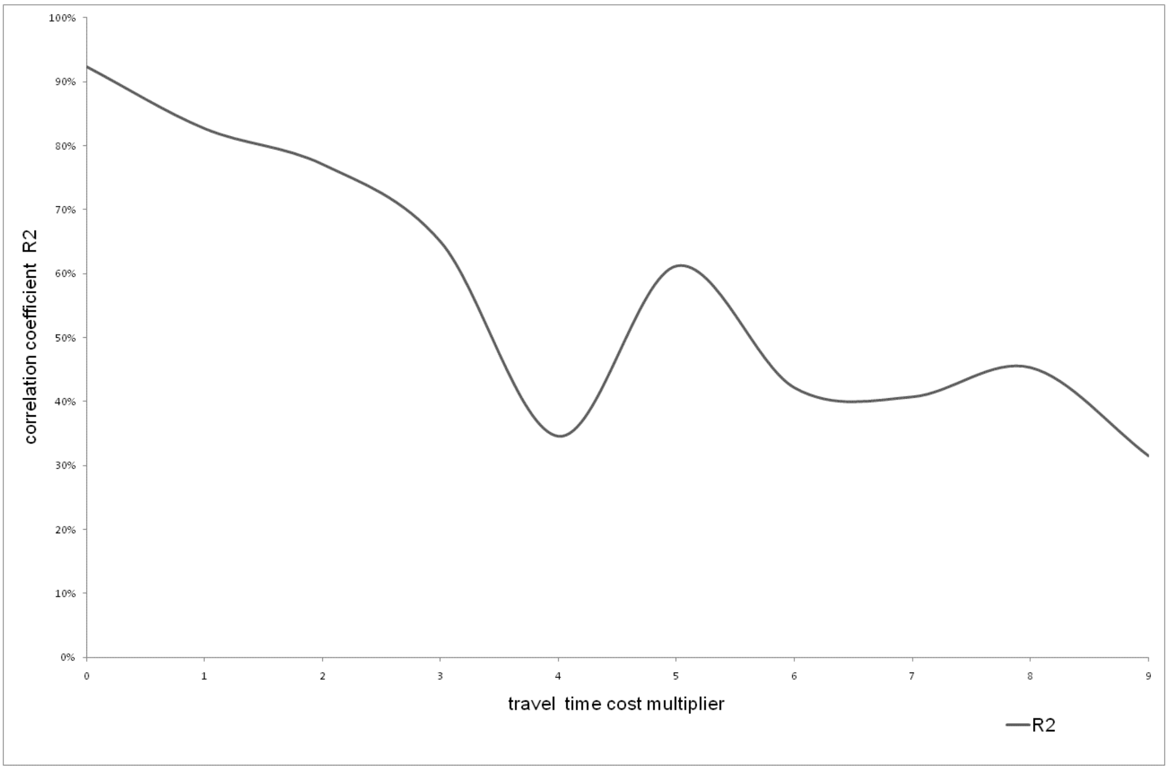
- the correlation coefficient R^2
- the root mean square error (%RMSE)
- the root mean square GEH ($RMGEH^2$)
- the GEH value of the screenline south of the Ngauranga interchange by direction
- the GEH value of the screenline south of the basin reserve by direction
- the GEH value of the HCV only screenline south of the basin reserve by direction.

R^2, %RMSE and $RMGEH^2$ were calculated over 151 count locations.

5.6 Results

Figures 5.1 to 5.3 show the network wide values of the correlation coefficient R^2, the root mean square error (%RMSE) and the root mean square GEH as a function of the travel time variability cost. The travel time variability is expressed as a ratio. For example, a ratio of 3.0 means the travel time variability cost is three times the travel time cost per unit of time.

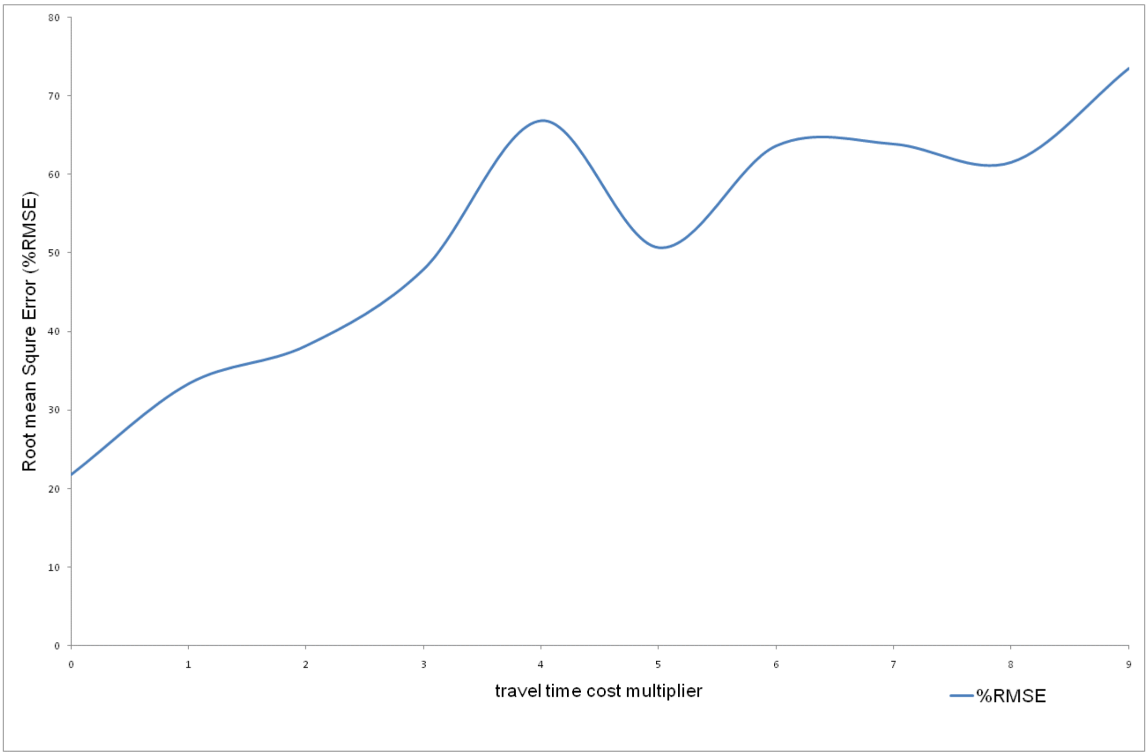
Figure 5.1 Correlation coefficient R^2 vs travel time reliability cost



Figures 5.1 to 5.3 show a similar pattern. The optimal value of the travel time variability cost is 0.0. Increasing values of travel costs showed a deteriorating value of the validation statistic R^2, %RMSE and the $RMGEH^2$ respectively. In this respect, even though the methodology outlined in sections 5.4 and 5.5 was demonstrated to be computationally successful and numerically stable, the inclusion of travel time reliability did not lead to an improvement in global validation statistics.

Careful examination of the validation statistics presented in figures 5.1 to 5.3 showed even though there was an overall deterioration in the validation statistics all three curves consistently showed a local maximum in the case of R^2, and minima in the case of RMSE and $RMGEH^2$ where the gradient of the curves was zero but where the validation was poorer than when the travel time variability cost was zero. At these locations of zero gradient, the value of the validation statistics was poorer than when the travel time variability cost was zero but these were locations where the deterioration in the validation statistics were better than the immediate surrounds.

Figure 5.2 Root mean square error %RMSE vs travel time reliability cost



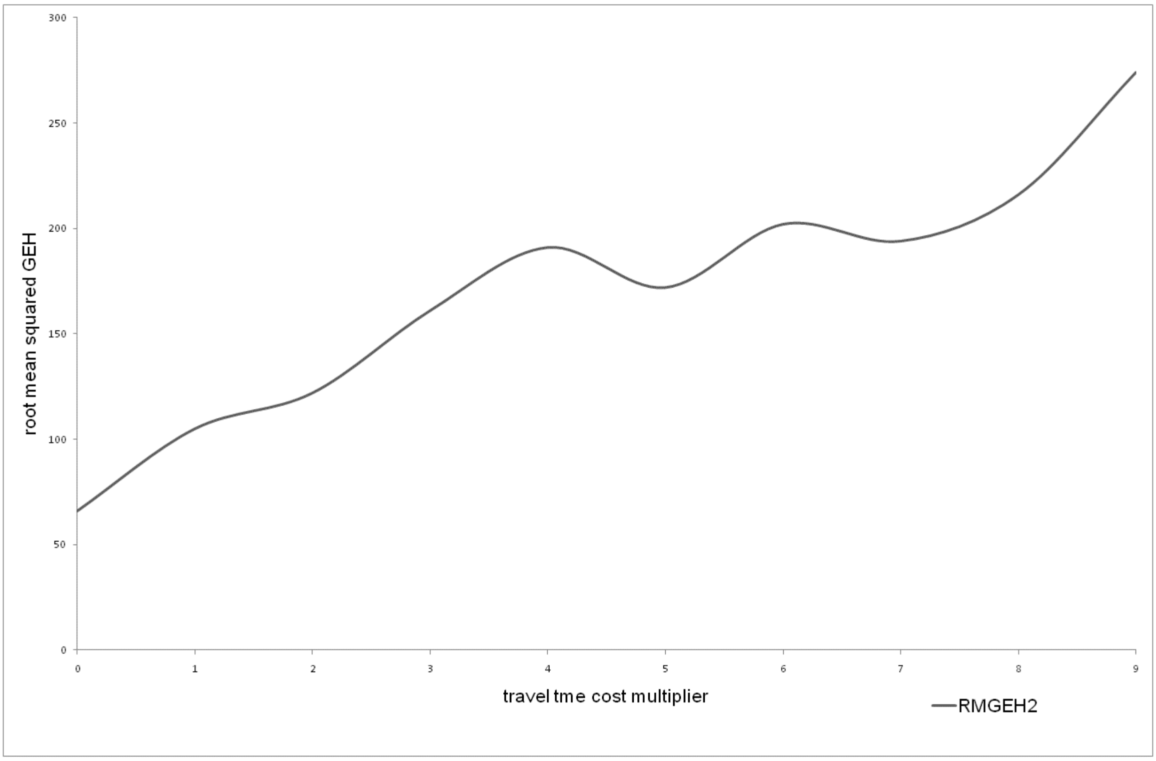
The impact of incorporating travel time variability varied across the network. This was because of the form of the standard deviation of travel time given in equation 5.4. Those links that have higher values of a volume to capacity ratio, causing the ratio of link travel time to free-flow travel time to approach ψ_1 = 1.4104, were affected more than those with low volume to capacity ratios.

Estimates of the locations of local zero gradient were calculated using equations 5.12 to 5.14. Details of these local zero gradient are summarised in table 5.1.

Table 5.1 Local zero gradient in the network wide validation statistics

	R^2	RMSE	$RMGEH^2$
Optimal travel time variability cost	5.1	5.1	4.9
Optimal validation statistic	61.4	51.0	171.7

Figure 5.3 Root mean square GEH RMSGEH[2] vs travel time reliability cost



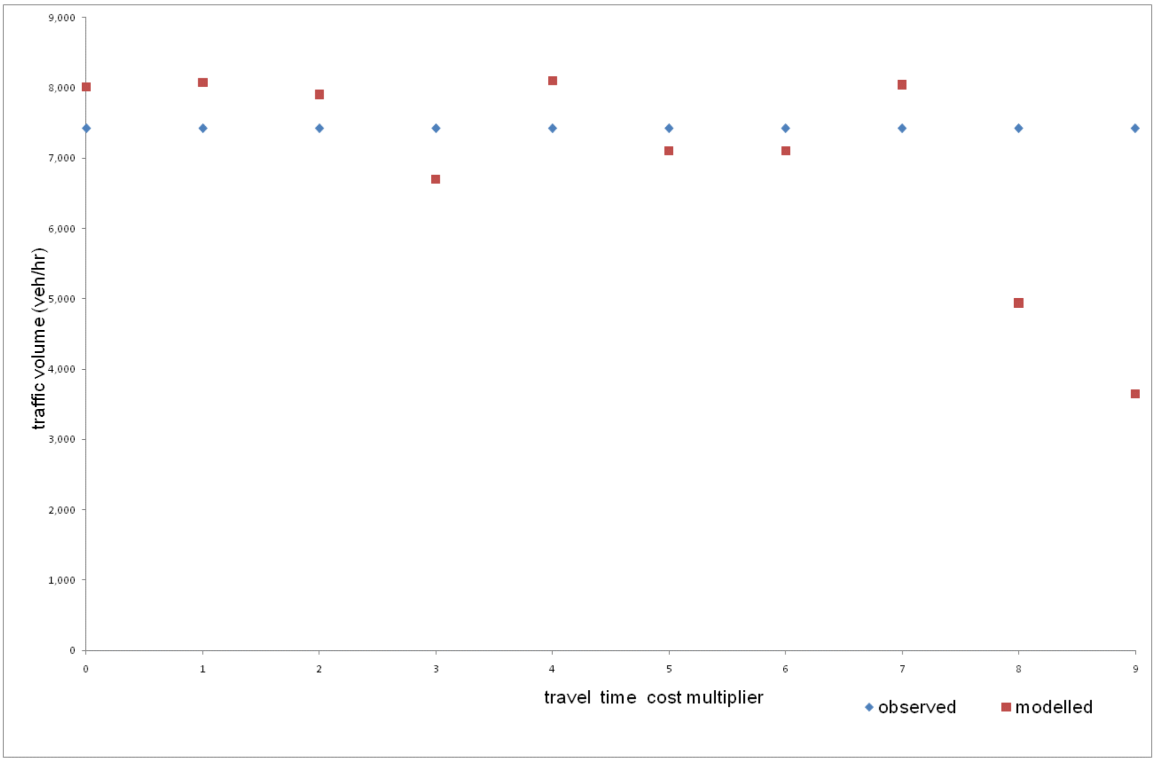
The results in table 5.1 indicate local optima in each of the network wide validation statistics where the travel time variability cost is in the range of 4.9 to 5.2 times the travel time cost. Nevertheless, improvements in the overall network validation statistics by incorporating travel time variability did not eventuate in this range as it is an area where there is less deterioration in the validation statistics.

The impact of modelling travel time reliability on a key screenline GEH is demonstrated in figures 5.4 to 5.12. These screenlines were chosen because they are high-volume screenlines that represent key entrances and exits to central Wellington. The first screenline is south of the Ngauranga merge of SH1 and SH2 and is examined separately by direction in figures 5.4 to 5.6. Again the travel time variability cost is expressed as a multiple of the travel time cost.

The second screenline is south of the Basin Reserve and is examined separately by direction in figures 5.7 to 5.9. Again the travel time variability cost is expressed as a multiple of the travel time cost. Figures 5.10 to 5.12 also shows the Basin Reserve screenline separately by direction except instead of showing GEH for all traffic it considers heavy commercial vehicles (HCV) only.

Figures 5.4 to 5.12 show the inclusion of travel time variability can lead to improved GEH values on individual screenlines even though the global validation statistics deteriorate. This suggests the inclusion of travel time variability is having a differential impact on screenlines across the network as discussed above related to the form of equation 5.4. In all cases considered, a travel time variability cost was found that produced a smaller screenline GEH than when the travel time variability cost was set to zero. It was found that if the travel time variability cost was further increased beyond this optimum point then the screenline GEH ultimately deteriorated rapidly.

Figure 5.4 Observed vs modelled traffic volumes at Ngauranga inbound



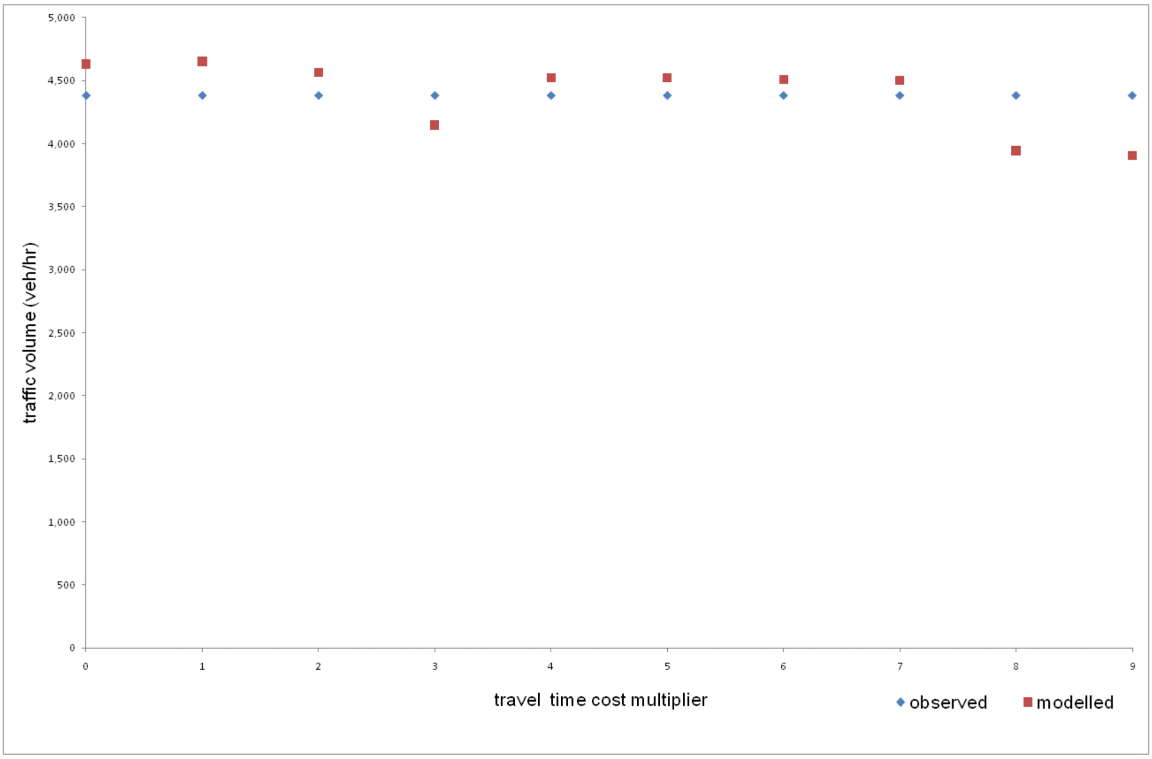
Figures 5.4 to 5.6 show the impact on the Ngauranga screenline. Figures 5.4 and 5.5 show the observed traffic volume and the modelled volume versus travel time variability cost by direction and figure 5.6 shows the respective GEH value versus travel time variability cost. It is noted that the form of the GEH value, shown in equation 5.15

$$GEH = (V_{obs} - V_{mod})^2/2(V_{obs} + V_{mod}) \quad \text{(Equation 5.15)}$$

means the GEH only ever touches the horizontal axis and never crosses the horizontal axis. Nevertheless, at the points where the GEH touches the horizontal axis the observed traffic volume equals the modelled traffic volume or $V_{obs} - V_{mod} = 0$. Numerically, a sharper estimate of the travel time variability cost where GEH is zero is obtained by examining where $V_{obs} - V_{mod} = 0$ rather than when GEH is zero but the zero points are the same.

Table 5.2 summarises these results from figures 5.4 to 5.6. Estimates of the locations of the points where GEH is zero can be found by fitting a quadratic of the form of equation 5.12.

Figure 5.5 Observed vs modelled traffic volumes at Ngauranga outbound



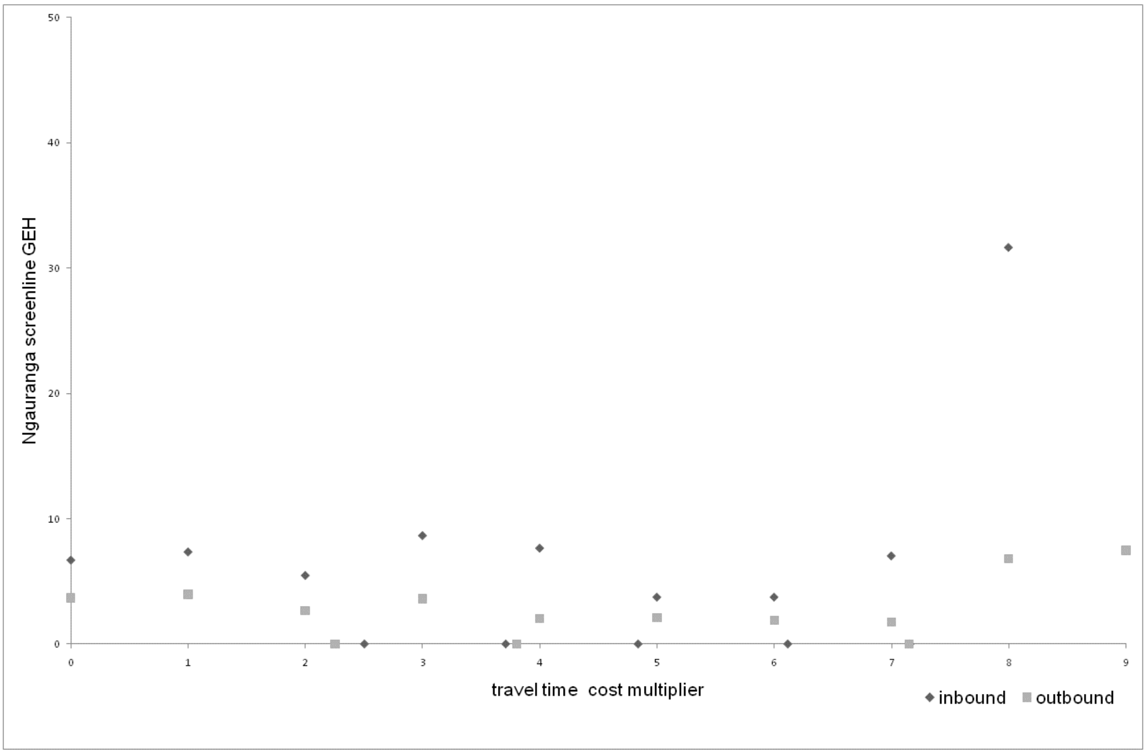
Using this approach it is possible to select a travel time variability cost where the GEH value is reduced to zero for each of the inbound and outbound directions separately. Alternatively, there are locations where the GEH value is significantly improved in both directions simultaneously but not reduced to zero. Such locations occur when travel time variability costs are in the range 2.1 to 2.6, 3.5 to 3.8, 4.5 to 6.5 and 7.1 to 7.2.

Of particular interest is the range of travel time variability cost of 4.9 to 5.2. From table 5.1 the network wide validation statistics are local optima. This means although the network wide validation statistics have deteriorated when compared with the case where travel time variability cost is zero, the network wide validation statistics have not deteriorated as much as at nearby travel time variability costs. This means the travel time variability cost range of 4.9 to 5.2 is an area where an improved Ngauranga screenline GEH can be found with a lesser deterioration in network wide validation statistics.

Table 5.2 Optimum GEH travel time variability costs at Ngauranga

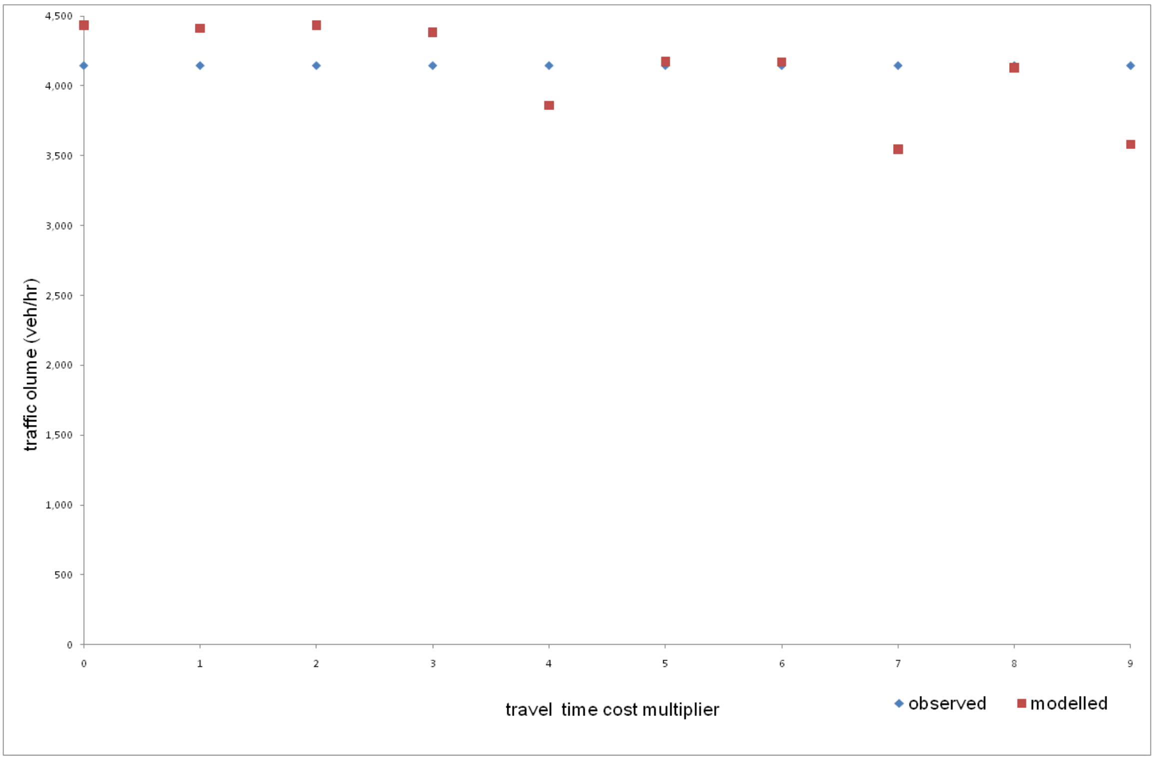
	Ngauranga inbound	Ngauranga outbound
Observed traffic volume	7427	4383
Traffic volume with zero travel time variability cost	8015	4631
GEH with zero travel time variability cost	6.7	3.7
Travel time variability cost multiplier where GEH = 0	2.50 3.71 4.84 6.12 7.16	2.25 3.80 7.15

Figure 5.6 Ngauranga screenline GEH vs travel time reliability cost



Figures 5.7 to 5.9 show the impact of travel time variability cost on the Basin Reserve screenline. Table 5.3 summarises these results from figures 5.7 to 5.9. Estimates of the locations of the points where GEH is zero can be found by fitting a quadratic of the form of equation 5.12.

Figure 5.7 Observed vs modelled traffic volumes at Basin Reserve inbound



Using this approach it is possible to select a travel time variability cost where the GEH value is reduced to zero for each of the inbound and outbound directions separately. Alternatively, there are locations where the GEH value is significantly improved in both directions simultaneously but not reduced to zero. Such a location is a travel time variability cost in the range 3.5 to 3.9. However, this location does not coincide with an area of local improvement in the network wide validation statistics. In the case of the Basin Reserve screenline improving the GEH value by selecting an appropriate value in travel time reliability cost will result in a significant deterioration in network wide validation statistics.

Figure 5.8 Observed vs modelled traffic volumes at Basin Reserve outbound

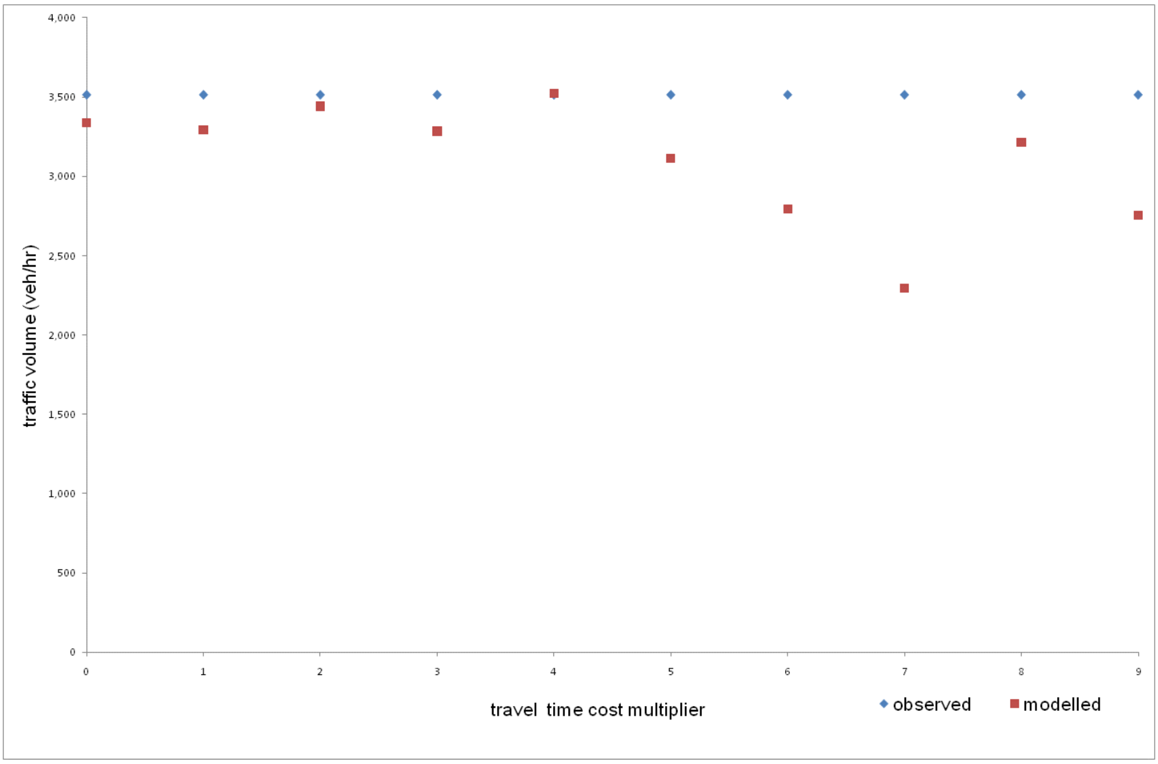
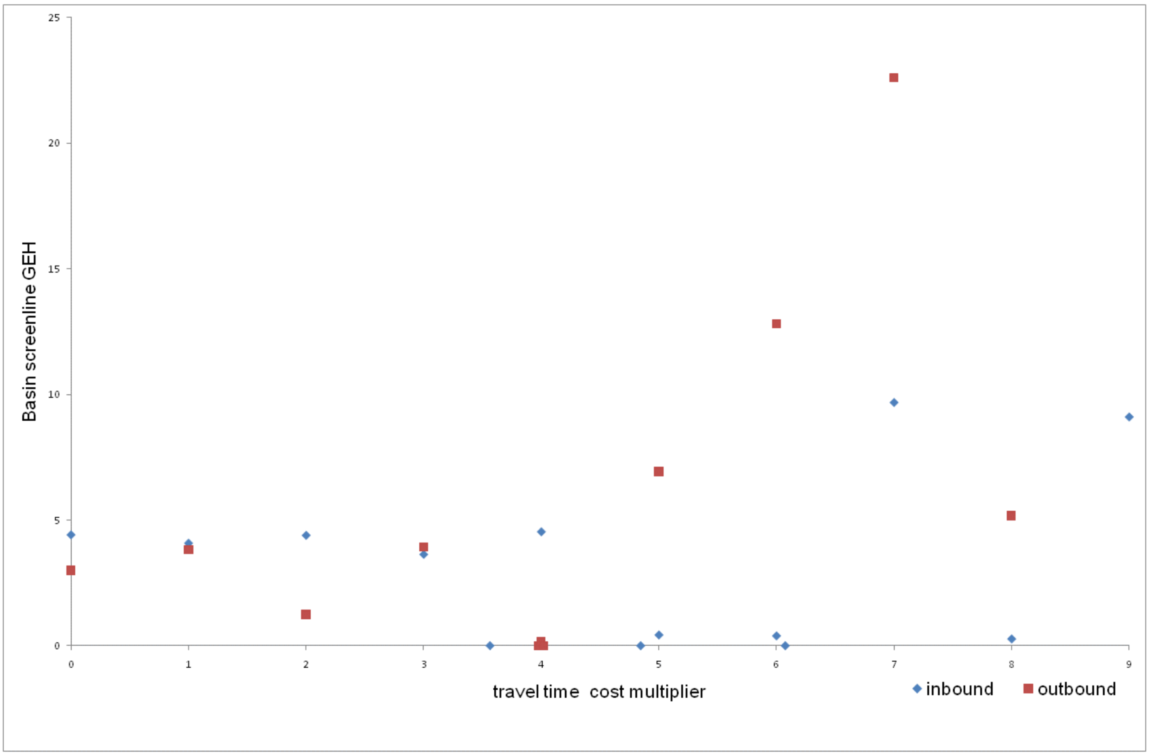


Table 5.3 Optimum GEH travel time variability costs at the Basin Reserve

	Basin inbound	Basin outbound
Observed traffic volume	4146	3514
Traffic volume with zero travel time variability cost (veh/hr)	4435	3338
GEH with zero travel time variability cost	4.4	3.0
Travel time variability cost multiplier where GEH = 0	3.56 4.85 6.07	3.98 4.02

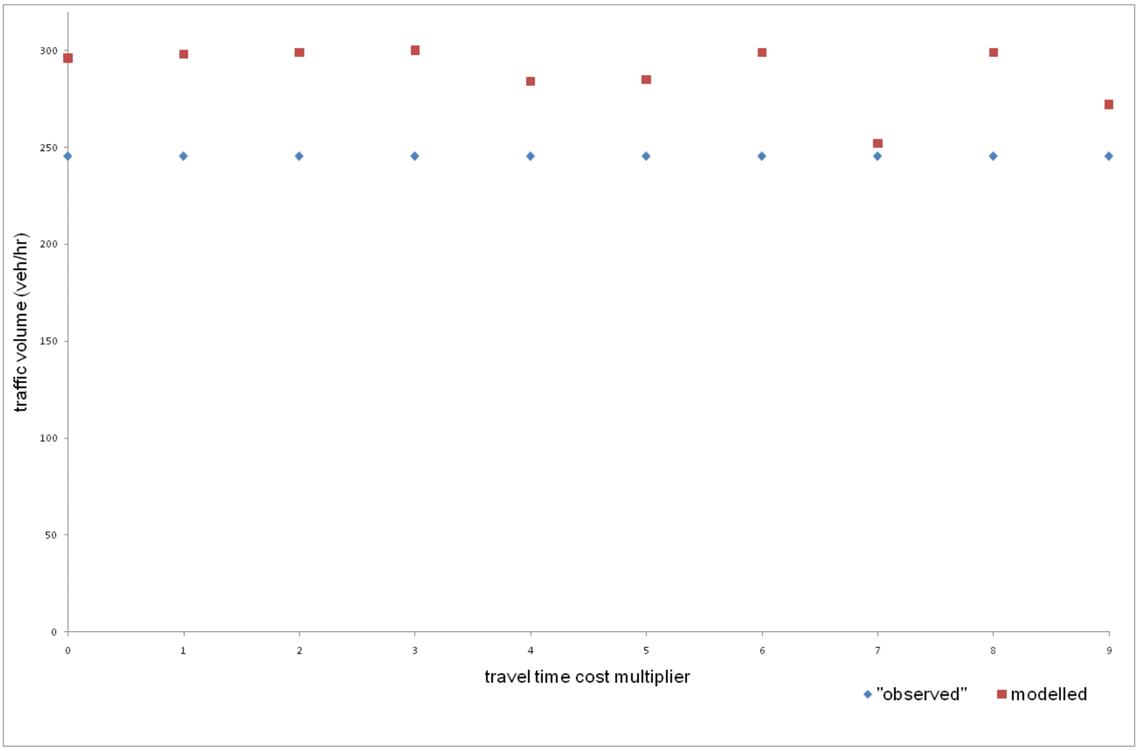
Figure 5.9 Basin Reserve screenline GEH vs travel time reliability cost



Figures 5.10 to 5.12 show the impact of travel time reliability cost on the Basin Reserve screenline again, but this time for heavy commercial vehicles only. It can be seen from figures 5.10 to 5.12 there are no values of travel time reliability cost where the GEH value becomes zero for either the inbound or outbound direction. This is because the modelled traffic volume curve, in each direction, never intersects the observed traffic volume.

However, even though the GEH values never reach zero there are ranges of travel time variability cost where the GEH is improved to be better than the GEH value when the travel time variability cost is zero.

Figure 5.10 Observed vs modelled traffic volumes at Basin Reserve HCV only inbound

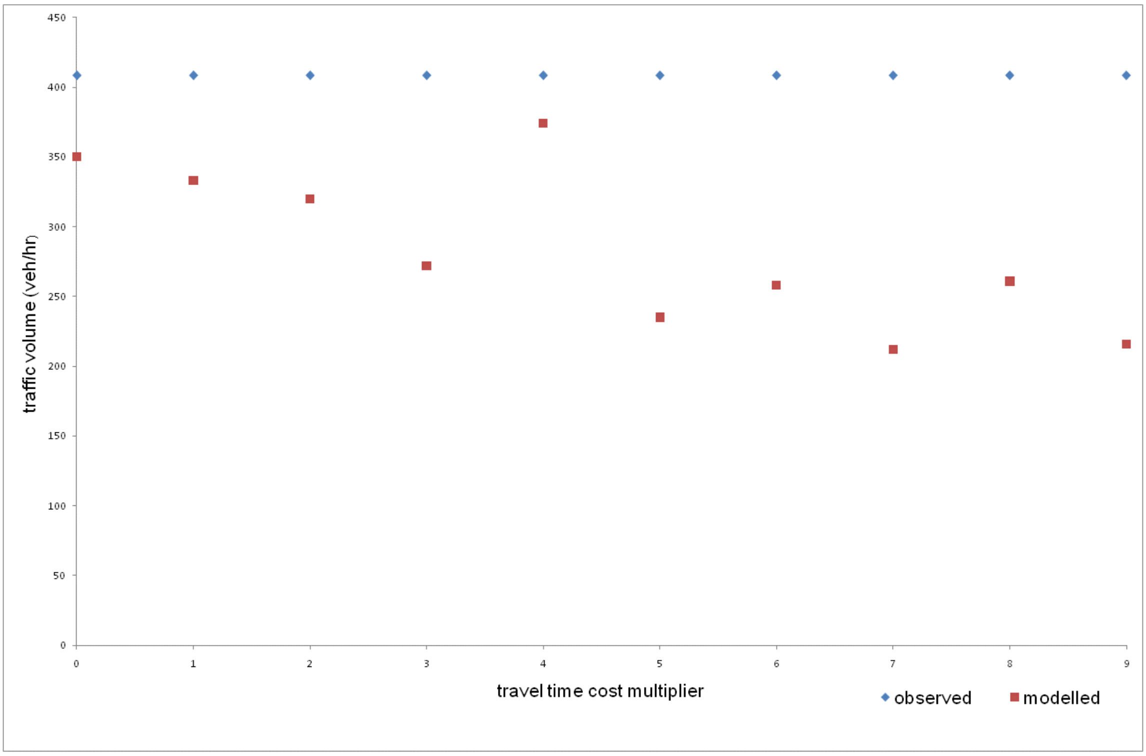


The points that correspond to the best GEH values correspond to the local extreme values where the derivative of the modelled traffic volume is zero. These points can be found using equations 5.12 to 5.14. This data is summarised in table 5.4.

Table 5.4 Optimum GEH travel time variability costs at Basin Reserve HCVs only

	Basin inbound	Basin outbound
Observed traffic volume	245	409
Traffic volume with zero travel time variability cost	296	350
GEH with zero travel time variability cost	3.1	3.0
Travel time variability cost multiplier for optimum GEH	4.42 7.00	3.92
GEH at optimum travel time variability cost multiplier	1.6 0.4	1.2

Figure 5.11 Observed vs modelled traffic volumes at Basin Reserve HCV only outbound



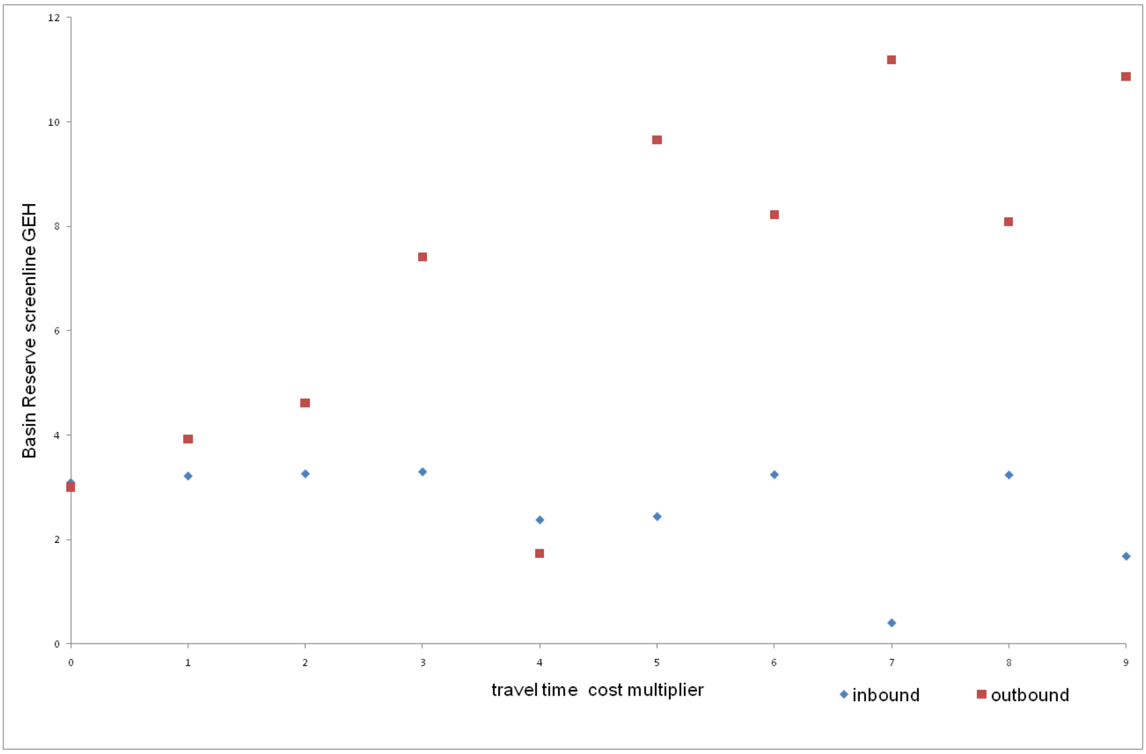
Again it is possible to choose a travel time variability cost that improves the GEH value for the screenline by direction separately. However, there is a range in travel time variability cost, being 3.7 to 4.2, where the GEH in both directions is improved simultaneously compared with a travel time variability cost of zero.

Examination of figures 5.4 to 5.12 and tables 5.2 to 5.4 shows there is a range of travel time variability cost where the GEH for both the Ngauranga and Basin Reserve screenlines and the Basin Reserve screenline for HCVs only are all improved in both directions simultaneously. This range is a travel variability cost of 3.7 to 3.8. However, at this location the global validation statistics have deteriorated significantly. This result suggests the improvement of the GEH values at these locations comes at the expense of the GEH values at other locations on the network.

The values of the travel time variability cost that improved GEH on the screenlines are broadly consistent with the findings of Margiotta (2006) that the travel time variability cost was two to six times the travel time cost.

It would be useful to examine models of other time periods in Wellington and of other cities to see if similar results are found. This could be the subject of further research.

Figure 5.12 Basin Reserve screenline HCV only GEH vs travel time reliability cost

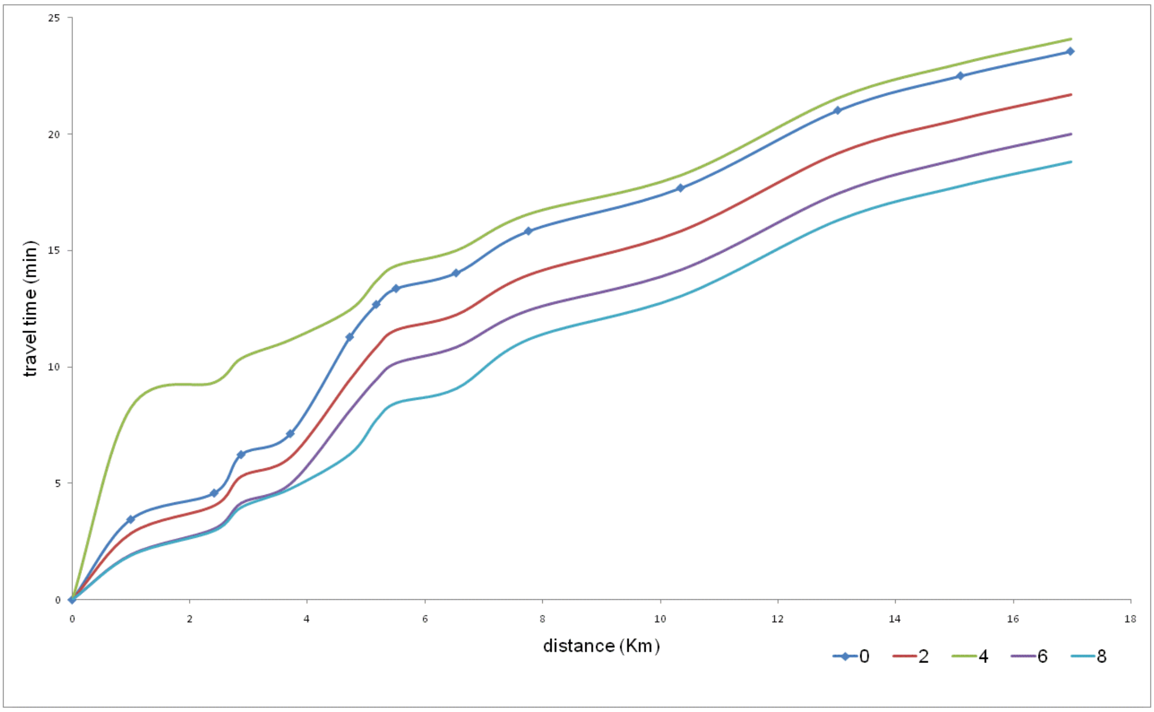


5.7 Impact of travel time variability on route travel time

Three routes were selected to investigate the impact of travel time variability on route travel time. Six examples in each direction were examined by the traffic model for the morning peak period in Wellington. The routes were as follows.

- Calabar Road to Newlands ramps using SH1
- Cobham Drive to Chaffers Street via Evans Bay and Oriental Parades
- Cable Street by Oriental Parade to Newlands ramps on SH1 via Jervois and Aotea Quays.

Figure 5.13 Travel time vs distance for various travel time reliability costs – Calabar to Newlands northbound



Travel times versus route distance for various travel time variability costs expressed as multiples of travel time cost are shown in figures 5.13 to 5.18. Figures 5.13 and 5.14 show the Calabar Road to Newlands ramps route using SH1 route in the northbound and southbound directions. Figures 5.15 and 5.16 show the Cobham Drive to Chaffers Street via Evans Bay and Oriental Parades route in the northbound and southbound directions. Figures 5.17 and 5.18 show the Cable Street by Oriental Parade to Newlands ramps on SH1 via Jervois and Aotea Quays route in the southbound and northbound directions.

Figures 5.13 to 5.18 only show the variation in route travel time for the travel time cost multipliers of 0, 2, 4, 6 and 8 even though the analysis has been undertaken for multipliers of 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. This has been done to improve the clarity of figures 5.13 to 5.18.

Examination of figures 5.13 to 5.18 show significant variation in travel times versus route distance except for figures 5.16 and 5.18. Figures 5.16 and 5.18 show little variation in travel time versus route distance for various travel time multipliers.

Figure 5.14 Travel time vs distance for various travel time reliability costs – Calabar to Newlands southbound

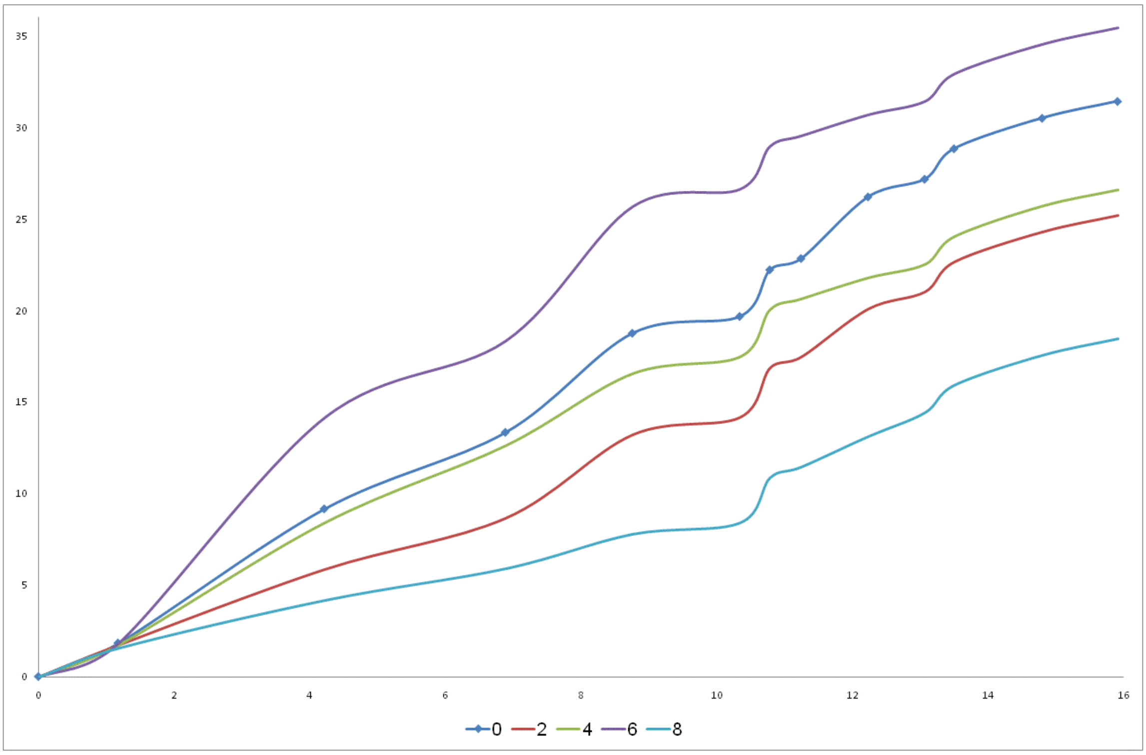


Table 5.5 shows the difference between the maximum and minimum route travel time for the travel time multipliers from 0 to 9, the mean route travel time and their ratio. These are provided as measures of variation in route travel time as a result of including travel time variability.

Table 5.5 Variation in route travel time resulting from travel time variability

Route	Travel time range (minutes)	Mean travel time (minutes)	Range/mean
Calabar-Newlands NB	8.5	22.3	0.382
Calabar-Newlands SB	19.9	27.5	0.725
Cobham-Chaffers NB	1.7	8.7	0.197
Cobham-Chaffers SB	0.2	8.4	0.018
Cable-Newlands SB	11.6	15.8	0.737
Cable-Newlands NB	0.6	10.6	0.056

Figure 5.15 Travel time vs distance for various travel time reliability costs – Cobham to Chaffers northbound

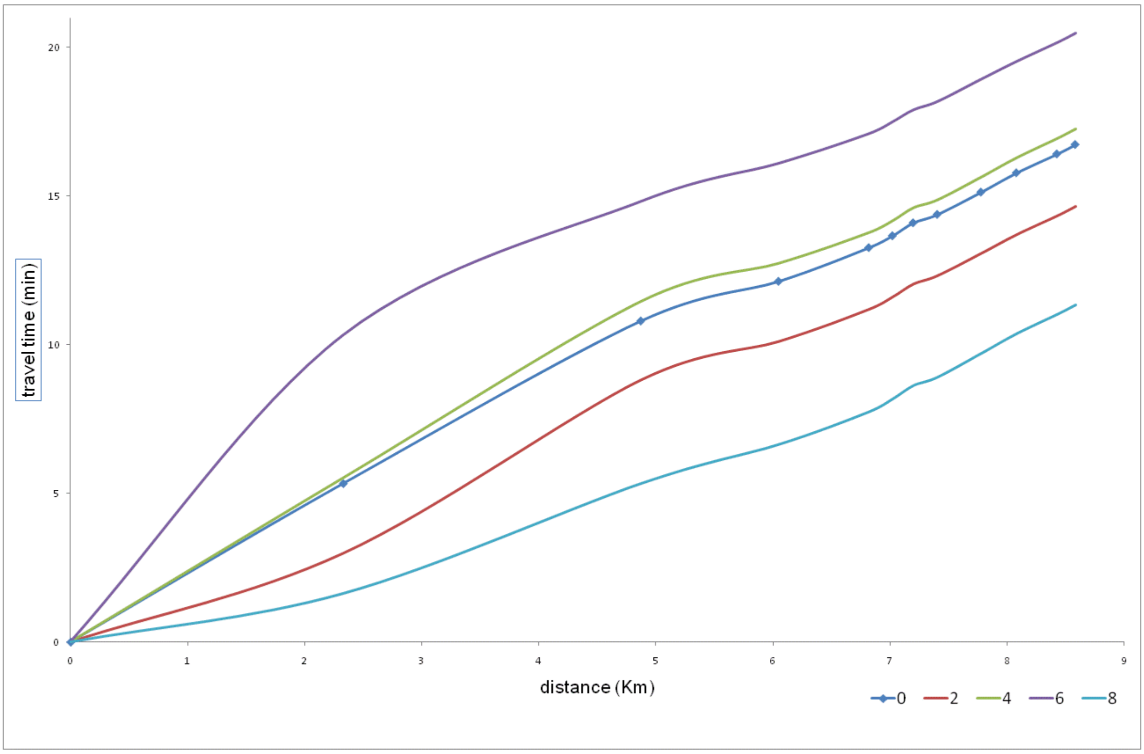


Table 5.5 presents measures of route travel time variation in response to travel time variability cost. All routes, with the exception of Chaffers Street to Cobham Drive southbound and Cable Street to Newlands northbound, show significant variation in route travel time in response to travel time variability cost. Cobham Drive to Chaffers Street southbound and Cable Street to Newlands northbound show significantly smaller travel time range and the ratio of travel time range to mean travel time ratios compared with the other routes. Table 5.5 is consistent with the patterns observed in figures 5.13 to 5.18.

Figure 5.16 Travel time vs distance for various travel time reliability costs – Cobham to Chaffers southbound

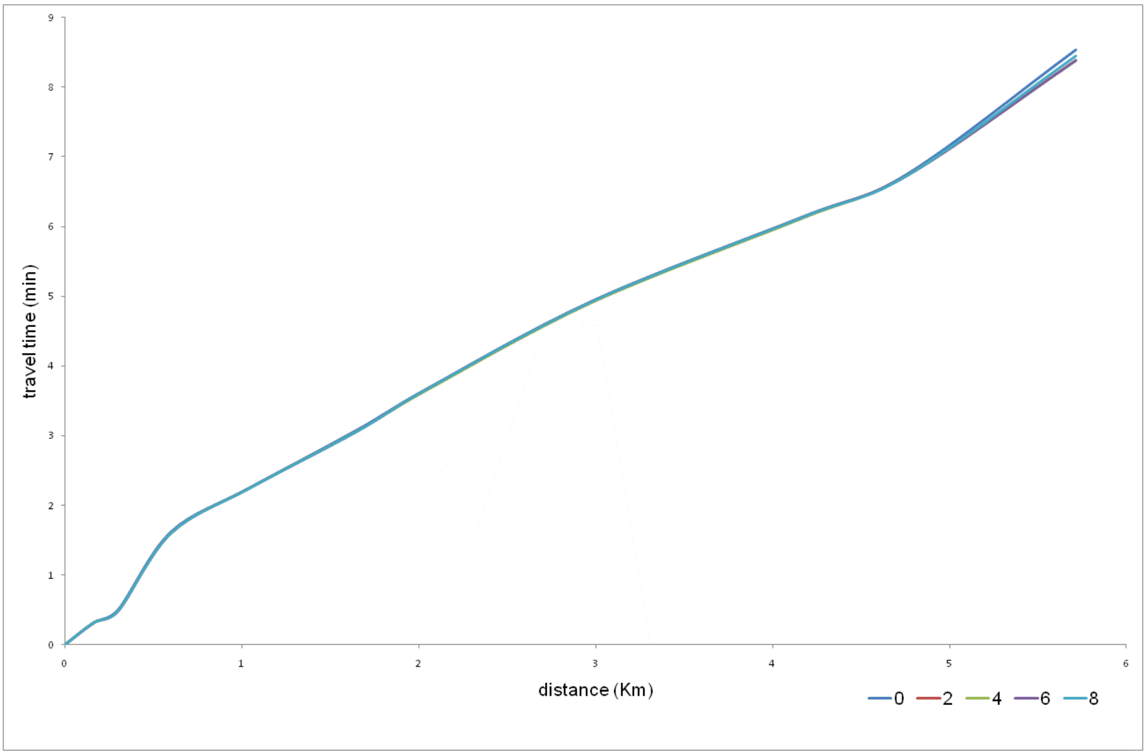


Table 5.6 shows traffic volumes and broad volume to capacity ratios at various locations on the Wellington network. This is to give some measure of how congested these routes are in order to understand whether this impacts on the spread of the time versus distance curves due to travel time variability cost.

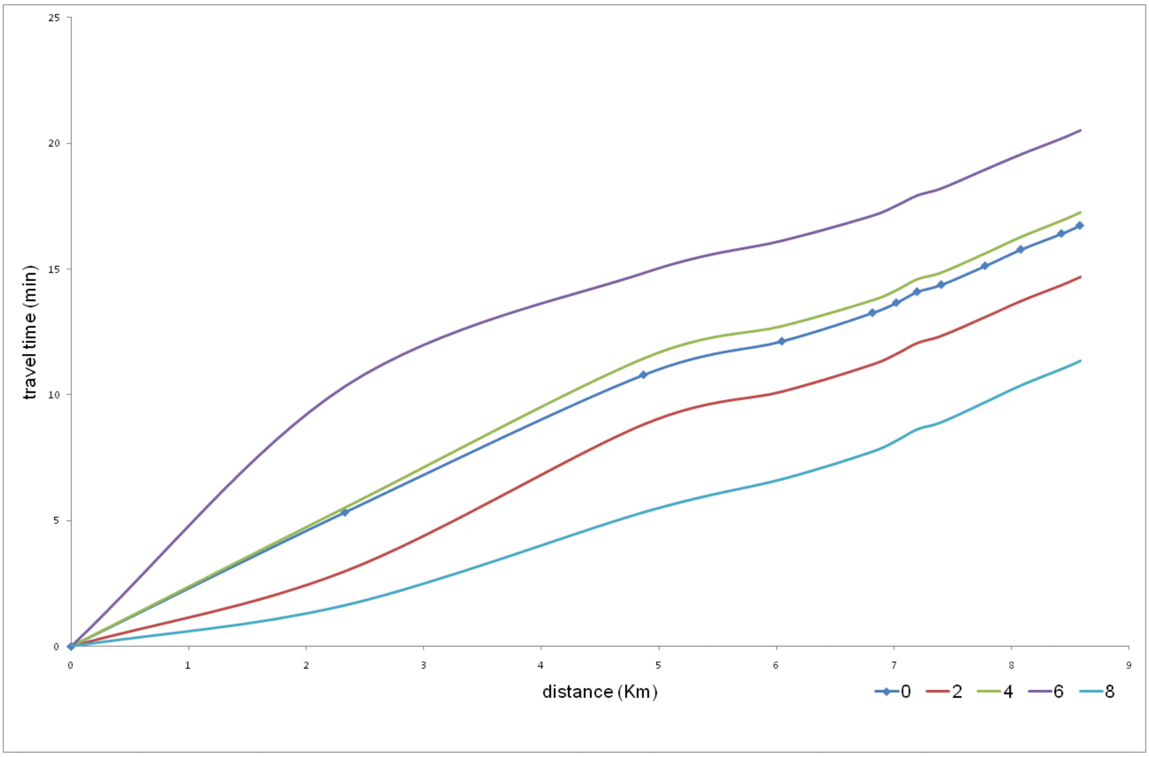
Table 5.6 Traffic volumes and broad volume to capacity measures

	AADT 2009	Lanes	AADT/lanes	Indicative V/C
Oriental/Evans	13,700	2	6850	0.42
Calabar	22,910	2	11,453	0.70
Cobham	34,520	4	8629	0.53
Ruahine	34,370	2	17,185	1.05
Paterson	38,960	2	19,480	1.19
SH1 (SH2-Aotea)	87,130	6	14,522	0.89
SH1 (Newlands)	71,830	6	11,972	0.73

AADT = annual average daily count

V/C = traffic volume to capacity ratio

Figure 5.17 Travel time vs distance for various travel time reliability costs – Cable to Newlands northbound



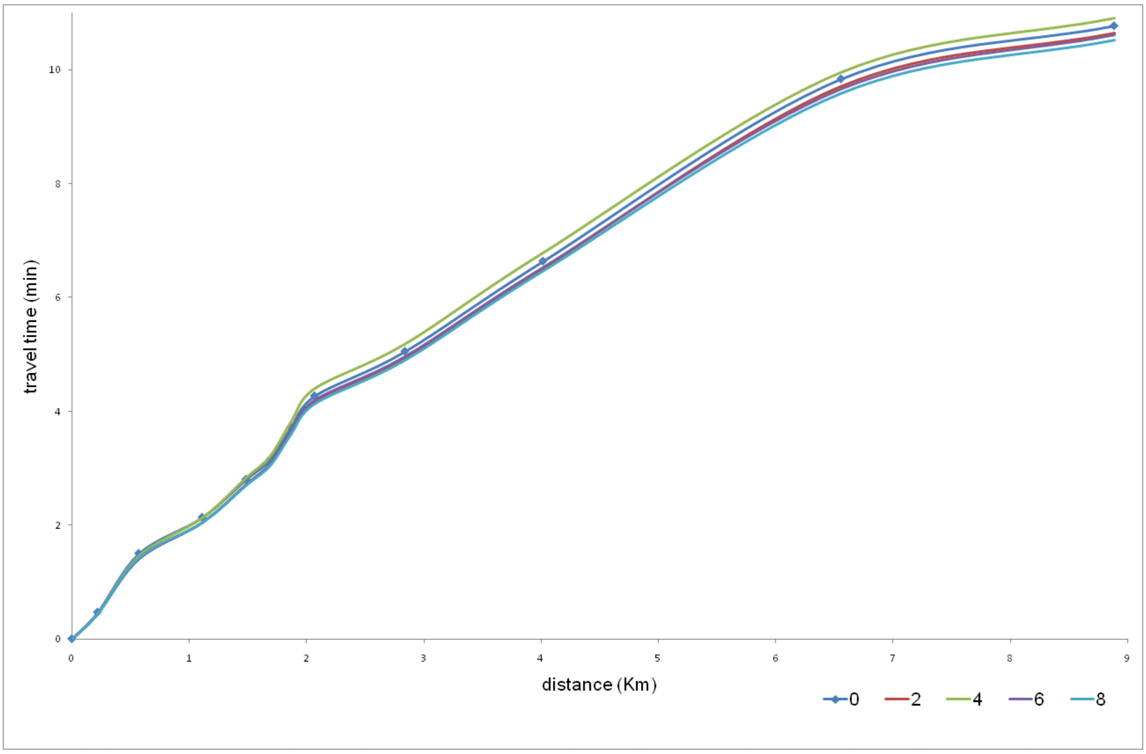
A broad estimate of the volume to capacity ratios at a selection of locations on the Wellington network good enough for relative comparisons is provided in the fifth column of table 5.6. This estimate has been obtained by taking the annual average daily traffic (AADT) measured in 2009 and divided by the total number of lanes on the road at that location. This is converted in a morning peak-hour volume to capacity ratio by assuming:

- the morning peak is approximately 11% of the daily traffic
- each lane is equally used
- lane capacity is 1800 vehicles per hour.

These indicative volume capacity ratios indicate Oriental and Evans Bay Parades and Cobham Drive are under little pressure, whereas Calabar Road and SH1 at Newlands are under moderate pressure, and Ruahine Street, Paterson Street and SH1 from the SH2 merge to the Aotea ramps experience congestion. No allowance for the difference in the peak direction traffic volume compared with the counter peak direction traffic volume during the morning peak has been made in this analysis.

From table 5.5, Cobham Drive to Chaffers Street southbound and Cable Street to Newlands northbound show significantly smaller travel time range and the ratio of travel time range to mean travel time ratios compared with the other routes. The Cobham Drive to Chaffers Street route is largely along Evans Bay Parade and Oriental Parade which from table 5.6 has a low indicative volume to capacity ratio.

Figure 5.18 Travel time vs distance for various travel time reliability costs – Cable Street to Newlands southbound



The Cable Street to Newlands northbound route uses the Wellington waterfront route and SH1 (SH2-Aotea) and SH1 (Newlands). It should be noted that the northbound route during the morning peak is in the counter peak direction and the nominal lane capacity of 1800 vehicles per hour is on the low side. This means the indicative volume to capacity ratios for SH1 in table 5.6 are likely to be on the high side. Hence it is concluded the real volume to capacity ratio is likely to be low.

The remaining routes in table 5.5 have road links that are likely to have high volume to capacity ratios using the data in table 5.6. These routes show significant variation in route travel time as a function of travel time variability cost.

This result is consistent with the travel time variability model outlined in equations 5.3 and 5.4. This result suggests the modelling approach using equation 5.6 to mimic the effect of travel time variability is behaving correctly.

5.8 Summary

A number of findings can be summarised from the analysis undertaken above. These include the following:

- The formulation of the standard deviation of travel time developed in this project based on a simple function of the travel time index, CI_t, appeared to be convenient for analysing the impact of travel time reliability in traffic assignment models.
- There are a number of models where traffic assignment includes the cost of travel time variability directly in the generalised cost function. These models offer a less cumbersome incorporation of travel time variability as they do it directly and are calibrated explicitly with travel time variability.

- Models, such as those based on a SATURN software platform and others, do not include travel time variability directly in the generalised cost function. By using an iterative approach these models are amenable to having their generalised cost function mimic travel time variability. Examination of the literature has not revealed any case studies where a similar approach has been attempted.
- The iterative approach used to mimic travel time variability appears to be numerically stable and converges quickly. This enables the effects of travel time variability to be modelled with a reasonable level of convenience and without requiring undue time.
- Simple numerical techniques can be used to estimate optimal travel time variability costs from a range of travel time variability costs.
- The incorporation of travel time variability into an assignment model did not improve network wide validation statistics in the case study undertaken. However, the model used already had a high degree of model validation. Examination of local maximisation of the R^2 statistic and the minimisation of the %RMSE and $RMGEH^2$ statistics across the network versus travel time variability cost showed the optimal travel time variability cost was in the range of 4.9 to 5.1 times the travel time cost.
- This was broadly consistent with the findings of Margiotta (2006) which is the only example found in the literature that provides guidance on the value of the travel time variability cost. The NZTA's (2010) *Economic evaluation manual* does not provide guidance on the value of the travel time variability cost. This was also the experience of Vincent (2008).
- The incorporation of travel time variability into an assignment model did improve the GEH values of some selected key screenlines compared with when the travel time variability cost was zero. On some screenlines it was possible to achieve a GEH value of zero in a particular direction which represents a perfect match of modelled and observed traffic volume
- It was possible to achieve an improvement in the GEH value simultaneously in both directions. This was achieved on all selected screenlines and produced a non zero GEH value in each direction whose value was an improvement on the GEH value when the travel time variability cost was zero.
- A travel time variability cost in the range of 3.7 to 3.8 was found to improve the GEH values of all three selected screenlines in both directions simultaneously. However, with those values the network wide validation statistics were worse than those for a travel time variability cost of zero.
- A travel time variability cost in the range of 4.9 to 5.2 was found to improve the GEH values for the Ngauranga screenline for both directions simultaneously and this coincided with the local optima of the network validation statistics. This represented a region of improved matching of traffic volumes at the Ngauranga screenline and where the deterioration in the network wide validation statistics was less significant. This was broadly consistent with the findings of Margiotta (2006).
- The examination of route travel times in response to travel time variability cost appeared to demonstrate the correct attributes using the modelling approach advocated in this report. This requires routes with low volume to capacity ratios to have travel times that are not sensitive to travel time variability cost whereas routes with higher volume to capacity ratios have travel times that are sensitive to travel time variability cost.

6 Conclusion

This study successfully achieved most of its objectives. The overall objective of this study identified in the introduction to this report was:

> *... to establish whether a convenient and robust methodology was available for the incorporation of travel time reliability in route choice algorithms of conventional assignment models.*

In order to achieve this overall objective four tasks were undertaken. Each of these tasks had a sub-objective designed to support the overall study objective.

The first task of the research was to survey the international literature on the subject of travel time reliability. This task successfully enabled:

- an understanding of how travel time reliability influenced driver's route choice
- documentation of international experience involving travel time reliability
- determination of a convenient means to mathematically describe travel time reliability
- identification of known functional relationships of travel time reliability with other variables.

A second task of this study was to analyse a New Zealand travel time survey data set. This analysis successfully:

- examined the usefulness of various mathematical descriptions of travel time reliability
- tested the validity of the functional relationships found in the literature review with New Zealand data
- determined a convenient formulation of travel time reliability for inclusion in assignment models.

A third task of this study was to gain insights about the effect of incorporating travel time reliability on traffic volume assignment and basic validation statistics with a simple synthetic network. This task was successfully completed and showed if optimal values of travel time variability cost existed, these could be found using numerical techniques. This task also showed the different responses that could be expected depending on the assignment routine and the shape of the trip matrix.

The final task of this study was to apply the methodology for incorporating travel time reliability to a real conventional assignment model. This showed that:

- incorporating reliability was feasible
- the proposed methodology was not unduly onerous and was numerically stable
- the network wide validation statistics did not improve although there were ranges of travel time variability cost where the deterioration in the network wide validation statistics was not as great
- it was possible to improve the matching of modelled and observed traffic volumes on individual or groups of screenlines but at the cost of worsening the overall network validation
- travel time reliability had a differential impact on route assignment depending on congestion levels.

Consequently, this study achieved its objectives except it was unable to show that the incorporation of travel time reliability would improve network wide validation in the particular case study undertaken. Nevertheless, the inclusion of travel time reliability in assignment models is thought to be useful as it does provide a means and rationale for improving the matching of modelled and observed traffic volumes on individual and groups of screenlines.

7 Recommendations

From the investigations undertaken as part of this research a number of recommendations can be made in relation to considerations regarding travel time reliability in transport planning and applications to traffic modelling. These recommendations include the following:

- The preferred measure of travel time variability should be the standard deviation of travel time.
- The standard deviation of travel time can be conveniently estimated from the coefficient of variation of travel time, which is the ratio of the standard deviation and the mean travel time, using simple mathematical relationships.
- Where estimation of the coefficient of travel time is being considered, simple functions of the ratio of mean travel time to free-flow travel time should be investigated.
- To improve the match of modelled traffic volumes and observed traffic volumes on an individual or group of screenlines, consider the incorporation of travel time reliability into an assignment model but note that this can also lead to worsening of the overall network validation.
- To include travel time reliability in an assignment model, the travel time variability cost should be set at a value between two to six times the travel time cost.

It is recommended further work be undertaken in the area of travel time reliability and its incorporation into assignment models. Several areas for further research may prove useful. These include:

- extension to other time periods for the Wellington case study – in particular the evening peak model for Wellington (without travel time variability incorporated) has assignment convergence difficulties. It would be interesting to see if travel time variability has an impact on convergence
- application to a model of a city other than Wellington
- recommendations immediately above could specifically consider whether the travel time variability cost that optimises link GEH values falls into the same range found in this research or is generally within the range found by Margiotta (2006)
- application of travel time variability to a model without a high degree of validation. A variety of tests could be undertaken to see which stage in the validation process travel time variability should be incorporated, ie from the beginning, pre matrix estimation or post matrix estimation
- further investigation of the optimal travel time variability cost for HCVs
- undertaking investigation of the independence of the variances of travel time on links in a network to consider whether the magnitude of the sum of the correlations is small compared with the sum of the variances which will highlight the validity or otherwise of the simplifications assumed in this study.

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Appendix A: Travel time survey data summary

Link	Direction	Time of day	Mean journey time (sec)	Journey time standard deviation (sec)	Maximum journey time (sec)	Minimum journey time (sec)
Waikanae – Whitford	SBD	AM	1854.5	53.57	1941	1805
		IP	1775.8	31.31	1798	1721
		PM	1877.4	51.89	1950	1823
	NBD	AM	1820.6	72.47	1916	1747
		IP	1789.2	40.15	1855	1752
		PM	2385.8	752.71	3716	1885
Whitford – Ghuznee	SBD	AM	1678.4	136.33	1884	1534
		IP	885.8	22.87	916	859
		PM	917.73	46.98	961	859
	NBD	AM	890.5	58.34	984	833
		IP	883.9	49.41	969	842
		PM	1060.4	154.39	1318	907
Ghuznee – Airport	EBD	AM	583.4	60.57	647	515
		IP	569.7	69.53	685	513
		PM	778.7	210.44	1120	600
	WBD	AM	686.1	240.8	1080	486
		IP	588.7	46.92	656	543
		PM	840.3	206.71	1168	634
Fergusson – Dowse	SBD	AM	1193.4	107.60	1333	1044
		IP	1077.3	36.22	1109	1035
		PM	1118.0	94.73	1219	992
	NBD	AM	1033.3	23.83	1069	1055
		IP	1057.3	67.14	1156	993
		PM	1115.9	92.83	1244	987
Dowse – Ghuznee	SBD	AM	1209.6	241.96	1456	819
		IP	575.0	21.29	608	548
		PM	792.3	318.19	1357	593
	NBD	AM	576.1	26.81	602	536
		IP	553.9	20.50	577	528
		PM	720.2	163.6	1004	601
Mungavin – Seaview	EBD	AM	1872.9	115.17	2023	1703
		IP	1729.3	22.26	1759	1704
		PM	2032.8	280.85	2424	1813
	WBD	AM	1927.4	91.08	2037	1787
		IP	1745.1	32.81	1772	1707

Link	Direction	Time of day	Mean journey time (sec)	Journey time standard deviation (sec)	Maximum journey time (sec)	Minimum journey time (sec)
		PM	1936.3	213.21	2298	1668
City – Island Bay	SBD	AM	754.3	61.51	847	683
		IP	819.3	101.81	993	743
		PM	1033.0	134.02	1151	809
	NBD	AM	984.4	185.51	1279	817
		IP	812.2	38.03	876	780
		PM	1053.4	158.09	1296	901
Featherston – Upper Hutt	SBD	AM	1739.6	60.35	1823	1679
		IP	1838.3	121.73	2043	1719
		PM	1852.4	152.87	2116	1730
	NBD	AM	1799.8	43.26	1859	1737
		IP	1674.0	97.15	1788	1551
		PM	1740.7	107.12	1912	1649
City – Karori	WBD	AM	672.5	61.27	756	595
		IP	628.1	30.31	680	600
		PM	869.8	260.58	1285	643
	EBD	AM	805.6	325.09	1367	610
		IP	650.1	27.85	680	615
		PM	721.5	76.23	841	637
Woburn – city	SBD	AM	1483.8	87.73	1633	1415
		IP	873.5	28.98	916	842
		PM	1027.1	185.72	1321	847
	NBD	AM	926.6	80.94	1038	845
		IP	811.2	43.91	879	775
		PM	1280.1	142.59	1459	1123
Seaview – Petone	WBD	AM	359.0	42.86	410	292
		IP	304.6	29.10	351	272
		PM	309.9	25.07	338	281
	EBD	AM	350.8	30.75	385	307
		IP	350.3	15.42	364	325
		PM	435.1	115.07	597	344

Appendix B: Validation experiments

Table B.1 Observed link flows

	E1	E2	E3	E4	E5	E6	E7	E8
Matrix	T_1	T_1	T_2	T_2	T_1	T_1	T_2	T_2.
Assign	AN	AN	AN	AN	Logit	Logit	Logit	Logit
1-2	2112	1727	2376	2372	2139	1719	2368	2368
1-4	873	475	1799	1803	841	481	1807	1807
2-1	2103	1760	2276	2068	2903	1770	2068	2281
2-3	2027	1177	2501	2497	2059	1519	2493	2493
2-4	2200	3448	2500	2500	2200	3100	2500	2500
3-2	1952	726	2400	2193	1943	720	2193	2406
3-4	1873	969	1925	2132	1883	1305	2132	1919
4-1	881	442	1894	2102	888	430	2102	1889
4-2	2317	3983	2501	2500	2320	4000	2500	2500
4-3	1747	467	1725	1728	1716	456	1732	1732

Key

E1, E2, experiment code

T_1, T_2 test matrix defined by equations 4.4 and 4.5

AN all or nothing assignment

Logit logit assignment

1-2, 1-4, road link from figure 4.1

2112, 873 observed link traffic volume

Appendix C: Quadratic fitting

Formulae are developed to fit a quadratic curve of the form

$$f = ax^2 + bx + c \qquad \text{(Equation C.1)}$$

through three points being (x_1,f_1), (x_2,f_2) and (x_3,f_3). This gives the set of three simultaneous equations

$$\begin{aligned} ax_1^2 + bx_1 + c &= f_1 \qquad \text{(Equation C.2)} \\ ax_2^2 + bx_2 + c &= f_2 \\ ax_3^2 + bx_3 + c &= f_3 \end{aligned}$$

with three unknown quantities a, b and c which we seek to solve for. This means the system of equations C.2 is solvable provided the determinant

$$\begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix} \neq 0 \qquad \text{(Equation C.3)}$$

In the system of equations C.2, if the second and the third equations are each separately subtracted from the first equation and the first equation is retained we obtain

$$\begin{aligned} ax_1^2 + bx_1 + c &= f_1 \qquad \text{(Equation C.4)} \\ a(x_1^2 - x_2^2) + b(x_1 - x_2) &= f_1 - f_2 \\ a(x_1^2 - x_3^2) + b(x_1 - x_3) &= f_1 - f_3 \end{aligned}$$

Using equations C.4, if the first two equations are retained and $(x_1 - x_2)$ times the third equation is subtracted from $(x_1 - x_3)$ times the second equation we obtain:

$$\begin{aligned} ax_1^2 + bx_1 + c &= f_1 \qquad \text{(Equation C.5)} \\ a(x_1^2 - x_2^2) + b(x_1 - x_2) &= f_1 - f_2 \\ [(x_1 - x_3)(x_1^2 - x_2^2) - (x_1 - x_2)(x_1^2 - x_3^2)] &= (x_1 - x_3)(f_1 - f_2) - (x_1 - x_2)(f_1 - f_3) \end{aligned}$$

The system of equations C.5 allows us to solve for a, b and c.

$$a = -\omega/\Delta \qquad \text{(Equation C.6)}$$

where

$$\omega = (x_1 - x_3)(f_1 - f_2) - (x_1 - x_2)(f_1 - f_3)$$

$$\Delta = (x_1 - x_3)(x_1^2 - x_2^2) - (x_1 - x_2)(x_1^2 - x_3^2)$$

$$b = [f_1 - f_2 - a(x_1^2 - x_2^2)]/(x_1 - x_2) \qquad \text{(Equation C.7)}$$

$$c = f_1 - ax_1^2 - bx_1 \qquad \text{(Equation C.8)}$$

It is noted that equations C.6 to C.8 cannot be used if Δ is zero which is identical to equation C.3.

To find a improved value of x where $df/dx \approx 0$, a local optimum value then we can use $f = ax^2 + bx + c$ where:

$$df/dx = 2ax + b \qquad \text{(Equation C.9)}$$

then we need to solve:

$$2ax_{imp} + b = 0 \qquad \text{(Equation C.10)}$$

or

$$x_{imp} = -b/2a \qquad \text{(Equation C.11)}$$

This result is independent of c.