Approaches to estimating regional input-output tables
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Input-output tables are a statistical summary of the flow of industry inputs into production and the subsequent use of that output – the focus is on the mechanisms of production. Regional input-output tables extend this, allowing regional differences in production and use – the focus is on both the technology of production and trade.

Accurate assessments of input-output tables (national or regional) require extensive and expensive surveying to acquire data. Regional tables are more effort to produce and require more surveying than national tables. New Zealand produces national tables every five years.

Because of the expense in producing tables from survey data, a number of methodologies have been developed to estimate national or regional input-output tables using a previously developed national table as a basis.

The commonly adopted approaches to estimate regional input-output tables strongly rely on 'bi-proportional balancing' methods for updating or regionalising national level input-output tables. The methods scale the rows and columns of the input-output table in order to satisfy a set of constraints, such as the column sums giving the total outputs. Recent results unify these approaches into a Bayesian optimisation problem. This formulation allows greater flexibility in application as it can be applied when the available data is relatively sparse, and it provides estimates of the uncertainty of the table's entries.

The statistical data New Zealand collects about production, consumption and trade is not sufficient to employ the commonly adopted approaches to producing regional input-output tables (that is, input-output tables that include flows of goods and services between regions and industries).

This paper presents a methodology for producing the most general form of regional input-output table with the data available, with error estimates. This methodology is novel. It is a Bayesian approach that requires 'best guesses' (priors) to be established for inter-regional trade and industry production technologies where those priors are updated in order to satisfy constraints identified from actual data.

The optimisation problem forming the core of the methodology requires constraints on the flows of intermediate inputs between industries and regions. We adapt a technique created to deal with 'cross-hauling' (concurrent import and export of the same goods) to identify constraints on bi-lateral regional trade. This approach is novel.

To employ the method will require:

- a data set to be assembled from Statistics NZ data and possibly the Longitudinal Business Database. This assembly should be designed to be automated, so updates of the tables can be done without great effort
- the development of a 'gravity model' of trade to establish a best guess for inter-regional trade proportions in New Zealand. Gravity models are well founded in economic theory and could be fitted using data from regional input-output models developed overseas
- a solution to the optimisation problem, again as a script that can be re-run without great effort.

Use

Once built, the regional input-output model could be made available as a public resource and automatically refreshed annually or quarterly.

The error estimates will provide intelligence on where effort might be needed to improve the understanding of regional trade and regional production technology.

Third-party data

We examined four sources of data that might be useful for improving the table's accuracy: Xero, eRUC, Marketview and Qrious data. In each case, there was significant work to be done to make the data useable for improving the regional input-output model. A key challenge in making use of eRUC and Qrious data is that the data does not describe monetised units, which is what is needed for input-output development.

Xero data was the most promising of the four third-party data sources, but it was probably not representative of the business population, being highly skewed to small-to-medium sized enterprises and 'not economically significant' businesses.

Marketview data was the most useful for estimating household consumption.

Application to transport

Regional input-output tables can be used to assess the economic impact of transportation on the economy in a number of ways. Since transportation input is accounted for as a cost, the amount of value added and 'absorbed' by a transportation industry can be used as an index of transportation efficiency – low amounts when trade is high indicate a reduced transport burden.

The indirect impact of transport can be measured by understanding how important a region is to the trade in value added to other regions. Important regions that are inefficient are a drag on the economy and targets for investment.

Regional input-output tables can be analysed to show the inter-dependencies between transport industries in the various regions, an understanding of which is useful for regional transport industry development.

Abstract

A methodology was developed with which to produce regional input-output tables for New Zealand. The methodology provides estimates of uncertainty for the entries of the table, allows the incorporation of third-party data, and makes best use of available data, be that official statistics, third-party data, or subject matter expertise.

The method contains a novel approach to estimating regional trade, allowing for cross-hauling.

In addition to the methodology for estimating regional input-output tables, this work develops indicators of the economic impact of transport and transportation industries using information available in regional input-output tables. The methodology is applied to the World Input-Output Database for the year 2000.

The work took place between July and December 2016.

1 Introduction

The development of input-output tables and analysis was pioneered by Leontief, work for which he received a Nobel Memorial Prize in Economics. It built on Stone's work on national accounts to provide a view of what role intermediate inputs play in final production.

Isard is credited with the development of regional input-output tables. Whereas national input-output tables are about giving greater granularity to the national accounts through an understanding of the technology of production, regional input-output tables are about regional variation in that technology and an understanding of the role of inter-regional trade in an economy.

Building national level input-output tables is highly resource intensive, requiring extensive surveying and data examination. It is generally not feasible to extend the approach to a regional level (though countries have done so, such as Finland and Japan), so that one of the central issues in developing regional input-output tables is how to use the data available (including the national level table and survey data and so-called 'superior' data) to estimate inter-regional trade and the regional variations in production technology.

The collection of techniques varies, from ad-hoc and unsupported by theory, through to complex, theory-driven models. Which ones can be applied to a given situation is largely up to what data is available and what purpose the regional input-output table would be put to.

We begin by surveying the literature to understand the types of regional input-output table, and the methods employed to estimate them. Each of these methods requires data, and the methods that might be employed in developing New Zealand regional input-output tables are constrained by the data that New Zealand collects.

Having surveyed the literature, we then take stock of what data New Zealand collects and proceed to develop a methodology that makes best use of this data, however patchy it might be. By 'best use' we mean a methodology that builds tables to any desired level of granularity, allows the use of data from a variety of sources and which provides error estimates of the table entries. For this reason, we immediately discard approaches that do not have these qualities.

Official statistical data is not the only data that might be useful for creating regional input-output tables. We take the view that a regional input-output table is both a set of accounts, giving amounts of inputs and amounts of production and consumption, and also a description of the trade, production, and consumption dynamic that is the New Zealand economy. Data that describes systems which are influenced by that dynamic can be statistically modelled in terms of any regional input-output table that we developed using the official statistical data – if the data can be made representative of the business population, then the results of the model can be used to improve the accuracy of the regional input-output table. We discuss this approach for four sources of data: Xero, Qrious, eRUC and Marketview. The case of Xero is most likely to be useful and it is that case that we most highly detail.

We conclude by developing a number of methods for using regional input-output tables to assess the economic impact of transportation. These methods are illustrated using tables from the World Input-Output Database (WIOD) (Dietzenbacher et al 2013).

2 Input- output tables

An (industry-by-industry)[1] input-output table (IOT) is an empirical description of an area's economy over a period in terms of:

- a breakdown of productive entities into industrial units
- the flow of monetised intermediate inputs between industries in the period in question
- the total output of each industry in the period
- the value added in production by each industry, often broken down into labour costs, consumption of capital, and operating surplus
- the final demand for each industry's production, including exports, household consumption, government consumption and capital investment
- the use of imports by industries and in directly meeting final demand.

The areas in question are typically countries and the period a calendar year; often an IOT represents a national economy over that nation's tax year.

Monetary values are typically in either purchasers' prices, producers' prices, or basic prices. Producers' prices differ from basic prices by net taxes on production; purchasers' prices differ from producers' prices by the cost of transport and the retail margin; value-added tax is removed in either case. Thus, when a table is in basic prices, the explicit transport margins have been removed from transactions and accounted for as a transaction between the purchasing industry and the transport industry. Similarly for retailing or wholesaling costs, basic prices represent the money the producer receives from a purchase before taxes or subsidies on production, whereas the purchaser's prices represent the money the purchaser pays, and the producer's prices are what the producer receives. The table below illustrates:

Table 2.1 Differences between purchasers', producers' and basic prices

Price	Production costs	Taxes on production	Transport, retail or wholesale costs
Purchasers'	Included	Included	Included
Producers'	Included	Included	Not included
Basic	Included	Not included	Not Included

Some industrial units produce in several industries. Such units will have a primary industry and (possibly several) secondary industries; the production that belongs to the secondary industries is called *secondary production*. In some instances, an IOT will account for secondary production by re-allocating the secondary production to the appropriate industry, so the IOT more accurately represents the use of goods and services in production, rather than representing the transfer of inputs between collections of businesses or consumers. Since 2007 New Zealand has not accounted for secondary production in this way, with the exception being transport, retail trade and wholesale trade services.

[1] There are also product-by-product input-output tables. These tables summarise the flow of products and services into intermediate production and the subsequent consumption of products and services by sectors of the economy. Arguably, regional product-by-product IOTs are more of interest in transportation planning and investment. Building regional product-by-product IOTs using methods analogous to those recommended in this work presents additional difficulties in that much of the statistical data useful for estimating is in terms of industries and not products.

Transport services may be provided by businesses that are not primarily in the transport sector; and retailing and wholesaling services may be provided by businesses that are not in the retail or wholesale trades. Nevertheless, the difference between the purchaser's price and the cost of production is the sum of the transport margin and the wholesale and retail margins. In the production of national IOTs this difference is split into the component margins to estimate the input of transport, wholesale and retail into production, regardless of whether that service is provided by businesses whose primary industry is transport, wholesale, or retail, respectively.

An additional accounting convention for national IOTs concerns the treatment of imported goods and services. As any product, imported products have transport, wholesale and retail margins, but they generally lack a record of what industry group produced the product and what industry purchased the product.

Imports are accounted for by assigning them to the industries that would produce the imported products. In the regional setting, it is as though there is an additional region called, 'rest of the world', whose industries can export only to the same industry in other regions of the country. Generally, the apportionment of an imported item produced by several industries is according to the relative levels of domestic production by those industries. Thus, the total output of an industry is a pool of products, some produced domestically and some imported – that pool can be considered net of exports or gross, leading to the notion of competitive and non-competitive imports.

Imports are of two types: competitive and non-competitive. Competitive imports are imports of goods and services which, availability allowing, could be substituted for domestic production. In some IOTs only non-competitive imports into industries are reported and competitive imports are subtracted from exports, so exports are actually a net figure. New Zealand does not account for imports in this way – instead, as described in the paragraph above, imported goods and services are allocated to industries proportional to the 'market share' an industry has for the product or service, regardless of whether they are competitive or non-competitive goods and services. The implication of this is the relative mix of intermediate inputs into an industry's total output (which includes imports) cannot be viewed as a production function.

2.1 The structure of an input-output table

Formally, an IOT consists of:

- a table of intermediate inputs, Z, where $Z = [Z_{ij}]$ and Z_{ij} is the amount of production by industry i used by industry j
- a table of final demand, Y. Y has typically four columns (exports, household consumption, government consumption, and capital investment and changes in stocks) and a row for each industry plus imports. Let $\boldsymbol{e}$ denote the vector of exports and $\boldsymbol{f}$ denote the vector of domestic final demand, an entry for each industry
- a table of value added, V. V typically has at least two rows (one for labour costs and other for total value added), and a column for each industry. Let $\boldsymbol{v}$ denote total value added by industry
- a table or vector of imports, M. If a vector, M records the imports by each industry and directly by final consumers; if a table, there will be an additional row for taxes on imports. Let $\boldsymbol{m}$ denote the row that is imports, an entry for each industry
- a vector of total production $\boldsymbol{x}$, with an entry for each industry.

As in the national accounts, there are accounting identities that must hold (supply equals demand):

$$x_i = \sum_j Z_{ij} + m_i + v_i \qquad \text{(Equation 2.1)}$$
$$x_i = \sum_j Z_{ji} + e_i + f_i$$

If we let $\boldsymbol{i}$ be the vector of ones (of the appropriate length) then this can be written in vector form as:

$$\boldsymbol{x} = \boldsymbol{i}^t Z + \boldsymbol{m} + \boldsymbol{v} \qquad \text{(Equation 2.2)}$$

$$\boldsymbol{x} = Z\boldsymbol{i} + \boldsymbol{e} + \boldsymbol{f}$$

2.2 Technical coefficients

The *technical coefficients* for an IOT are of the form:

$$a_{ij} = \frac{Z_{ij}}{x_j} \qquad \text{(Equation 2.3)}$$

so that a_{ij} is the amount of input required from industry i in order to produce a unit of production by industry j. Often the collection of technical coefficients is thought of as defining a production function with constant returns to scale – this is sensible only when imports are only non-competitive imports. Indeed, suppose the output of an industry consists of two products, one produced domestically and the other imported. If some imports are competitive (that is, the domestically produced product) then doubling intermediate inputs and doubling imports produces relatively more domestic products than non-competitive imports, so the return to scale is not constant. When some imports are competitive an adjustment should be made when calculating technical coefficients to reallocate the competitive imports to exports, possibly producing negative export values. In practice, the information required to do this is not available (especially for services) and one must make recourse to estimates or settle for production functions that do not have a constant return to scale.

Note that $A = [a_{ij}]$ satisfies:

$$\boldsymbol{x} = A\boldsymbol{x} + \boldsymbol{f} + \boldsymbol{e} \qquad \text{(Equation 2.4)}$$

which is equivalent to:

$$(I - A)\boldsymbol{x} = \boldsymbol{f} + \boldsymbol{e} \qquad \text{(Equation 2.5)}$$

The matrix $(I–A)$ has an inverse[2] provided every industry has either some imports or provides some value added, which is generally the case. It is generally denoted L, and called the *Leontief inverse* or the *requirements matrix* – so called because L_{ij} can be interpreted as the required input by i when there is an additional unit of final demand for industry j.

The Leontief inverse provides a measure of impact the economy receives when an industry's final demand has a unit increase. This is called the industry's *multiplier.* Multipliers are used in industrial analysis, and also when assembling IOTs to guide which industries need special survey focus. (Industries that use little imports and add relatively small amounts of value to production have high multipliers).

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[2] (I-A) has an inverse provided that the eigenvalues of A have modulus less than 1. The eigenvalues of A are the same as the eigenvalues of A^t. Let y be an eigenvector of A^t with eigenvalue λ - scale y so that the entries sum to 1. Then $\lambda = \lambda \sum y_i = \sum_i (A^t y)_i = \sum_{ij} a_{ji} y_j \leq \sum_i y_i = 1$, with equality only when every industry i has no imports and has no value added.

IOTs are the result of combining *supply and use tables*. To assemble a supply or use table one needs to specify a classification of goods and services into products; a supply or use table has a row for every product and a column for every industry. A supply table records the value of an industry's production of each product. A use table records the value of the products (including imports) used by each industry in production. Generally, there are more products than industries, so some industries will supply (and use) multiple products.

Suppose U is a use table and V a supply table. If the number of industries was equal to the number of products and each industry produced a single product (so that V is a diagonal matrix of total outputs), then the obvious way to create the inter-industry transactions portion of the IOT is as $Z = U$, with $A = UV^{-1}$. When V is not diagonal or the number of industries is not equal to the number of products, then this prescription no longer makes sense.

The problem is to derive an industry's input into another industry's production by assigning the first industry's production to the other industry's use. When more than one industry produces a product, there is an infinitude of ways to do this.

In the EuroStat manual for IOTs (Eurostat 2008), there are two methods proposed for producing industry-by-industry tables.

1 *The fixed industry-sales structure model.* In this approach, it is assumed each industry has its own fixed sales structure, regardless of its product mix. Each industry 'decides' how much of each of the products it produces that it is going to sell, regardless of whether it produces enough to cover that. This sales structure is stipulated by choosing a left inverse for V, call it H and forming a matrix T by $T_{ip} = \sum_q V_{qi}H_{ip}$. T is the result of scaling each row of H by the total output of the appropriate industry – when V is diagonal T is the identity matrix. Otherwise it is a measure of how much of product p industry i provides.

   The matrix Z is then:

$$Z = TU \qquad \text{(Equation 2.6)}$$

   The reader can check the column sums of Z equal the row sums of U, that is, Z accurately reflects the total use. However, total supply may not be accurately reflected. Being a left-inverse for V implies $\sum_p V_{pi}H_{ip} = 1$, so heuristically, $H_{ip} \simeq 1/(|P_i|V_{pi})$, where P_i is the set of products that industry i produces. With this notion, the proportion of use of product p by industry j that comes from industry i is the ratio between the average amount of production per product that industry i does and the amount of production by industry i of product p. A diversified industry therefore can contribute less than an industry that produces only one product. It is possible under this model for industries to supply more than they produce, leading to negative entries in the IOT. In general, this is a very flawed model.

2 *The fixed product-sales structure model.* In this model, an industry has a market share for a product in proportion to the amount of the product it produces. Precisely, we form the industry-by-product matrix:

$$T_{ip} = \frac{V_{pi}}{\sum_j V_{pj}} \qquad \text{(Equation 2.7)}$$

   and set:

$$Z = TU \qquad \text{(Equation 2.8)}$$

   and:

$$A = Z\Delta(U^t\boldsymbol{i})^{-1} \qquad \text{(Equation 2.9)}$$

Where we use the notation $\Delta(x)$ for the diagonal matrix with x on the diagonal.

This model is well-defined and does not lead to negative entries in the IOT. This is the model Statistics New Zealand employs when producing IOTs.

For additional information regarding the construction and analysis of IOTs we refer you to Miller and Blair (2009); ten Raa (1994; 2010); Dietzenbacher and Lahr (2013); Angel Tarancon et al (2008); De Mesnard (2002a); 2004c; 2007; 2009; 2011); Dietzenbacher (1995; 2006); Dietzenbacher et al (2013); Jansen (1994); Jiang et al (2010); Kim et al (2015); Koopman et al (2014); Liew (2000; 2005); Los et al (2016); Mahajan (2006); Oosterhaven (1984; 1996); Polenske (1995); Rampa (2008); Rose and Allison (1989); Schwarm et al (2006); Viet (1994); Wolff (1994); and Wood and Lenzen (2009).

3 Regional input- output

If national IOTs concern production and final consumption, regional IOTs concern production, consumption and trade. The scale is set by the instance of application: it could be nations and states or provinces; it could be cities and boroughs; it could be a set of nations with open borders such as the EU.

The term 'regional IOT' can mean several things, and is often confused with a number of other terms (which are themselves often used incorrectly or ill-defined).

We set some definitions that we will adhere to throughout this document:

- A *regional IOT* is an IOT for a region within a larger area (say a country) in which a distinction is made between overseas trade involving the region and trade between the region and its complement in the larger area. We use the acronym RIOT to refer to a regional IOT.
- An *inter-regional IOT* is an IOT in which each industry within a region is considered to be a separate industry, so that monetary flows are recorded spatially (between regions) and industrially. We refer to such tables by the acronym IRIOT.

  Note that in an IRIOT there is full specification of regional and industrial inputs into a regional-industry's production, not just aggregated totals by region or industry. We also note there need not be the same industry classification in each region, so different regions will have different numbers of industries and the industries in one region may not cleanly map onto or into the industries in another region.
- A *multi-region IOT* is in a sense an approximation to an IRIOT, in which instead of a full specification of regional-industry flows one has:
  - for each region, a record of the amount of input needed from each industry (regardless of source region) in order to create a unit of production by a given industry – this is called a *regional technical coefficient*
  - for each industry, a record of how much production from that industry in a region is intermediate input into production in any other region.

  We refer to a multi-region IOT as a MRIOT.

There are numerous examples of RIOTs and MRIOTs, though few IRIOTs. For examples, see Bhattarai (2007); Boomsma and Oosterhaven (1992); Chenery (1953); Deng et al (2014); Dietzenbacher et al (2013); Eding et al (1999); Flegg and Tohmo (2013); Gilchrist and St Louis (1999); Kipnis (1976); Lenzen (2001); Morrison and Smith (1974); Oosterhaven and Escobedo-Cardeñoso (2011); Stoeckl (2012); Zhang et al (2015).

There is a lot of flexibility in these definitions in how trade between regions is accounted for. For example, the WIOD (Dietzenbacher et al 2013) is an example of the most general form of IRIOT in the sense that industry imports for intermediate production and consumption are tracked by industry and sector of consumption. In the WIOD, the regions are nations and the industry classification is consistent across all the nations. We will revisit the WIOD in later sections where we develop approaches to measuring the impact of transport on economies.

Other IRIOTs are less general. For example, Finland produces an IRIOT in which the regional industry inputs into national industry production and national sectoral consumption are recorded, but production and consumption of industrial production is not accounted for at a regional level. Thus, as an example, the amount of output produced by the agriculture, hunting and related services industry in the Uusimaa region that is consumed by Finnish households is accounted for, but the regions in which that consumption occurs is not specified.

3.1 A remark on transport, wholesale and retail services in the regional context

In a national IOT in basic prices, the transport and retail margins in the transfers between industries are allocated to the transport or retail industries. When we consider regional input-output we need to consider the location of the industry's economic units, which for retailing is without complication, but for transportation is not as straightforward. When a delivery is made between two regions by a transport business whose headquarters is in a third region and which has depots in all these regions and four other regions, where should the supply of transportation be allocated? In other industries where the means of production might be owned by an entity in a different region it is the practice to allocate the production to the region in which production occurs, regardless of where the profits and taxes end up. In transportation, the 'means of production' can be mobile and not readily assigned to any one region.

In Statistics NZ's methodology for developing regional GDP (Statistics NZ) these issues are discussed in terms of two principles of allocation – the residence principle (where the physical and legal residence of producer is allocated the production), and the territory principle (where the location of activity is allocated the production). In many instances, the physical and legal residence of the producer is the same region as where the production occurs, but transport is one of the industries where that is not the case, and the practice adopted differs across different types of transport, for example air transport is distributed across regions whereas shipping is allocated to the base of operations.

Yet in input-output accounting, transport is accounted for as a margin on a financial transaction, so in the regional setting it is appropriate a transport margin is accounted for as the provision of transportation services from the region where the seller was located to the region where the buyer was located. So there is conflict between the accounting conventions for regional GDP and those of the input-output framework should we extend that framework to regional input-output.

If a RIOT is to be useful for analysing the economic impact of transportation one would ideally want the transportation costs allocated on a territory basis. Under this scheme, to each amount of transfer between two regional industries there would be an amount transferred by the transport industry in the region of origin to the regional industry of delivery. From this one could see whether some regional industries were incurring larger transport costs per unit of production than others and whether this was because they were physically far away from their suppliers or simply were subject to greater costs due to poor infrastructure or a lack of competition. If the transportation costs are recorded as a transfer between the residence region of production and the regional industry of delivery, then it would be possible to detect when a regional industry has high transportation costs but not whether it was due to distance from suppliers or the quality of transport infrastructure at destination or origin.

We now introduce some notation that we will employ to make these notions precise and show how a MRIOT provides an approximation to an IRIOT.

3.2 Key notation

We extend the notation used to describe IOTs to the regional setting. For ease of exposition we will assume each of the R regions has its industrial output classified into the same N industries.

In the following list of definitions it should be understood i and j refer to industries and r and s refer to regions.

- z_{ij}^{rs} is the amount of input from industry i in region r provided to industry j in region s as intermediate inputs.
- f_i^{rs} is the amount of domestic final demand in region s for production by industry i in region r. Similarly, we define e_i^r and Y^{rs} – a matrix of final demand analogous to Y.
- x_i^r is the total production by industry i in region r.
- v_i^r is the value added in production by industry i in region r.
- m_i^r is the amount of imports from abroad into industry i in region r.

For a variable of the form g_{ij}^{rs} or g_i^r or g_i^{rs} the use of a 'bullet' in an index location indicates a summation over the range of that index, eg:

- $g_{ij}^{\cdot s} = \sum_r g_{ij}^{rs}$
- $g_{i\cdot}^{r\cdot} = \sum_{js} g_{ij}^{rs}$
- $g_i^{\cdot s} = \sum_r g_i^{rs}$.

3.3 A note on error and uncertainty

There is error and uncertainty in any IOT. The survey-based tables will have uncertainty due to the sample size and inherent variability of the survey population; in addition, the lack of specification of who buys what from whom leads to error in the table over all. When tables are updated or produced with non-survey techniques more error and uncertainty is introduced.

The key issue is that generally there is little or no 'gold standard' data with which to evaluate the accuracy of an IOT, regional or otherwise. Researchers tend to focus on sensitivity analyses, to understand how robust a table is to small perturbations in the construction. There are numerous schemes for assessing sensitivity or for comparing two constructions.

The recently developed Bayesian approaches to IOT estimation are particularly attractive as the uncertainty estimates are 'built-in' and it is feasible to perform IOT analysis that incorporates the uncertainty. See section 5.2 for more details.

For more on error and uncertainty in (regional) IOTs see Butterfield and Mules (1980); Denman (1966); Jensen (1980); Lahr and Stevens (2002); Roy (2004); Stover (1994); Temurshoev (2015).

3.4 Problem formulation

With this notation, we see that:

- A RIOT for a region r requires the estimation of:
  - z_{ij}^{rr} intra-regional inter-industrial flows
  - x_i^r regional industry total output
  - v_i^r regional industrial value added
  - f_i^{rr} final regional consumption of regional production

- m_i^r regional overseas imports
- e_i^r regional exports to overseas
- $f_i^{r\cdot}$ regional domestic exports to final consumption
- $Z_{i\cdot}^{r\cdot}$ regional domestic exports (and internal use) for intermediate use
- $Z_{i\cdot}^{\cdot r}$ regional domestic intermediate use
- $f_j^{\cdot r}$ regional domestic final consumption.

- An IRIOT for a set of regions and industries requires the estimation of:
  - Z_{ij}^{rs} inter-regional inter-industrial flows
  - x_i^r regional industrial final production
  - v_i^r regional industrial value added
  - f_i^{rs} final consumption
  - m_i^r overseas imports
  - e_i^r exports.
- And finally, a MRIOT requires:
  - Z_{ij}^{rr} intra-regional inter-industrial flows for each region
  - $Z_{ij}^{\cdot r}$ intermediate inputs into industry *j* in region *r* ignoring the source region
  - $Z_{i\cdot}^{rs}$ intermediate input from industry *i* in regions *r* into region s, regardless of the destination industry
  - v_i^r value added
  - $f_i^{\cdot s}$ final demand for production from industry *i* in region s
  - m_i^r overseas imports
  - e_i^r exports.

Thus, *regional technical coefficients* as introduced in the description of MRIOTs are matrices:

$$A^r = [A_{ij}^r] \qquad \text{(Equation 3.1)}$$

so that:

$$A_{ij}^r = \frac{Z_{ij}^{\cdot r}}{x_j^r} \qquad \text{(Equation 3.2)}$$

In other words, A_{ij}^r is the amount of input obtained from industry *i* in order to produce a unit of production by industry *j* in region *r*.

This is different from the regional input coefficients matrices $A^{rs} = [A_{ij}^{rs}]$ for IRIOTS, where:

$$A_{ij}^{rs} = \frac{Z_{ij}^{rs}}{x_j^s} \qquad \text{(Equation 3.3)}$$

The regional technical coefficients are much more likely to resemble the national technical coefficients, provided industrial production is homogenous. The regional input coefficients are related to the technical coefficients but are influenced by the inter-regional patterns of trade. A key issue in developing IRIOTs and MRIOTs is to understand inter-regional trade.

Trade coefficients relate the regional technical coefficients to the regional input coefficients. Described most generally, trade coefficients are of the form $0 \leq T_{ij}^{rs} \leq 1$, where:

$$A_{ij}^{rs} = T_{ij}^{rs} A_{ij}^{s} \quad \text{(Equation 3.4)}$$

Note this formulation allows domestic imports (that is, intermediate input by industry i in region r into production by industry j in region $s \neq r$) to be treated differently from overseas imports. If overseas were a region r and imports were accounted for as in a national IOT then $T_{ij}^{rs} = 1$ if and only if $i = j$, since industrial output is treated as a pool. However, if information exists to track the industry of the importer we need not impose this restriction on the trade coefficients.

A complication in estimating trade coefficients is the phenomenon of *cross-hauling*, in which there is concurrent trade by an industry between regions – that is, a good produced in one region is exported and a similar good imported into the same region. This is related to the heterogeneity of production within industries and is the focus of a later section.

To use a MRIOT to approximate an IRIOT the trade coefficients are approximated by:

$$T_{ij}^{rs} = T_{i}^{rs} = \frac{Z_{i\cdot}^{rs}}{Z_{i\cdot}^{\cdot s}} \quad \text{(Equation 3.5)}$$

This is the same as assuming industrial outputs are pooled and then distributed, as in the treatment of overseas imports in an IOT. We estimate A_{ij}^{rs} and $Z_{i\cdot}^{rs}$ by:

$$A_{ij}^{rs} = T_{i}^{rs} A_{ij}^{s} = \frac{Z_{i\cdot}^{rs}}{Z_{i\cdot}^{\cdot s}} \frac{Z_{ij}^{\cdot s}}{Z_{\cdot j}^{\cdot s}} \quad \text{(Equation 3.6)}$$

$$Z_{i\cdot}^{rs} = T_{ij}^{rs} Z_{i\cdot}^{\cdot s} \quad \text{(Equation 3.7)}$$

This model for trade coefficients, where it is assumed the industry of destination is not a determinant of trade, is referred to as the *Chenery-Moses* model after its creators (Chenery 1953; Moses 1955). When the industry i is a transportation services industry, this model assumes all industries in a region use transportation according to the relative needs for transportation, regardless of whether some industries are more dependent on products of certain regions than others. Thus, the Chenery–Moses model is unlikely to estimate transport service imports accurately. However, the Chenery-Moses model has the attraction of generally being implementable with the data available.

3.5 Leontief’s balanced regional model

The output of some industries is likely to be consumed in the region of production, whereas for other industries, the output is more likely to be used nationally or exported. Leontief developed an approach to producing MRIOTs that relied on a classification of industries into regional industries and national industries, with the former being mostly consumed within the regional of production and the latter not. Examples of regional industries would be wholesaling, retailing, real estate, local government, or takeaways. National industries might be electricity generation, education (as some people attend boarding schools or university away from their region or country of residence, or study via correspondence), central government, and road transportation.

The ingredients for the approach are:

- a decomposition of the set of industries into regional industries and national industries
- for each region s a vector p^s that provides the proportion of each national industry's output produced in s
- for each region s the vector $\mathbf{f}^{R(s)}$ of final demand in region s for the outputs by the regional industries
- a national technical coefficients table A and national industrial output $\mathbf{x}$, final demand $\mathbf{f}$ and exports $\mathbf{e}$.

Re-index the set of industries so the first r are the regional industries and the final $R–r$ are the national industries. With this, a national matrix of input coefficients A takes the block form:

$$A = \begin{bmatrix} A^{RR} & A^{RN} \\ A^{NR} & A^{NN} \end{bmatrix} \quad \text{(Equation 3.8)}$$

the vector of output x can be split as:

$$x = \begin{bmatrix} x^{R} \\ x^{N} \end{bmatrix} \quad \text{(Equation 3.9)}$$

and the vector of final consumption f can be split as:

$$f = \begin{bmatrix} f^{R} \\ f^{N} \end{bmatrix} \quad \text{(Equation 3.10)}$$

Regional industries should have no exports, so we abuse notation and refer to $\mathbf{e}$ as the vector of exports by national industries – some entries to $\mathbf{e}$ may still be zero as there could be national industries that do not export (such as, in New Zealand, road or rail transport).

The relation $x = Ax + f$ still holds and in this notation gives us the following two equations:

$$(I - A^{RR})x^R - A^{RN}x^N = f^R \quad \text{(Equation 3.11)}$$

$$-A^{NR}x^R + (I - A^{NN})x^N = f^R + e \quad \text{(Equation 3.12)}$$

The production of regional industries in region s consists of the production in regional industries to meet final demand in that region and the intermediate input needed to meet that region's share of national industry production, that is:

$$x^{R(s)} = (I - A^{RR})^{-1}f^{R(s)} + (I - A^{RR})^{-1}A^{RN}x^{N(s)} \quad \text{(Equation 3.13)}$$

$$= (I - A^{RR})^{-1}f^{R(s)} + (I - A^{RR})^{-1}A^{RN}px^{N} \quad \text{(Equation 3.14)}$$

The production of $x^{N(s)}$ also requires input from other national industries in other regions, totalling $A^{NN}x^{N(s)}$, and imports from abroad. Hence we can estimate the imports $m^{N(s)}$ for the national industries in s as:

$$m^{R(s)} = x^{N(s)} - v^{N(s)} - A^{NN}x^{N(s)} \quad \text{(Equation 3.15)}$$

Where $v^{N(s)}$ is the value added by the national industries in region s.

The balanced regional model does not provide estimates for the trade coefficients. For a national industry in a given region, the model only provides information about how much intermediate input comes from other regions, not broken down by region of origin. Some other technique (say location quotients, which we will present shortly) must be brought to bear to provide an origin for the regional imports. Nevertheless, Leontief's regional model can be used to establish constraints on amounts of inter-regional imports and exports.

4 The general approach to estimating regional input- output tables from a national input-output table.

Producing IRIOTs, MRIOTS or IOTs from survey data is extremely resource intensive and is done by few countries. If the survey approach is not being used, the missing information needs to be obtained elsewhere or estimated. This section discusses the general approaches that have been employed in estimating IRIOTs and MRIOTs using available data. This data needs to supply estimates of regional technical coefficients, regional trade coefficients and regional consumption coefficients.

There are approaches to estimating inter-regional trade through quantitative models, such as the gravity model approach to be discussed later. These approaches are hampered by a lack of data on which to build, tune or validate the models, and hence we view them as essentially rules-based approaches to assigning values to unknown trade coefficients or similar. The remaining approaches are based on developing regional technical coefficients based on having a national table, and then estimating trade so pools are output and distributed in such a way as to satisfy regional supply and demand.

We assume a national table is available and consider the approaches that use a national table as a starting point or prior. If the national table is out of date, it may be necessary to update the national table in some fashion.

Whether the task is to develop RIOTs, MRIOTs or IRIOTs, the approaches found in the literature are all similar and differ only in specific methods used in each of the four steps.

1 Update the national table (which may be out of date) to be in accord with current or new data.

2 Regionalise the national technical coefficients matrix to form regional technical coefficients.

3 Estimate trade coefficients.

4 Estimate regional exports and imports and consumption.

The GRIT method, which stands for generation of regional input-output tables, was developed in Australia by Jensen and West, (see Hewings and Jensen 1986; Jensen et al 1979; and West 1990). Originally developed to produce RIOTs, it has gone through several stages of development, proceeding from GRIT to GRIT II and finally to GRIT III in which multi-regional tables can be produced.

Like many updating methods, there is a reliance on so-called *superior data*. Superior data is poorly defined in the literature, but reading from context it is data that has not been used to produce a national table and pertains to production or consumption or trade by region, industry, or sector. It may exist for a subset of regions, or a subset of industries, or a subset of sectors of final consumption. An example would be a survey about domestic tourism, estimating the amounts of tourism between any two pairs of regions.

The procedure consists of five main steps (where ‘adjustments’ consist of matrix balancing techniques to be presented shortly):

1 Adjustments to the parent table:

   a Select parent IOT

   b Adjust for updating

   c Adjust for international trade

2 Adjustments for regional imports:

   a Calculate non-competitive imports

   b Calculate competitive imports

3 Definition of regional industries:

   a Insert disaggregated superior data

   b Aggregate industries

   c Insert aggregated superior data

4 Derivation of prototype table:

   a Derive initial transaction values

   b Adjust to derive transaction table

   c Make consistency checks, analysis of sensitivity and coefficient significance.

   d Derive inverses and multipliers.

5 Derivation of final transactions table:

   a Make final superior data insertions and other adjustments

   b Derive final transactions table

   c Calculate inverses and multipliers for final table.

The DEBRIOT method, developed in Holland, is the official Dutch method for developing MRIOTs and IRIOTs. The name is an acronym of 'double entry bi-regional input-output tables' – the approach applies to the case of two regions. In practice, one region is much bigger than the other, as the regions will be an actual region within a country and the 'rest of the country' (see Boomsma and Oosterhaven 1992; Eding et al 1999; and Oosterhaven 1981). Based on regional supply and use tables and heavily dependent on survey data of one of the regions to understand trade coefficients, the method has the following steps for two regions r and s:

1 Compute $Z^{nr} = \left[Z_{ij}^{\cdot r} - Z_{ij}^{rr}\right]$ for region r and industry j in r. This amounts to apportioning the use of products by industry j in r to the various national industries that produce those products. Do this for Z^{ns} as well.

2 Compute regional final demand $F^{nr} = [f_i^{\cdot r}]$ as well as F^{ns}.

Estimate the *domestic sales matrix* for region r, Z^{rn}. The estimation requires the *regional sales coefficients* t_i^{rs} which gives the proportion of domestic sales by industry i in region r that is transferred to s. With these regional sales coefficients (which in practice are estimated via survey of region r) we can compute the *regional domestic sales coefficients* as a weighted average

$$S_{ij}^{rn} = t_i^{rs} \frac{Z_{ij}^{ns}}{Z_{i\cdot}^{ns} + f_i^{ns}} + (1 - t_i^{rs}) \frac{Z_{ij}^{nr}}{Z_{i\cdot}^{nr} + f_i^{nr}} \qquad \text{(Equation 4.1)}$$

so we can estimate the entries Z_{ij}^{rn} as:

$$Z_{ij}^{rn} = S_{ij}^{rn}(x_i^r - e_i^r) \qquad \text{(Equation 4.2)}$$

1 Construct Z_{ij}^{rs} as a constrained mathematical programming problem.

The procedure is as follows:

a Preparatory step: Some input-coefficients Z_{ij}^{rr} or Z_{ij}^{rs} will be better known than others – label these as *specified* coefficients; the rest are *non-specified*.

b Base step: Let $M_{ij}^{rr} = \min(Z_{ij}^{nr}, Z_{ij}^{rn})$ and set the constraint $Z_{ij}^{rr} \leq M_{ij}^{rr}$. This simply states that the total input between industries *i* and *j* is not bigger than the national input into *r* nor the national supply from *r*. Observe then that:

$$Z_{ij}^{rs} = Z_{ij}^{rn} - Z_{ij}^{rr} \geq Z_{ij}^{rn} - M_{ij}^{rr} \quad \text{(Equation 4.3)}$$

$$Z_{ij}^{sr} = z_{ij}^{nr} - z_{ij}^{rr} \geq z_{ij}^{nr} - M_{ii}^{rr} \quad \text{(Equation 4.4)}$$

c For the specified entries of the form z_{ij}^{rr} set $z_{ij}^{rs} = z_{ij}^{ns} - z_{ij}^{rr}$, and analogously if z_{ij}^{rs} is specified. If they are both specified, then they may need to be balanced to ensure consistency.

d For each *i* and *r* define:

$$h_i^r = \frac{\sum_j M_{ij}^{rr} - (1 - t_i^{rs}) Z_{i\cdot}^{rn}}{\sum_j M_{ij}^{rr}} \quad \text{(Equation 4.5)}$$

h_i^r measures how much the intra-regional trade in *i* differs from the maximum; $h_i^r = 1$ corresponds to ubiquitous cross-hauling in region *r* for industry *i*: all the production (if any) is exported and all the input is imported.

e For each non-specified input-coefficient, set $z_{ij}^{rr} = (1 - h_i^r) M_{ij}^{rr}$ or $z_{ij}^{rs} = z_{ij}^{rn} - (1 - h_i^r) M_{ij}^{rr}$.

f Finally, set:

$$Z_{ij}^{rs} = Z_{ij}^{nr} - Z_{ij}^{rr} \quad \text{(Equation 4.6)}$$

$$Z_{ij}^{ss} = Z_{ij}^{nn} - Z_{ij}^{rr} - Z_{ij}^{rs} - Z_{ij}^{sr} \quad \text{(Equation 4.7)}$$

In subsequent chapters we present a method for finding constraints on amounts of inter-regional trade that is analogous to this but which deals with more than two regions.

5 Updating and regionalising methods

Updating and regionalising is the core of the hybrid approach to developing regional IOTs. ‘Updating’ refers to transforming an existing matrix of technical coefficients or inter-industry flows to conform to data for a different time or a different place. ‘Regionalising’ refers to taking data that exists only at a national level and disaggregating it in some fashion to provide regional values.

There is a possibly surprising amount of controversy in the literature about these methods, particularly around the use of location quotients.

For more details on updating and regionalising methods, beyond what we cover here, see Comer and Jackson (1997); Gilchrist and St Louis (1999); Jackson (1998); Jackson et al (2006); Lahr (2001); Madsen and Jensen-Butler (1999) and Temurshoev et al (2011).

5.1 Bi-proportional matrix balancing

The simplest way to transform a matrix is to scale its columns and rows independently. When the scale is chosen so the resulting matrix has prescribed row and column sums, this is referred to as *bi-proportional matrix balancing*. Since the inception of the approach there has been much development of generalisations and extensions – in this section we present both the original form and the extensions, leading in to possibly the most important idea in this review: the Bayesian information-theoretic approach.

5.1.1 The RAS method

To motivate this section, suppose we are presented with a matrix $A = [A_{ij}]$ and the problem of finding a matrix B, ‘close’ to A, so the row and column sums of B are prescribed.

That is, there are vectors x and y so $B\mathbf{e} = x$ and $B^t\mathbf{e} = y$, and we are looking for the solution B - the space of which, if B is square of dimension n is of dimension $n^2 - 2n$ which is ‘close’ to A.

The RAS approach begins by supposing there are vectors r and s of dimension n so if we set $R = \Delta(r)$ and $S = \Delta(s)$:

$$B = RAS$$

The method takes its name from this common naming convention.

The vectors r and s are solutions to a set of $2n$ equations in $2n$ unknowns, and hence if there is a solution it is unique. The solution can either be found numerically or, more commonly, using an iterative technique in which approximate matrices B_i are produced by alternately scaling the rows or columns of B_{i-1} to achieve row sums or column sums of x or y, respectively. If a solution does not exist, then the sequence of matrices does not converge.

Bacharach (1970) developed an equivalent formulation that demonstrates the relationship between the RAS method and *cross-entropy* or more accurately the *Kullback-Leibler divergence*.

For two probability functions p and q the *cross-entropy* of p with q is

$$I(p,q) = -\sum_x p(x)\log(q(x)) \qquad \text{(Equation 5.1)}$$

and the Kullback-Leibler divergence of p with q is:

$$KL(p;q) = -\sum_x p(x)\log\frac{p(x)}{q(x)} \qquad \text{(Equation 5.2)}$$

The Kullback-Leibler divergence of p with q is the difference between the cross-entropy of p and q and the entropy of p (which is the cross-entropy of p and p). The Kullback-Leibler divergence is a measure of how different p and q are.

As a heuristic to understand the Kullback-Leibler divergence, we consider the problem of developing an efficient code, where frequent events are encoded by short words. Suppose the number of characters required to encode an event is linearly related to the logarithm of the frequency of the event. If we have two assessments of event frequency (that is, two probability densities p and q), then the relative change in code length when coding with p over q is $\log(p(x)/q(x))$. Therefore the Kullback-Leibler divergence is the expected extra number of characters needed when coding with p instead of q – it is a measure of information change.

Bacharach's approach was to observe that when A and B are technical matrices then the entries are analogous to probabilities, and so the Kullback-Leibler divergence provides a way to quantify the notion of 'nearness' to A. Thus, finding B amounts to optimising this distance from A under the constraints on the row and column sums. This naturally leads to the use of Lagrange multipliers.

For quantities a_{ij} and b_{ij} we define a functional:

$$\mathcal{L}(B) = -\sum_{ij} b_{ij}\log\left(\frac{b_{ij}}{ea_{ij}}\right) + \sum_i \lambda_i \sum_j (b_{ij} - x_i) + \sum_j \sigma_j \sum_i (b_{ij} - y_j) \qquad \text{(Equation 5.3)}$$

where λ_i and σ_j are Lagrange multipliers, and we want to optimise subject to the constraints that the ith row sums to x_i and the ith column sums to y_i. The symbol e denotes the base of the natural logarithm.

Optimising $\mathcal{L}$, we find that:

$$\log\left(\frac{b_{ij}}{a_{ij}}\right) = \lambda_i + \sigma_j \qquad \text{(Equation 5.4)}$$

so that if we set $r_i = e^{\lambda_i}$ and $s_i = e^{\sigma_i}$ we find that $b_{ij} = r_i a_{ij} s_j$ just as in the RAS method. Since any solution to the RAS method is unique, this solution is identical to the RAS method solution.

5.1.2 RAS method application

The application of the RAS method is for the generation of updated technical or inter-industrial transfer matrices, or the generation of intra-regional input matrices when one has information about the row and column sums.

For example, if one has a current national technical coefficients matrix and knows the regional intermediate inputs and total purchases by industry, then the RAS method can produce an estimate of the regional input coefficients. Other combinations work as well: knowing total production, value added and final demand, for example.

Or, if one has an out-of-date national technical coefficients matrix and has updated vectors of intermediate input and total purchases then the RAS method can produce an estimate of a current national technical coefficient matrix.

Note that error and uncertainty in either the constraints or the prior technical coefficients can lead to a lack of solution under the RAS method.

In section 5.2 we will consider extensions to the RAS method that concern situations where there are multiple linear constraints on the entries of B, which are possibly ill-defined due to data quality or uncertainty in the entries of the prior matrix A. These methods use the what the literature refers to as the 'cross-entropy' approach, though it would be more appropriately named after the Kullback-Leibler divergence.

For more on the RAS method and its application see Byron (1978); Cole (1992); De Mesnard (1997); De Mesnard and Miller (2006; Dietzenbacher and Miller (2009); Lenzen et al (2009); Lenzen et al (2012); Lenzen et al (2007); Minguez et al (2009); Temurshoev et al (2013); Temurshoev and Timmer 2011); and West (1990).

5.2 Cross-entropy and more matrix balancing

There are a number of extensions and generalisations to the RAS method, all of which seek to alter an existing matrix so the new matrix is both 'near' to the original matrix and a number of constraints on the matrix entries are met.

The *modified RAS* (MRAS) method is applicable to the situation in which in addition to the row and column sums of the new matrix being constrained, some set of matrix entries are also constrained. Because RAS preserves the cells that are zero, it is straightforward to apply the RAS procedure to the matrix obtained by setting the fixed cells equal to zero and afterwards altering the cells to be the required values.

Of course, setting some matrix entries to be fixed values is an example of a linear constraint. The *three-stage RAS* (TRAS) method (Gilchrist and St. Louis 1999; 2004) applies to instances where you want to find a matrix B so the entries of B satisfy a collection f_i of linear constraints with value c_i – since row and column sums are linear functions, this problem formulation includes RAS. Though not introduced in this fashion – TRAS was introduced as an algorithm with three steps to it – the approach is to optimise the following function:

$$\sum_{ij} b_{ij} \log\left(\frac{b_{ij}}{ea_{ij}}\right) - \sum_{k} \lambda_k (f_k(B) - c_k) \qquad \text{(Equation 5.5)}$$

When row and column sums are included in the set of constraints, we know any solution will be of the form $b_{ij} = r_i a_{ij} s_j$ with additional constraints on r_i and s_j.

Some IOTs have non-positive entries, which leads to a lack of solution for the RAS method. The *generalised RAS* method (Junius and Oosterhaven 2003; Lenzen et al 2007) addresses this situation by applying the above optimisation approach to the functional:

$$\sum_{ij} |b_{ij}| \log\left(\frac{b_{ij}}{ea_{ij}}\right) - \sum_{i} \lambda_i f_i(B) \qquad \text{(Equation 5.6)}$$

as you might expect.

Finally, the *konfliktfrei RAS* (KRAS) (Lenzen et al 2009) generalises GRAS to cover the situation in which the constraints on the matrix entries are in conflict. In this situation, the numerical optimisation of the GRAS functional may not converge; the KRAS algorithm is to 'tweak' the constraint values c_k when convergence is stalled. The amount of change depends on how close the current iteration is to meeting the constraints and the amount of variance known to exist in the constraint values; thus if the constraint is quite well estimated the new constraint will be set to be equal to the value of f_k on the current iteration, and if it is not well estimated it will be altered by some fixed proportion of the standard deviation.

For more on the use of information-theoretic ideas in matrix balancing, see Batten and Martellato (1985); Batten (1982); Canning and Wang (2005); Cole (1992); Golan et al (1994); Golan and Maasoumi (2008); Macgill (1978); Snickars and Weibull (1977); Wilson (1970).

5.2.1 The Bayesian approach to matrix balancing

Recently, all these approaches have been integrated into a single general Bayesian framework (Rodrigues 2014). The paper is not without error, but the basic results are sound – we have taken some trouble to rectify the errors that we noticed. We are going to go into this framework in some detail because it presents an approach to developing regional IOTs that allows the use of data from many sources (which may themselves be uncertain or unreliable) and provides assessments of the reliability of the final results.

In the Bayesian statistical paradigm all variables and parameters are random variables. In particular, the probability distribution of a random variable is itself a random variable and has its own probability distribution. To make this more concrete, consider a random variable with a probability distribution p that depends on a set of parameters θ. Then:

$$p(x) = \int p(x|\theta)p(\theta)d\theta \qquad \text{(Equation 5.7)}$$

If there exists observations D of the random variable, then the probability distribution p can be updated:

$$p(x|D) = \int p(x|\theta)p(\theta|D)d\theta \qquad \text{(Equation 5.8)}$$

The distribution p is the *prior* for the random variable, and $p(x|\mathrm{D})$ is the ***posterior***.

Note that by Bayes' theorem, the posterior for θ is related to the prior for θ by:

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} \qquad \text{(Equation 5.9)}$$

This approach allows the estimation of uncertainty in the values of the random variable.

If we consider the entries of the matrix Z of intermediate inputs as a vector $\mathbf{q}$, then a set of linear constraints on the entries of Z translates into a matrix G and a vector c with:

$$0 = G\boldsymbol{q} + \boldsymbol{c} \qquad \text{(Equation 5.10)}$$

Note that these constraints imply that the mean m of q satisfies:

$$0 = \mathbb{E}(Gq) + \mathbb{E}(c) \qquad \text{(Equation 5.11)}$$

and with regard to the second moments of q:

$$Cov(c) = G\Sigma G^t \qquad \text{(Equation 5.12)}$$

where Σ is the covariance matrix of q.

In practice, c are statistics obtained through survey or administrative data, generally means or sums. Though it may not always be the case, we can take c to be multivariate normal and hence so is Gq. The distribution of multivariate normal random variables is entirely determined by the mean and covariance matrix, and hence so is the distribution of q.

The essential problem is that the constraints imposed by c are not sufficient to determine the distribution of q. Additional constraints must be imposed on the distribution in order to specify it. The heuristic is that the distribution of q should not be too different from the distribution of the entries in a previous or known matrix of intermediate inputs, where we measure difference using the Kullback-Leibler divergence.

We suppose we have an observation $\mathbf{q}_0$ drawn from a prior distribution $\pi(\mathbf{q})$ – if $\mathbf{q}_0$ is based on survey data then π is a multivariate normal distribution or truncated multivariate normal distribution with known mean and covariance matrix. The update problem can be reformulated as seeking a probability distribution $p(\mathbf{q})$ so:

- p is 'close' to π.
- the expected value $\boldsymbol{m} = \mathbb{E}(q)$ satisfies $G\boldsymbol{m} + \bar{c} = 0,$ where $\bar{c} = \mathbb{E}(c)$
- the covariance $\Sigma = Cov(q, q)$ satisfies:

$$Cov(c) = G\Sigma G^t \qquad \text{(Equation 5.13)}$$

Using the Kullback-Leibler divergence to measure closeness, this amounts to optimising the following functional:

$$\mathcal{L}(p) = p(q)\log \int \left(\frac{p(q)}{\pi(q)}\right) dq + \sum_i \lambda_i \left(\sum_j G_{ij} \int q_j p(q_j) dq_j - \bar{c}_i\right) + \nu\left(\int p(q)dq - 1\right) + \qquad \text{(Equation 5.14)}$$

$$\sum_{ij} \sigma_{ij}\left(\sum_{kl} G_{ik}G_{jl} \int q_i q_j \left(p(q_i, q_j) - p(q_i)p(q_j)\right) dq_i dq_j - Cov(c_i, c_j)\right) \qquad \text{(Equation 5.15)}$$

The three additional terms (and associated Lagrange multipliers) concern the mean and covariance of $\mathbf{q}$ and ensure p is a probability distribution.

Rodrigues shows that with certain assumptions about $Cov(c)$, certain numerical approaches to finding an optimum give the RAS, GRAS, TRAS, and KRAS methods.

5.3 Techniques used to 'regionalise' intermediate inputs, technical matrices and estimate trade coefficients

It is common to have aggregate measures of a quantity, such as industry output at a national level (not broken down by region) or household consumption at a regional level (not broken down by industry). The problem is to disaggregate the quantity to provide additional granularity.

The most common and simplest approach is through *simple location quotients*. In general, a location quotient is a set of non-negative weights assigned to regions or pairs of regions and industries. Simple location quotients are the most obvious construction.

Let E be a quantity (such as salary and wages expenditure or FTE workers or gross output). For a region r and an industry i let E_i^r denote the total value of E for the industry i in r; let E_i denote the total value of E for the industry i nationally; let E^r denote the total value of E for the region r; and let E^T denote the total value of E across all regions and all industries. Define:

$$LQ_i^r := \frac{E_i^r}{E_r}\frac{E_T}{E_i^T} \qquad \text{(Equation 5.16)}$$

Then LQ_i^r measures the relative magnitude (with respect to E) of the industry i in r with respect to the industry nationally. If the industry is the same proportion of E in the region as it is nationally then $LQ^r_i = 1$; when it is less than 1 the value on the region is under-represented, and over-represented when the value is greater than 1.

In practice, simple location quotients are used to alter values that are expected to be less than the national average. For example, if $A = [A_{ij}]$ is a national table of technical coefficients then for a region r we can form:

$$A_{ij}^{rr} = \begin{cases} A_{ij} & LQ_i^r \geq 1 \\ A_{ij}LQ_i^r & LQ_i^r \leq 1 \end{cases} \quad \text{(Equation 5.17)}$$

The amount of inputs needed by industry j in region r to produce a unit of production supplied by industry i in region r is adjusted according to the relative size of industry i in region r – with size measured according to the quantity E.

If LQ_i^r is greater than 1 then under the rule all the industry i input for industry j in region r would be met by regional production; when it is less than 1 there are domestic imports to the amount of $A_{ij} - A_{ij}^{rr}$ for each unit of production. Clearly this does not address cross-hauling.

There are a number of criticisms to the use of simple location quotients for the estimation of regional input coefficients. The technology of simple location quotients has no particular basis in theory, and can be sensitive to the choice of quantity E and to statistical variation in the estimates of the various aggregations of E; small regions can be poorly estimated. After the regional input coefficients and domestic and overseas imports are estimated, the resulting RIOT will in general not be balanced, necessitating an application of a RAS-type procedure.

A result of the approach is that $A_{ij}^{rr} \leq A_{ij}$ for all $i, j,$ and r. This need not be the case, of course, as for some industries in some regions the regional supply might be larger than the national average.

Numerous variations of the location quotient (LQ) method have been developed, but they are generally met with similar criticisms.

- The PLQ (purchases location quotient) method computes a LQ as:

$$PLQ_i^r = \frac{E_i^r}{\sum_j E_j^r I(a_{ij} > 0)} \frac{\sum_{j,r} E_j^r I(a_{ij} > 0)}{\sum_{j,r} E_i^r I(a_{ij} > 0)} \quad \text{(Equation 5.18)}$$

  so that PLQ only makes its comparisons based on industries that use industry i as an input.

- The *cross-industry location quotients* **CIQ** are defined on pairs of industries as follows:

$$CIQ_{ij}^r = \frac{LQ_i^r}{LQ_j^r} \quad \text{(Equation 5.19)}$$

  CIQ_{ij}^r can be used to estimate regional input coefficients in much the same way as LQ_i^r:

$$A_{ij}^{rr} = \begin{cases} A_{ij} & CIQ_{ij}^r \geq 1 \\ A_{ij}CIQ_{ij}^r & CIQ_{ij}^r \leq 1 \end{cases} \quad \text{(Equation 5.20)}$$

  A criticism of this use of CIQ is that A_{ii}^{rr} is always equal to A_{ii}, as the ‘diagonal’ of CIQ is always 1. Thus, a modification is often made to adopt LQ_i^r on the diagonal, which gives:

$$A_{ij}^{rr} = \begin{cases} A_{ij} & CIQ_{ij}^r \geq 1, i \neq j, or\ i = j\ and\ LQ_i^r \geq 1 \\ A_{ij}CIQ_{ij}^r & CIQ_{ij}^r \leq 1, i \neq j \\ A_{ij}LQ_i^r & i = j, LQ_i^r < 1 \end{cases} \quad \text{(Equation 5.21)}$$

- The *semi-logarithmic quotient* SLQ due to Round (1978) is an attempt to build a LQ that takes into account the size of the buyers, the size of the buying region, and the size of the purchasers. To this end we define:

$$SLQ^r_{ij} = \frac{LQ^r_i}{\log_2(LQ^r_j)} \quad \text{(Equation 5.22)}$$

  However, in practice, SLQ has not been seen to provide any advantages over simple or cross-industry quotients.

- The FLQ method due to Flegg et al (1995) is an attempt to modify cross-industry quotients to take into account both the buying and purchasing industries as well as the relative size of the region. Define:

$$FLQ^r_{ij} = \left(\log_2(1 + \frac{E^r}{E^T})\right)^\delta CIQ^r_{ij} \quad \text{(Equation 5.23)}$$

  where $0 \le \delta \le 1$ is a parameter – in a study using Finnish data it was found that $\delta \approx 0.3$ [110].

- Finally, a further variant is the *augmented FLQ* AFLQ, which adjusts the FLQ according to the size of the buying industry (Flegg and Webber 2000), defined as:

$$AFLQ^r_{ij} = \begin{cases} \log_2(1 + LQ^r_j)FLQ^r_{ij} & LQ^r_j \ge 1 \\ FLQ^r_{ij} & LQ^r_j \le 1 \end{cases} \quad \text{(Equation 5.24)}$$

  Note there is not much evidence that AFLQ performs better than FLQ.

For more information about the construction and use of location quotients (and the occasional controversy that surrounds this), see Bakhtiari and Dehghanizadeh (2012); Brand (1997; Chiang (2009); Flegg and Tohmo (2013); Flegg et al (1995); Flegg and Webber (1997; 2000); Harris and Liu (1998); Isserman (1977); Swanson et al (1999) and Zhao and Choi (2015).

5.3.1 The supply-demand pool method

Suppose regional industries produce according to the national technical coefficients table. Then from x^r_i and final regional demand $f^r_{\cdot k}$ by demand industry k, we have:

$$\tilde{x}^r = \sum_j A_{ij} x^r_j + \sum_k C_{ik} f^r_{\cdot k} \quad \text{(Equation 5.25)}$$

where A_{ij} is the usual national technical coefficient and C_{ik} is the demand for industry i output by final demand industry k (such as households, central government and the like). The output required $\tilde{x}^r$ may be different from the region's production x^r_i : define *the regional commodity balance*:

$$b^r_i = x^r_i - \tilde{x}^r_i \quad \text{(Equation 5.26)}$$

If the balance is positive, there is no evidence the national technical coefficients are not reasonable technical coefficients for r – there may indeed be domestic or overseas exports that make up the difference.

However, when the balance is negative there is evidence the national technical coefficients do not provide a reasonable estimate for the regional technical coefficients. In this situation, the *supply-balance pool method* is to estimate A^{rr}_{ij} and C^{rr}_{ij} as follows:

$$A_{ij}^{rr} = A_{ij} \frac{x_i^r}{\tilde{x}_i^r} \quad \text{(Equation 5.27)}$$

and:

$$C_{ik}^{rr} = C_{ik} \frac{x_i^r}{\tilde{x}_i^r} \quad \text{(Equation 5.28)}$$

With this, we have:

$$\sum_j A_{ij}^{rr} x_j^r + \sum_k C_{ik}^{rr} f_{\cdot k}^{\cdot r} = \sum_j A_{ij} x_i^r \frac{x_i^r}{\tilde{x}_i^r} + \sum_k C_{ik} f_{\cdot k}^{\cdot r} \frac{x_i^r}{\tilde{x}_i^r} = \tilde{x}_i^r \frac{x_i^r}{\tilde{x}_i^r} = x_i^r \quad \text{(Equation 5.29)}$$

The regional production is now balanced for regional inputs and final demand.

This is another LQ, with quotient equal to $x_i^r/\tilde{x}_i^r$. Like most of the other LQ techniques, it does not deal with cross-hauling. In particular, its use can only estimate net exports; it is unable to estimate exports and imports separately.

5.3.2 Regional purchase coefficients

In the DEBRIOT method, use was made of *sales coefficients*: t_i^{rs} is the proportion of production by industry i in region r that is sold to region s. The flip-side is *regional purchase coefficients*, which provide the proportion of inputs into industry i in region r that is met by purchases from region r.

As we have covered, just as there is a technical matrix at the national level, there will be a technical matrix for each region with the country. These technical matrices will in general be different. Input into industry j in region r can be sourced within region r or externally, though still domestically. If $A^r = [A_{ij}^r]$ denotes the technical coefficients for r then we can write:

$$A^r = A^{rr} + \sum_{r \neq s} A^{sr} \quad \text{(Equation 5.30)}$$

A^{rr} is called the matrix of intra-regional input coefficients.

Under the regional purchase coefficients model, there is a set of coefficients p_i^r for each region and industry that relate A^r and A^{rr} as:

$$A^{rr} = \Delta(p^r) A^r \quad \text{(Equation 5.31)}$$

which is to say $\Delta(p^r)$ provides a simple estimate of the trade-coefficients. Generally, p_i^r is defined as:

$$p_i^r = \frac{Z_{i\cdot}^{rr}}{Z_{i\cdot}^{\cdot r}} \quad \text{(Equation 5.32)}$$

Thus p_i^r is completely determined by Z_i^{rr} and $Z_{i\cdot}^{\cdot r}$.

Much effort has been put into estimating these fractions. Results indicate that the use of regional purchase coefficients is superior to LQs (Stevens et al 1989).

5.3.3 The CHARM method

The *CHARM method* stands for *Cross-hauling adjusted regionalisation method*. It is a variation of the supply-demand pool method where an attempt is made to account for cross-hauling, the concurrent import and export of the same industrial output. The CHARM method is due to Kronenberg (2009). Flegg and Tohmo (2013) evaluated the CHARM method by comparing Finnish RIOTs (which are survey based) to those obtained applying the CHARM method to national tables – the CHARM method performed well. Cross-hauling can be measured for each industry i as q_i where:

$$q_i = e_i + m_i - |e_i - m_i| \qquad \text{(Equation 5.33)}$$

with e_i being the exports and m_i the imports for the industry (for the moment, we suppress regional superscripts, and assume exports and imports includes overseas and domestic destinations and sources). Note that q_i is always non-negative and bounded above by $2e_i = 2m_i$. If we express $v_i = e_i + m_i$ as the ***trade volume*** and $b_i = e_i - m_i$ as the ***trade balance***, then we can write $v_i = |b_i| + q_i$. Note that q_i is zero when there are no imports or exports and reaches its maximum when exports and imports are equal. For more on cross-hauling, see Nakano and Nishimura (2013) and Többen and Kronenberg (2015).

Kronenberg argues cross-hauling is a function of heterogeneity of industry outputs – if output by industries was independent of the region of origin then there would be no cross-hauling. To make this concrete Kronenburg introduces a variable h_i which captures the degree of heterogeneity in the production of industry i, (independent of region) and posits that q_i is a function of total production, intermediate use, final domestic consumption and h_i. Moreover, he assumes q_i is proportional to h_i with form:

$$q_i = h_i(x_i + Z_{i\cdot} + f_i) \qquad \text{(Equation 5.34)}$$

This leads to the expression:

$$h_i = \frac{v_i - |b_i|}{x_i + Z_{i\cdot} + f_i} \qquad \text{(Equation 5.35)}$$

which can be evaluated using the data present in a national IOT.

Now choose a region r and an industry i. We assume we have values for $x_i^r, Z_{i\cdot}^{\cdot r},$ and $f_i^{\cdot r}$. Using these we estimate q_i^r as:

$$q_i^r = h_i(x_i^r + Z_{i\cdot}^{\cdot r} + f_i^{\cdot r} \qquad \text{(Equation 5.36)}$$

Then using the supply-demand pool method we obtain estimates for net exports (overseas and inter-regional) b_i^r which together with q_i^r gives an estimate for trade volume v_i^r. Finally, we obtain imports (overseas and inter-regional) m_i^r and exports e_i^r using $m_i = (v_i - b_i)/2$ and $e_i = (v_i + b_i)/2$.

6 Estimating regional trade

As we stated earlier, estimating regional trade is essential for assembling regional IOTs. We have already seen some approaches to estimating trade in that the regional purchase coefficients, regional sales coefficients are forms of trade coefficients. In this chapter we look at other methods of estimating regional trade, focusing in particular on gravity models.

For further information on estimating regional trade (aside from the gravity model) see Bachmann et al (2015); Falocci et al (2009); Giarratani (1980); Isserman (1980); Jiang et al (2010); Park et al (2009) and Stadler et al (2014).

6.1 The gravity model of trade

A *gravity* model of trade is a model of the flows of a (set) of goods and services between categories where the amount of trade reduces with 'distance' between the categories. More precisely, and using the notation we have already developed, a gravity model for Z_{ij}^{rs} or $Z_{i\cdot}^{rs}$n its simplest form is:

$$Z_{ij}^{rs} = G\frac{P_r^{\alpha_1}P_s^{\alpha_2}}{d_{rs}^{\alpha_3}} \qquad \text{(Equation 6.1)}$$

where α_1,α_2, and α_3 are parameters (exponents) that need to be specified, G is a constant of proportionality (which may depend on j or s or r), P_r is a variable that encodes information about the supply of production from i in region r and P_s is a variable that encodes information about the demand for production from i in industry j in region s, and d_{rs} is a notion of distance between r and s – this notion is conceptual and could be defined in terms of spatial distance, ease of trade, cost of transport, cultural similarities, length of a common border, or some combination of these or others.

We refer the reader to Byers et al (2000); Evans (2003); Mayer (2014); Sargento et al (2012); Sargento (2007) and Sohn (2004).

Practical application of the gravity model approach requires the practitioner to define variables and identify parameters. Generally, there is no data available to identify the parameters analytically – that is, fitting the model parameters using actual trade data. Some countries have developed RIOTs using the survey approach (Japan and Finland, for example, or using the WIOD) and parameters can be identified by using this data to build a gravity model with the desired variables. If these parameters can be modelled in terms of quantitative characteristics of the instances providing the data (say, information about the countries and its trading partners) one could apply this model in the new context. It is also common in the literature to simply select parameters (all equal to 1 is a popular), but this provides a rules-based model rather than anything based on data.

However, even as a rules-based approach it is not entirely without merit as the structure of the gravity model does have a theoretical foundation.

6.2 Theoretical foundations of the gravity model

In this section we closely follow Costinot and Rodriguez-Clare (2013).

Suppose we have a set of R regions or countries that each produces a single product, and region i is naturally endowed with a quantity Q_i of production. These products are substitutable, in that the utility provided by one region's product can be met to an extent by other regions' products. We assume the elasticity of substitution of one product for another is constant, $\sigma \geq 0$ (though in practice σ will be at least 1).

We assume each region is represented by a purchasing agent who is seeking to maximise a utility function – a constant elasticity of substitution utility function. If C_{ij} is the demand for product i in region j then the utility of region j has the form:

$$C_j = \left(\sum_i \left(\frac{C_{ij}}{\psi_{ij}} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\sigma/(\sigma-1)} \quad \text{(Equation 6.2)}$$

where ψ_{ij} is a parameter representing the preference that country j has for product i.

The associated consumer price index is:

$$P_j = \left(\sum_i \psi_{ij}^{1-\sigma} P_{ij}^{1-\sigma} \right)^{1/(1-\sigma)} \quad \text{(Equation 6.3)}$$

where P_{ij} is the price of product i in region j.

Trade between regions is subject to iceberg costs, in which the sale of 1 unit of product i in region j requires $t_{ij} \geq 1$ units of i. Clearly, $t_{ii} = 1$. To avoid arbitrage opportunities $P_{ij} = t_{ij}P_{ii}$.

If Y_i is the total income of region i then $P_{ii} = \frac{Y_i}{Q_i}$ so that:

$$P_{ij} = t_{ij} \frac{Y_i}{Q_i} \quad \text{(Equation 6.4)}$$

With $X_{ij} = C_{ij}P_{ij}$ the expenditure by j on product i we can optimise C_j subject to the constraint that $\sum_j X_{ij} = Y_i$ to obtain:

$$X_{ij} = \left(\frac{\psi_{ij} P_{ij}}{P_j} \right)^{1-\sigma} E_j \quad \text{(Equation 6.5)}$$

where $E_j = \sum_i X_{ij}$ is the total amount of purchases by j. This can be re-written as:

$$X_{ij} = G \frac{E_j Q_i^{\sigma-1}}{\left(\psi_{ij} t_{ij} Y_i \right)^{\sigma-1}} \quad \text{(Equation 6.6)}$$

where $G = P_j^{\sigma-1}$. Thus we see the parameters in the gravity model are related to the elasticity of substitution between products, and the distance measure is related to product preference, trade costs and the total production of the exporter.

7 The regional input- output model: a modelling approach to building regional input- output tables.

7.1 The dual nature of regional input-output tables

An IRIOT is a collection of technical matrices, a trade matrix, a vector of total production, a vector of imports, and a decomposition of final consumption into household expenditure, government expenditure, capital investment and exports. There is a duality to the technical and trade matrices in that on one hand they give, with the vectors of production, imports and consumption, the flows of intermediate outputs between industries, providing more nuanced reporting of national production; on the other hand, the technical and trade matrices can be thought of as sets of parameters describing the dynamics of production and trade.

We take the approach that unbiased and representative data on regional and industrial supply and use can directly inform the entries in the technical and trade matrices, but data that is not representative might be used to implicitly improve the accuracy of the estimates by using the technical and trade matrices as parameters in models built to explain this data.

The approach we outline has three steps. The first step (sections 7.2 and 7.3) is to formulate the problem of creating an IRIOT as an optimisation problem, extending the Bayesian cross-entropy approach. To employ this approach it is necessary to have a set of priors (best guesses) for the quantities to be estimated and a set of constraints that are functions of these quantities.

The second step (section 7.4) is to use data sourced from Statistics NZ or third-party data to develop regional technical matrices and to construct constraints on and estimates of the trade matrices.

The third step (chapter 8) is to model systems for which we have non-representative or biased data in terms of these matrices, so using Bayesian methods to fit these models provides a posteriori estimates of the trade and technical matrices. We illustrate the third step using accounting data from Xero and, to a lesser extent, the freight movement data from eRUC.

Our presentation will follow this breakdown. In section 7.2 we will present the model formulation in detail. Following that we will consider how the necessary priors and constraints might be established using available data from Statistics NZ or the third-party sources discussed in chapter 8.

7.2 Extending the Bayesian approach to estimating regional input-output tables

Consider the most general problem of developing an IRIOT for R regions and N industries. What is required is a set of technical matrices, trade matrices, and estimates of final consumption and regional sources of that consumption.

We adapt and extend Rodrigues's approach to estimate technical coefficients, trade coefficients and trade in final consumption. The first step is to formulate the problem as a cross-entropy optimisation problem, and following that, to establish priors and constraints using available data.

7.2.1 Updating the national technical matrix

Updating a national technical coefficients matrix requires national data about production and consumption. To employ the RAS method or one of its generalisations requires $2N$ bits of data, though not every set of data with that cardinality will result in a solution – for example if we choose these $2N$ points of data randomly from the inter-industrial flow matrix (which is otherwise unknown), call them $Z_{i_k j_k}$ $k = 1, \dots, N$, then the RAS problem in this situation is to find r_i, s_i so that:

$$Z_{i_k j_k} = r_{i_k} A_{i_i j_k} s_{j_k} \, x_{j_k} \qquad \text{(Equation 7.1)}$$

Clearly for there to be a solution the data points need to have a representative for each row and each column of the inter-industrial flow matrix. To employ this formulation, using the technical matrix and the vector of outputs, requires knowing the vector of outputs and if that is lacking then one would resort to working with the old national table that we are trying to update.

More generally, if we have $2N$ linear constraints on the entries of the inter-industrial flow matrix of the form:

$$c_i = \sum_{jk} G^i_{jk} Z_{jk} \qquad \text{(Equation 7.2)}$$

for some matrix of coefficients G^i_{jk} – when the constraints are simple aggregates and all the entries are either 0, 1 or -1.

If we think of the coefficient matrices G^i_{jk} as vectors of length N^2 then a necessary constraint on the set of G^i is that they be linearly independent; this is not sufficient, however. One can use Macauley's multivariate resultant[3] to characterise which coefficients lead to RAS solutions for a given national table to be updated.

Linear independence of the G^i requires that within the set of non-zero entries there is complete representation of each row and column in Z.

While $2N$ datapoints (or N datapoints and the vector of total output) are necessary to pin down a solution under the RAS method, the Bayesian cross-entropy approach applies regardless of the number of datapoints, whether fewer or greater than $2N$ or even greater than N^2. Obviously, the greater the number of consistent data points the more likely the updated table is to resemble the actual table. Fewer than $2N$ datapoints is likely to leave some rows or columns poorly estimated.

In New Zealand we do not have official data on the total intermediate use of a national industry's production, except for when that information is obtained to produce IOTs. But we can make use of the basic use identity:

$$(I - A)\boldsymbol{x} = \boldsymbol{f} + \boldsymbol{e} \qquad \text{(Equation 7.3)}$$

where $\boldsymbol{f}$ and $\boldsymbol{e}$ are the vectors of final consumption and exports, respectively. The basic supply identity allows us to constrain A in terms of total output, imports, and value added.

$$\boldsymbol{x} = \boldsymbol{i}^{\boldsymbol{T}} A \Delta(\boldsymbol{x}) + \boldsymbol{m} + \boldsymbol{v} \qquad \text{(Equation 7.4)}$$

[3] The resultant of two polynomials is an algebraic notion. It is a polynomial that contains information about the intersection of the curves defined by the two polynomials; the polynomials have a common zero if and only if the resultant is zero. The resultant is easily computed using the polynomials' coefficients. Macauley's multivariate resultant is a generalization to polynomials in n variables, and has the same property that it is zero if and only if the polynomials have a common zero. It is, however, more complex to define and compute, and a precise description is outside the scope of this work.

By dividing through by total output, these two sets of equations form the set of constraining equations G. Thinking of $\bar{A}$ as a vector of length $(N+k) \times N$ so that:

$$G\bar{A} = \boldsymbol{c} \qquad \text{(Equation 7.5)}$$

we obtain the familiar constraints on the mean and variance of $\bar{A}$:

$$G\mathbb{E}(\bar{A}) = \mathbb{E}(\boldsymbol{c}) \qquad \text{(Equation 7.6)}$$

and:

$$Cov(\boldsymbol{c}) = G\Sigma G^t \qquad \text{(Equation 7.7)}$$

$$Cor(c_i, \sum_{jk} G_{ijk}\bar{A}_{jk}) = 1, \forall i \qquad \text{(Equation 7.8)}$$

In this formulation we are assuming the entries of G are not random variables – in practice, this would not be strictly true as, because we are adopting $\bar{A}$ over Z, the entries in G are functions of total output $\boldsymbol{x}$, which will have non-zero variance. Treating G as a random vector is best done by Monte Carlo methods rather than analytically.

Thus updating the national table is completely analogous to Rodrigues's (2014) approach, but modified so the technical matrix is sought from the optimisation problem rather than the inter-industrial flow matrix. By choosing the augmented technical matrix $\bar{A}$ to be distributed as Dirichlet, we lose the requirement that the density integrate to unity, but otherwise the functional form of the optimisation problem is the same.

7.2.2 From the national technical matrix to the regional case

In the regional case, it is appropriate to work with the regional input coefficient matrices A_{ij}^{rs}. If we know the regional input coefficient matrices, then we can work out the inter-industry trade matrices and the regional technical coefficient matrices. Not all trade is between industries: there is also trade for final consumption. In our IRIOT formulation this f_i^{rs}.

To deal with f_i^{rs} we introduce a set of coefficients C_i^{rs}, which is the proportion of final consumption of industry i product in region s which is supplied by region r. Hence:

$$f_i^{rs} = C_i^{rs} f_i^{\cdot s} \qquad \text{(Equation 7.9)}$$

Just as in the problem of updating the national technical matrix, applying the cross-entropy approach requires data on regional total output and consumption, and constraints on the probability density of the regional input-output matrices.

One of the issues with regional data is that even when it is available it may be aggregated over industries or regions. We will discuss this now, but for the sake of exposition, consider regional technical matrices rather than input coefficient matrices.

Suppose a national technical matrix exists for the period for which the regional estimate is required. Updating methods such as RAS require data about production and use at the industry level; often the regional data in New Zealand is lacking in granularity – having say fewer industries than the national table or aggregating over several regions.

Continuing with the notation used previously, we have for a region r an $(N+k)\times N$ technical matrix $\bar{A}^{r}$, a vector of final production $\boldsymbol{x}^r$, and a matrix of intermediate inputs $Z^{\cdot r}$. If A^r is the $N \times N$ submatrix of $\bar{A}^r$ that neglects imports and value added, then these quantities are related by:

$$Z^{\cdot r} = A^r \boldsymbol{x}^r \qquad \text{(Equation 7.10)}$$

Now suppose the N industries have been classified into K collections of industries $\sigma_1, \dots, \sigma_K$. There is a vector of final production $\boldsymbol{x}_\sigma^r$ that we need to use to update the national table to estimate A^r. Define a $N \times K$ matrix π_σ^r so that:

$$\boldsymbol{x}^r = \pi_\sigma^r \boldsymbol{x}_\sigma^r \qquad \text{(Equation 7.11)}$$

So that $\pi_{\sigma\,ji}^{r}$ is the proportion of output by industry collection σ_j that was done by industry $j \in \sigma_i$. These proportions will not be known, but a prior can be constructed.

Making reference to the previous discussion on regional statistical data, we see that priors on such proportions would be needed for the following quantities:

- value added
- gross fixed capital formation
- total outputs.

Exports and value added can be found at the industrial level both nationally and regionally. Exports in goods and services are available at a product level and the national supply tables could be used to apportion that to industries. Regional exports could then be apportioned according to regional industrial shares of production or through more complex methods. This would be similar for imports.

New Zealand's economy is dominated by a few large industries, and priors for the regional supply and use by these industries would benefit from directly obtaining pertinent data from the industries, as in the *National freight demand study* (Ministry of Transport 2014).

7.2.3 Formulating the Bayesian cross-entropy optimisation problem

The ingredients into the Bayesian cross-entropy optimisation problem are:

- a prior on the technical coefficients matrices
- a prior on the trade coefficient matrices
- a prior on the consumption trade coefficients
- a prior for each instance in which a regional statistic reports an aggregate of industries
- a prior for each instance in which an industrial statistic reports an aggregate of regions
- a prior on each of the estimates of regional foreign imports, regional foreign exports, regional household consumption, regional central government consumption, regional local government consumption, and regional gross capital formation
- a set of constraints on values of technical coefficients matrices
- a set of constraints on the variance of the technical coefficients
- a set of constraints on the values of the trade coefficients – both industrial and for consumption
- a set of constraints on the variance of the trade coefficients – both industrial and for consumption.

7.2.3.1 Dirichlet distributions and 'cross- entropy'

To mimic the notation used in Rodrigues's (2014) work, let q, as a vector, denote $\bar{A}$ and C. q is a composite of Dirichlet variables. A prior on q is given by a set of parameters $\bar{\theta}$ so $\pi(q) = p(q \mid \bar{\theta})$. We want to describe a functional that must be optimised, analogous to Rodrigues's work. To do so we need to consider the case of a single Dirichlet random vector.

What does it mean to be a random vector X of dimension n that is distributed as Dirichlet? Each entry of X belongs to the open unit interval and the sum of the entries is 1. Being Dirichlet means there is an n-dimensional vector of positive real numbers θ, so that the probability of X taking the values $\boldsymbol{x}$ is given by:

$$p(X = x) = K(\theta) \prod_i x_i^{\theta_i - 1} \qquad \text{(Equation 7.12)}$$

Where $K(\theta)$ is a normalising constant – it can be given explicitly in terms of Gamma functions. Classically, a variable that is Dirichlet describes the likely probabilities for rolling the various faces of an n-sided die. In modelling a column of $\bar{A}$ as Dirichlet we are assuming the businesses in the corresponding industry are obtaining intermediate inputs in proportions obtained by selecting a random die from the Dirichlet distribution.

The Kullback-Liebler divergence between $p(X|\bar{\theta})$ and $p(X \mid \theta)$ can be shown to be:

$$KL(\theta, \bar{\theta}) = \ln\left(\frac{\Gamma(\theta)}{\Gamma(\bar{\theta})} \prod \frac{\Gamma(\theta_i)}{\Gamma(\bar{\theta}_i)}\right) + \sum_j (\theta_j - \bar{\theta}_j)\psi_0(\theta_j) - \psi_0(\Sigma\theta_j) \sum_i (\theta_i - \bar{\theta}_i) \qquad \text{(Equation 7.13)}$$

where $\Gamma(x)$ is the Gamma function and ψ_0 is the digamma function (the logarithmic derivative of the Gamma function).

The expected value of X is a vector with entries $\mathbb{E}(X_i) = \frac{\theta_i}{\Sigma\theta_j}$. If we set $\theta_0 = \Sigma\theta_i$ then the covariance of X_i and X_j is given by:

$$Cov(X_i, X_j) = -\frac{\theta_i \theta_j}{\theta_0^2(\theta_0 + 1)} \qquad \text{(Equation 7.14)}$$

and the variance is:

$$Var(X_i) = \frac{(\theta_0 - \theta_i)\theta_i}{\theta_0^2(\theta_0 + 1)} \qquad \text{(Equation 7.15)}$$

The point being that the function to be optimised for Kullback-Leibler divergence can be expressed as a function of the various Dirichlet parameters for the regional input coefficients and the consumption trade coefficients as follows in this special case where we have a single Dirichlet variable. Let $\boldsymbol{x}$ be a vector – in practice $\boldsymbol{x}$ would be obtained from the regional output vectors and the regional consumption vectors.

$$\mathcal{L}(\theta) = \ln\left(\frac{\Gamma(\theta_0)}{\Gamma(\bar{\theta}_0)} \prod \frac{\Gamma(\bar{\theta}_i)}{\Gamma(\theta_i)}\right) + \sum_j (\theta_j - \bar{\theta}_j)\psi_0(\theta_j) - \psi_0(\theta_0)(\theta_0 - \bar{\theta}_0) + \qquad \text{(Equation 7.16)}$$

$$+\Sigma\lambda_i(\frac{G_i\Delta(\boldsymbol{x})\theta}{\theta_0} - \bar{c}_i(\theta)) + \sum_{ij} \sigma_{ij}(\frac{G_i\Delta(\boldsymbol{x})\theta^t\theta\Delta(\boldsymbol{x})G_j^t}{\theta_0^2(\theta_0 + 1)} - Cov(c_i(\theta), c_j(\theta))) \qquad \text{(Equation 7.17)}$$

Where G_i is the ith row of G and $\Delta(\boldsymbol{x})$ is the diagonal matrix with entries $\boldsymbol{x}$. Due to the presence of the dilogarithm function there is no analytical solution, so numerical methods will be required to optimise.

7.2.3.2 The fully specified problem

In our situation, where q consists of a number of Dirichlet variables, the functional breaks into sums over the components. There are a number (at least $2NR$) Dirichlet variables in q, and let I_k denote the set of indices that belong to the k-th variable. Let θ_0 be a vector the same dimension as θ with the entries belonging to I_k being the sum $\sum_{\mathrm{I}_k} \theta = \theta_0^k$. Similarly organise $\bar{\theta}$. Let $\boldsymbol{x}$ be a vector of the same dimension as θ with entries from $x_i^r, f_i^{\cdot r}$ or equal to 1 when the corresponding entry in θ is for a variable that disaggregates an aggregated quantity – $\boldsymbol{x}$ is a function of θ because of the disaggregating variables.

With this, the functional becomes:

$$\mathcal{L}(\theta) = \sum_k \left(\ln\left(\frac{\Gamma(\theta_0^k)}{\Gamma(\bar{\theta}^k{}_0)} \prod_{i \in \mathrm{I}_k} \frac{\Gamma(\bar{\theta}_i)}{\Gamma(\theta_i)} \right) + \sum_{j \in \mathrm{I}_k} (\theta_j - \bar{\theta}_j)\psi_0(\theta_j) - \psi_0(\theta_0^k)(\theta_0^k - \overline{\theta^k}{}_0) \right) \quad \text{(Equation 7.18)}$$

$$+ \sum_i \lambda_i (G_i \Delta(\mathbf{x}(\theta)) \Delta(\theta_0)^{-1}\theta - \bar{c}_i(\theta)) \quad \text{(Equation 7.19)}$$
$$+ \sum_{ij} \sigma_{ij} (G_i \Delta(\mathbf{x}(\theta)) \Delta(\theta_0)^{-1} \theta^t \theta \Delta(\theta_0)^{-1} \Delta(\theta_0 + \mathbf{1})^{-1} \Delta(\boldsymbol{x}(\theta)) G_j^t$$
$$- Cov(c_i(\theta), c_j(\theta)))$$

Where $\Delta(\boldsymbol{x})$ is the square matrix with $\boldsymbol{x}$ on the diagonal, and G_i is the ith row of G. The constraints are functions of θ because of the approach we used to find constraints on the industrial and consumption trade matrices, which required the use of the regional technical matrices.

To make it explicit, the vector q consists of the (extended) regional input-coefficients, the trade consumption coefficients, the proportions used for creating regional estimates of national statistics, and the proportions used for 'converting' aggregated statistics to disaggregated statistics – these two last components of q do not have constraints in the same way as the other components of q, but appear in the optimisation problem in the constraint values $c_i(\theta)$.

7.3 Priors and constraints

7.3.1 A prior on the trade coefficient matrices

Unlike the technical coefficients there is no national table on which to base a prior for the trade coefficients matrices. We propose a prior be set through the use of a gravity model.

A gravity model could be fitted using Finnish or Japanese RIOTs, or even the WIOD. This would give an estimate for the trade coefficients and their variances. This gravity model could be informed by New Zealand specific data such as that obtained in the *National freight demand study* (Ministry of Transport 2014).

This prior, together with the national technical coefficients matrix, forms the prior on the set of regional input coefficient matrices.

For the consumption trade coefficients C_i^{rs} we propose a non-informative prior, so that:

$$\mathbb{E}(C_i^{rs}) = \frac{1}{R} \quad \text{(Equation 7.20)}$$

for all r, i and s (and where R is the number of regions). Arguably, a prior that favours consumption of local production might be closer to the 'true' distribution but the non-informative prior allows the possibility of trade between distant regions, which is entirely plausible with internet commerce. There is clearly a lot of scope for setting a prior on consumption trade coefficients, and such a study could benefit a lot from courier and postal information, and data about the regions and industries of retail suppliers.

7.3.2 Constraints on trade coefficient matrices

The CHARM method can be used to put constraints on the trade coefficient and technical coefficient matrices.

Consider the rest-of-the-world as additional region, W, so that exports are part of $f_i^{r\cdot}$, and imports are part of $Z_{i\cdot}^{\cdot r}$ as $Z_{i\cdot}^{Wr} = m_i^r$. Suppose that we have regional technical coefficient matrices A^r.

The trade balance for industry i in region r, which is the difference between what industry i in region r exports and what it imports, is:

$$b_i^r = Z_{i\cdot}^{r\cdot} - Z_{i\cdot}^{\cdot r} - m_i^r + f_i^{r\cdot} - f_i^{\cdot r} \qquad \text{(Equation 7.21)}$$

and the trade volume, which is the sum of imports and exports (and hence always non-negative) is:

$$v_i^r = Z_{i\cdot}^{r\cdot} + Z_{i\cdot}^{\cdot r} - 2Z_{i\cdot}^{rr} + f_i^{r\cdot} + f_i^{\cdot r} - 2f_i^{rr} + m_i^r \qquad \text{(Equation 7.22)}$$

In the national case (where there are only two regions – the rest of the world and New Zealand) this reduces to the usual representation as the difference between exports and imports and the sum of imports and exports, as we would expect.

With these notions, the amount of cross-hauling for industry i in region r is then:

$$q_i^r = v_i^r - |b_i^r| \qquad \text{(Equation 7.23)}$$

To apply the CHARM method, we need estimates of the industry's product homogeneity as used by Kronenberg:

$$h_i^r = \frac{q_i^r}{x_i^r + Z_{i\cdot}^{\cdot r} + f_i^{\cdot r}} \qquad \text{(Equation 7.24)}$$

These measures h_i^r can be estimated at the national level (so they are the same for each region) either using the definition, or by calculating h_i at the product level and using the national supply table to produce the measure for an industry as an appropriate weighted average of product-level measures.

Having estimated h_i^r we are able to estimate q_i^r from total production, total requirements (using the regional technical matrix) and regional final consumption. From q_i^r and b_i^r we can recover v_i^r. And finally, from b_i^r and v_i^r we can identify total imports (foreign and domestic) and total exports (foreign and domestic) as:

$$I_i^r := \frac{v_i^r - b_i^r}{2} = Z_{i\cdot}^{\cdot r} - Z_{i\cdot}^{rr} + f_i^{\cdot r} - f_i^{rr} + m_i^r \qquad \text{(Equation 7.25)}$$

and:

$$X_i^r := \frac{v_i^r + b_i^r}{2} = Z_{i\cdot}^{r\cdot} - Z_{i\cdot}^{rr} + f_i^{r\cdot} - f_i^{rr} \qquad \text{(Equation 7.26)}$$

We can estimate $Z_{i\cdot}^{\cdot r}$, $f_i^{\cdot r}$ and m_i^r, which gives $Z_i^{rr} + f_i^{rr}$. And since $Z_{i\cdot}^{r\cdot} + f_i^{r\cdot} = x_i^r$, for which we have estimates, from X_i^r we obtain another estimate of $Z_{i\cdot}^{rr} +$. Following the chain of estimates one sees these two estimates are the same.

Hence the CHARM method provides constraints on total regional self-supply, conditional on estimates for final consumption (including exports), regional imports and regional technical matrices. This gives estimates for the regional purchase coefficients:

$$p_i^r = \frac{Z_{i\cdot}^{rr} + f_i^{rr}}{Z_{i\cdot}^{\cdot r} + f_i^{\cdot r}} \qquad \text{(Equation 7.27)}$$

In terms of trade and technical coefficients the constraint on total regional self-supply can be expressed as a constraint:

$$\sum_j T_{ij}^{rr} A_{ij}^r x_j^r + C_i^{rr} f_i^{\cdot r} \qquad \text{(Equation 7.28)}$$

We can repeat this analysis in the situation where we 'create' a region as a union of two regions, and consider the new situation where the two original regions are discarded and replaced by their union. The regional technical matrix for the union of two can be obtained as a weighted sum of the individual regional technical matrices by weighting the columns according to the regional output in the appropriate industries. The preceding analysis then produces estimates of:

$$Z_{i\cdot}^{ss'} + Z_{i\cdot}^{s's} + f_i^{ss'} + f_i^{s's} \qquad \text{(Equation 7.29)}$$

for any two regions s and s'. Repeating this, creating artificial regions with three, four, up to $R-1$ actual regions provides $2^R - 2$ linear equations in $\frac{R(R-1)}{2}$ unknowns of the form $Z_{i\cdot}^{ss'} + Z_{i\cdot}^{s's} + f_i^{ss'} + f_i^{s's}$. This data could be used to establish means and variances of $Z_{i\cdot}^{ss'} + Z_{i\cdot}^{s's} + f_i^{ss'} + f_i^{s's}$, to be used as constraints for the optimisation problem. In terms of the regional input-coefficient matrices the constraint on $Z_{i\cdot}^{ss'} + Z_{i\cdot}^{s's} + f_i^{ss'} + f_i^{s's}$ is a constraint of the form:

$$\sum_j (\bar{A}_{ij}^{ss'} x_j^{s'} + \bar{A}_{ij}^{s's} \ x_j^{s}) + C_i^{ss'} f_i^{\cdot s'} + C_i^{s's} f_i^{\cdot s} = c \qquad \text{(Equation 7.30)}$$

which is a linear equation in $\bar{A}$ and C assuming an estimate of $f_i^{\cdot s'}$ and $f_i^{\cdot s}$. The values of c depend on the values of h_i^r and the regional technical coefficients, as well as the estimates for $x^r, f_i^{\cdot r}, m_i^r$, and e_i^r. We choose h_i^r to minimise the variance in c while requiring that the h_i^r combine appropriately to give the national value of h_i – without this requirement, the variance in c is minimised when h_i^r is zero for all regions r.

7.3.3 Proportional estimates in the Bayesian setting

When creating an estimate of a regional value by pro-rating a national value, we can think of that estimate as the mode of a prior on the regional value. Hence, if we estimate the amount of gross capital formation in a region as the product of total gross capital formation and the proportion of total production that comes from that region, we can think of this as a prior on the vector of capital formation as a proportion of total regional output.

More precisely, if v is a vector with an entry for each national industry, and q is a vector with an entry for each region, we can form for each region r the vector:

$$v^r = v \, \frac{q_r}{\Sigma q_s} \qquad \text{(Equation 7.31)}$$

that is, proportionally scaling v according to the relative size of the entries in q. Examples of this would be in estimating gross capital formation (in which case v is national gross capital formation and q is the vector of regional output or regional operating surplus or regional consumption of capital plus additive change in total output, for example).

More generally, q could be a matrix with a row for each industry and a column for each region, and we could form the vector with entries:

$$v_i^r = v_i \frac{q_{ir}}{\Sigma q_{is}} \quad \text{(Equation 7.32)}$$

An example of this would be estimating exports by apportioning the amount of exports by industry i in region r according to the proportion of production by industry i done in region r.

Regardless of how they are calculated, the vectors v^r can be thought of as the product of v and a random variable that is distributed as Dirichlet.

7.4 Statistics NZ held data

A number of sources of regional and industry data are described in *Regional statistics at a glance* (Statistics NZ 2015). These statistics are commonly produced by Statistics NZ and there will be information in the Longitudinal Business Database (LBD) and the Linked Employer-Employee Database (LEED) that could be used to provide regional or industry data not listed.

Some data sources provide data at a territorial authority level, which is more granular than regional, but the 67 territorial authorities do not each belong to only one of the 16 regions – some territorial authorities overlap regions. When using data at a territorial authority level we need to apportion appropriately to the various regions and then aggregate.

Several of the data sources are described as having ‘partial’ regional coverage. In order to protect confidentiality or to improve the robustness of the estimates, the data from several regions has been combined into one or more ‘virtual’ regions for reporting purposes. In using this data to develop RIOTs we are faced with a choice of either using the published data, which presents a technical challenge of inferring disaggregated regional information from the aggregated statistics, or return to the source data and, where privacy issues allow, extract regional statistics and estimates of their uncertainty.

In addition to the published data we need the sample standard deviations as well, which are not published but surely will be available from Statistics NZ if requested.

Another recourse to the published data is to assemble data from the LBD. This has the benefit of being able to assess the covariances of the various statistics – for example, value added, total output, and even exports should be identifiable by region and industry and the businesses that contribute provide the link that allows the covariance to be established.

In the following sections we note which Statistics NZ’s publications might be of use for the various pertinent statistics, namely total output, value added, consumption, exports, and imports. Some is regional and some is national – we include the national data as it will be useful for updating the national technical coefficients.

7.4.1 Regional production

Regional GDP estimates are reported for 15 regions (with Tasman and Nelson combined) and 19 industry collections (in contrast to the 106 industries used in the national IOTs). These estimates are in producer’s prices. The data is produced annually with the year ending in March.

Productivity statistics provide annual estimates of the year-on-year percentage change in input and output of 28 industry collections. They are useful for updating national tables and identifying imports and exports (insomuch that these balance the difference between inputs and outputs and domestic consumption).

National account data on GDP(I) gives operating surplus and salary and wages annually for 55 industry groups. It can be used to estimate value added at a national level to update national accounts.

GDP(P) data gives total output, total inputs and value added for 55 industry groups nationally.

7.4.2 Regional consumption and final use

Regional final household consumption can be estimated on a pro-capita basis from the national IOTs, with prices updated using CPI data, which is reported semi-regionally as well as nationally. Regional deviations from national pro-capita averages can also be taken into account, albeit imperfectly, using survey (Household Economic Survey) and census data on income and household spending.

There is a wealth of information on local and central government finances and expenditure, though the published central government data does not identify the location of expenditure nor link the products purchased to the industry categories used in the national IOTs. Some work would need to be done to carefully assemble the published local and central government finances and expenditure tables into a useable framework.

The LBD and LEED could be used to identify the regional consumption by central government, pro-rating the total government purchases (not consumption) according to staffing numbers in the regions.

Central government consumption could be updated nationally from survey data to reflect changes in prices, and apportioned to regions in a manner appropriate to the product consumed - transport infrastructure consumption according to where the roads are; education based on where the schools, universities and polytechnics are; healthcare and hospitals by capita; residential housing operations according to where the state housing is located. This information is available, but needs to be systematically collated and made use of. Work in this area has been done, see NZIER (2013).

National accounts data on GDP(E) gives gross fixed capital formation annually for 55 industry groups, and these could be regionalised proportional to regional GDP by industry.

7.4.3 Regional overseas exports and imports

Recent work by the author of this report (Holt 2016) could be used to estimate regional exports. The work built a model to predict which businesses in the LBD were exporters. When businesses are exporters the best estimate for export revenue is the amount of zero-rated GST – this would be in producer's prices. The model was based on grouped economic units, not geographic economic units, so some modification might need to be done to employ it for identifying regional exports. Because the model provides a probability of any given company being an exporter, when a company's region is identified in the LBD the expected amount of exports the company is responsible for can be calculated and allocated the regional exports for the industry the company belongs to.

Alternatively, exports could simply be apportioned to regions based on the national proportion of the industry's production that occurred in that region, making the assumption that exports are sourced uniformly across the locations of production. This is equivalent to using total national exports and disaggregating parameters whose prior is set to be proportional to the regional industry's share of the industry's national GDP – see section 7.2.2 for the sense of this statement.

Different regional industries may need more or less imports as part of their total outputs. The total amount of imports by HS10 classification is published regularly by Statistics NZ and hence using the supply table one can approximate the amounts of imports by industry – granted, this only covers merchandise imports, and a large proportion of imports is consumed or is for capital formation, with the industry mix of these imports not specified.

The following table summarises the use of data (Statistics NZ-held and third party) in setting priors and constraints:

Table 7.1 Data sources for setting priors and constraints

Parameters	Data for setting priors	Data for defining constraints
Regional technical coefficients	National IOT, Xero data, Longitudinal business frame	Regional GDP, national accounts, productivity statistics.
Trade coefficients	Gravity model, freight demand study, eRUC data, Xero data.	Extended CHARM method
Consumption trade coefficients	Gravity model, freight demand study, Marketview data, tourism satellite account.	Extended CHARM method
Regional disaggregations	LBD data, IDI data, census data.	Depends on variable being disaggregated.
Industry disaggregations	LBD data, LEED data.	Depends on variable being disaggregated.
Regional o/s exports	Regional GDP, Ports data, LBD data, national Supply table.	Exports data, LBD data.
Regional o/s imports	Gravity model, Ports data.	Imports data.
Regional household consumption	Population demographics, income survey data, national IOT, CPI data, Marketview data, IDI data.	
Regional government consumption	CPI data, government administrative data, IDI data, national IOT.	Administrative data, Treasury/MBIE data.
Regional gross capital formation	National accounts data, regional GDP data, productivity statistics, Xero data.	Productivity statistics, LBD data.

8 Incorporating third- party data

The methodology detailed in the preceding section allows data collected as part of the official statistics system to maximally inform RIOTs, and to measure where (in terms of inter-regional and inter-industrial flows and consumption) the uncertainty lies in the table's values. More data would allow the uncertainty to be reduced, but at additional expense due to the collection and processing costs.

Third-party data exists and offers the potential to improve the accuracy of the regional input-output model. In this chapter we discuss four sets of third party data: Xero data on financial transactions, eRUC data on road freight transport movements, Qrious data on cellular telecommunications devices movements, and Marketview data on electronic card transactions.

All four of these datasets are examples of dynamical systems in which objects (money or vehicles or devices) change – either in terms of their location, ownership or category. The reason for thinking they might inform regional input-output modelling is that the parameters in the regional input output model can be thought of as analogous to probabilities: the probability of trade between two regional industries or the probability of input from one industry into another. The analogy is not strictly fair as the technical coefficients are proportions of monetised input required – so the challenge in making use of many of these data sources is to turn probabilities into proportions of monetised inputs, or to only make use of the data to improve our understanding of inter-regional trade.

Unlike survey data, the third-party data is biased and unrepresentative. For this reason, the avenue we have pursued is not to try to extract direct estimates of key statistics, but rather to model the data as a parametrised stochastic dynamical system in such a way that either:

1 The parameters include (some subset) of the parameters of the regional input-output model or the regional input coefficients, or

2 The parameters can provide observations of the parameters or the regional input coefficients, so that the parameters can be improved by Bayesian update.

Each of the first two sections that follow have the same basic structure – the third section, on Qrious data, is different because of the particular challenge that Qrious data presents. We begin by describing the data, its source, what is known about its coverage and issues of bias, and challenges that would need to be overcome in terms of data preparation or the restrictions the owner might have on data access. Following this, we discuss the dynamics of system described by the data and how those dynamics relate to the regional input-output model, if at all. In some cases considerable detail is provided where we believe the link is stronger, as with Xero data.

8.1 Xero data

Xero is a cloud-based accounting software package that is owned, developed and managed by a company of the same name. The product is aimed at small-to-medium enterprises, and offers to simplify the accounting process by drawing data directly from the customer's banks so the customer need only code the financial transactions according to their code of accounts and process employee expenses that are not present in the bank data. There are additional modules to support invoicing and payroll, but they are not core to the product.

At the time of writing Xero has 200,000 New Zealand customers, though a substantial (but unquantified) number of these would be deemed not 'economically significant' by Statistics NZ (under $30,000 per annum in revenue).

In addition to the accounts data, Xero captures the New Zealand Business Number (NZBN) and address information. This means that it is possible to link the Xero customers to the information held in the New Zealand Companies Register. This holds information on addresses, business type, directors and shareholders. And in recent years, new companies have been required to nominate their industry classification, but for older businesses this data will be missing. By the end of 2016, Xero is anticipating having built a link between their data and the Companies Register data through the NZBN application program interface (API).

The most comprehensive and reliable source of industry classification data is the Accident Compensation Corporation (ACC) – with the caveat that businesses might choose to classify themselves in order to minimise their ACC levy. Should the Ministry of Business Innovation and Employment (MBIE) integrate the Companies Register data with ACC business classification data (or fill the gaps in the industry coding data in some other manner) the Xero data could be attributed to industry and region of head-office operation.

Information about the location of suppliers, buyers and business operations could be inferred from bank data even without the coding of financial transactions. Every New Zealand bank account number encodes the name of the bank and the branch. If we assume small businesses and people bank locally to their place of operations or residence then this allows us to identify where operations are (according to where salary and wages or dividends are paid), where suppliers are located (according to where business expenses or capital costs are paid), and where customers are located (according to where sales are paid from).

When there are financial transactions between two Xero customers and there is industry classification data we obtain data about inter-industry inter-regional flows – either intermediate inputs or final consumption.

Accurately measuring the retail or wholesale margin is a challenge. (This margin is the gross output by the retailer or wholesaler in the input-output framework). A retailer will have a number of different suppliers and may have a different margin with each; data on sales will not stipulate the relative quantities coming from each supplier. Similarly, for wholesalers. The ‘expenses’ of retailers includes the inputs required to ‘produce’ retail services and the costs of the goods sold. The inputs into retailing are those that are not the costs of goods sold, and the retail margin is the difference between sales and expenses apportioned amongst the goods sold. This apportioning is an unknown factor. It is problematic when a retailer fails to code their stock purchases as ‘cost of goods sold’ or similar.

Transportation of goods and services is frequently (but not always) arranged by the seller and charged to the buyer as part of the sale price. The default category for freight expenses in Xero does not differentiate between transport mode or between freight and courier: it is all ‘freight and courier expenses’. As in retail margin, the freight and courier expenses will be a set of financial transactions without any link to the purchasers of the goods and their locations – there will be total expenses for freight and courier by region and total sales by region, but we will need to infer the appropriate amount of freight and courier expenses to subtract from the total sales (along with retail and wholesale expenses) to obtain the actual intermediate inputs. This is a different (and harder) problem than for retail or wholesale margins because freight costs change with distance between buyer and seller to an unknown degree. In retail and wholesale margins it is arguably more reasonable to apportion the cost according to the costs of the goods sold by region and industry.

The coding of financial transactions is very flexible, so it is difficult to systematically identify what product or product type a financial transaction concerns. Some common categories such as ‘domestic travel’ include transportation, accommodation, and food and beverage costs, or other incidental expenses as might be incurred when travelling. To separate this into individual product categories in line with those used in the supply and use tables is not feasible without a large amount of effort involving text mining and semantic classification.

However, the coding is set out to allow the automatic generation of financial statements and IR10 returns, and the broad ledger categories of expenses, assets, liabilities, equity and revenue are well defined. Transaction codes to expenses should be largely purchases and salaries and wages. Assets is net capital formation. Revenue includes sales.

Understanding exports is problematic. There will be instances where a business sells directly to an overseas buyer, in which case the transaction will be GST zero-rated and the transfer of money will be from a foreign bank account or an intermediary such as PayPal. But many businesses may not sell directly, but instead sell to a wholesaler or retailer who exports and engages in domestic sales. In this situation it is not possible to know the actual goods exported if the exporter sells a range of products from a range of producing industries. Another issue would be exporters having their overseas sales transferred onto foreign credit cards and using these cards to make payments in New Zealand, but this is contrary to tax regulations and is unlikely to be widespread.

The data takes some time to stabilise as accounts are subject to revision up to seven years after tax filing. In practice the annual data to the year ending 31 March stabilises after the filing of business income taxes. Thus while it might be feasible to use the Xero data to better understand how intermediate inputs and final consumption change over a year (say quarterly), it would be inadvisable to use that data until it is a year to eighteen months old.

In summary, the Xero-held data has the potential to provide a lot of information on inter-industrial, inter-regional flows to (and occasionally from) small-to-medium enterprises. In order to realise this potential, the following needs to occur:

1 The attribution of industry codes to NZBN in the Companies Register needs to be more comprehensive.
2 An assessment needs to be made of how representative the group of Xero customers is in the larger group of economically significant companies. For example, what is the distribution of business demographic data for the Xero customers in each region so we can compare this to the Statistics NZ business demographic data and understand the relative over- and under-representation in the Xero data.
3 It must be feasible to predict what industry the businesses transacting with Xero customers are in, making use of the data available: bank account associated to the transaction, ledger codes of transactions associated to that bank account, industries commonly associated with the region in which the bank branch is located.
4 There needs to be a sufficient number of Xero customers from each of the wholesale and retail industries to provide information about how retailers and wholesalers source and distribute goods. Without this it is not possible to use the Xero data to estimate inter-regional, inter-industrial trade.

8.1.1 A statistical model of Xero data incorporating regional input-output matrices

Because frequently information about industry or region or the nature of the transaction will be missing from the Xero data, regardless of how the data is used to inform the development of RIOTs there will be a need to impute this missing data, drawing on the power of the information that is present. In this section we outline a statistical model for the Xero transactional data. In the following sections we describe two ways in which the data (both actual and inferred) could be used for RIOT development.

In the model we present, an observation is a financial transaction. Each financial transaction is described by data, some of which is available to us and some of which is unknown. The data that describes a transaction is as follows:

1 S_I – the industry code of the seller.

2 S_R – the region in which the seller operates.

3 B_I – the industry code of the buyer.

4 B_R – the region in which the buyer operates.

5 N – the nature of the transaction, be it imports, exports, intermediate inputs, household consumption, government consumption, gross capital formation, salary and wages, or tax expenses.

6 A – the nominal amount of the transaction.

7 R – the tax rate of the transaction.

8 T – the freight/transport margin of the transaction.

9 V_T – the transport mode of the freight.

10 M – the retail/wholesale margin of the transaction.

11 V_M – the margin type, either a variety of retail or wholesale. This is an industry category from the supply and use tables.

12 C – the code of the transaction from the chart of accounts, or the ledger category of the code.

13 S_A – the address of the seller.

14 B_A – the address of the buyer.

15 S_{BA} – the region of the bank branch of the seller's bank account.

16 B_{BA} – the region of the bank branch of the buyer's bank account.

The data we have depends to an extent on whether the buyer or the seller (or both) is a Xero customer, and whether information like industry codes are known for every company with an NZBN. We will always have the nominal amounts and the tax rate; sometimes we will be certain of the nature of the transaction, such as salary and wages or tax expenses. Even these known values can be erroneous due to user error or tax non-compliance.

When we do not have data we need to infer the missing values by leveraging our understanding of how these variables co-vary. Figure 8.1 presents a possible causal diagram of the system with observable variables appearing in a square and unobservable (latent) variables appearing in a circle. There is a directed line from one variable to another when the originating variable has a causal effect on the destination variable.

Thus, from figure 8.1 we can read off the factorisation of the joint probability distribution of the 16 variables above and the associated aggregates (total expenses, total sales and transportation expenses by mode, which might be assessed knowing the proportions of mode use for the supplier industry). A fully specified model requires the distributions of these variables to be described, generally in terms of standard distributions and parameters (and hyper-parameters in the Bayesian approach). The Bayesian approach is then to update the hyper-parameters using the data, while the classical approach would be to estimate the parameters by optimisation of the likelihood function or the entropy.

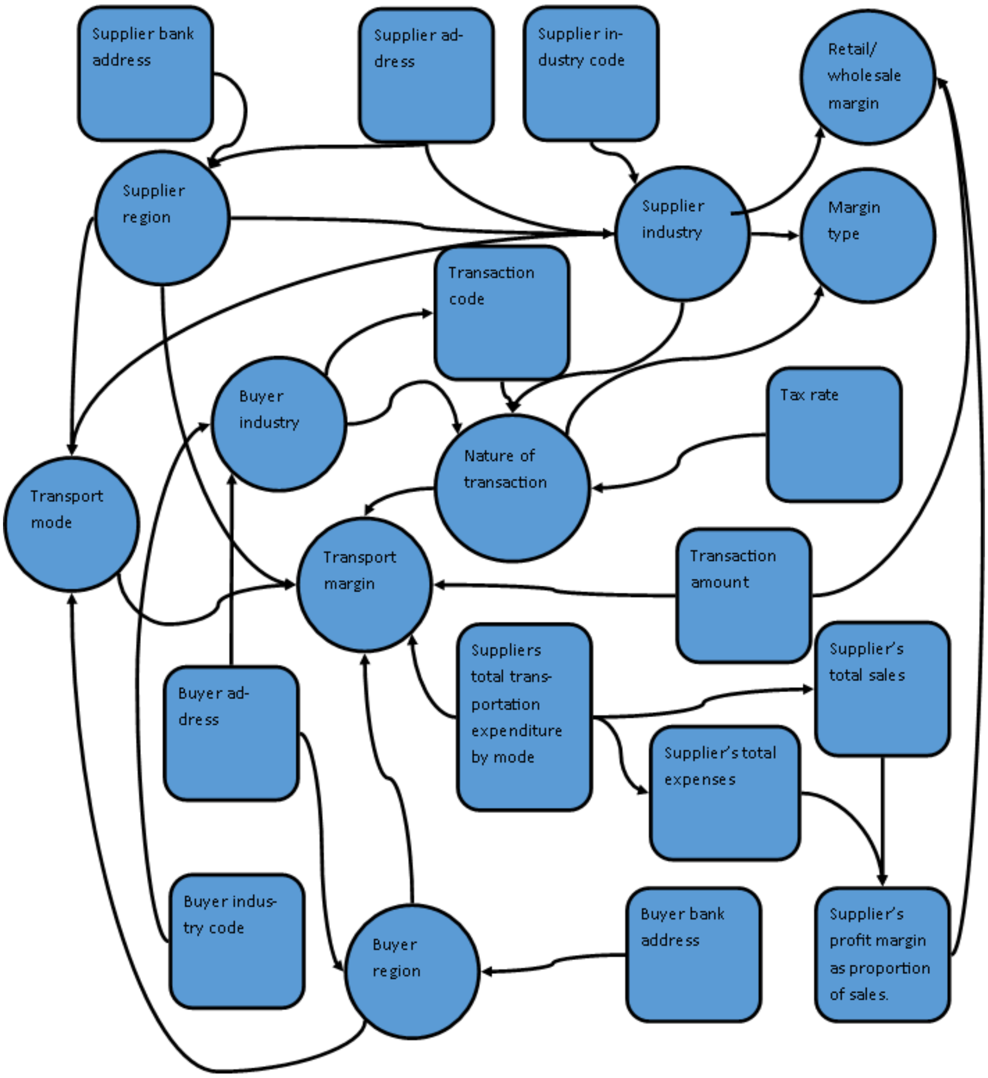
The probabilities estimated will be in terms of the frequency in which values occur in the transaction data. This is in contrast to the input-output framework where the technical and trade coefficients are proportions of total outputs or inputs, respectively, over entire industries or regional industry pairs.

Consider the trade coefficient T_{ij}^{rs}. By definition, it is the quotient of the amount of input from industry i in region r by the amount of input from industry i required by industry j in region s. This input could be a direct transfer or it could be through wholesale or retail industries.

Retail and wholesale industries are service industries through which flows much production and many imports. When a purchase is made from a retail or wholesale industry it may comprise goods from a number of industries, but we have no direct observations of what these goods may be. This is problematic as it obscures the apportionment of supplier to buyer. We stated there needs to be a sufficient number of retailers and wholesalers in the Xero customer base so we can attempt to address this by estimating the proportions of retail/wholesale sales by regional industry of production.

Using the model, every transaction, where the supplier is in the retail or wholesale industry and the nature of the transaction is 'sales', is a collection of 'virtual transactions', one for each region-industry pair. We will make this more explicit.

Figure 8.1 Depicts a possible causal structure relating the variables listed above. This causal structure translates into a factorisation of the probability density. This factorisation specifies the modelling problem.



8.1.2 Virtual transactions

Fix a transaction where the supplier industry is in the wholesale or retail industry, call it w. Let u be the region of the supplying wholesaler/retailer. Because we have assumed there are sufficient Xero customers from industry w in region u (which is unlikely to be the case for large retailers), we can estimate a matrix D that describes how input from regional industries is sourced and dispersed. That is:

$$D_{js}^{ir} = \frac{\mathbb{E}(A - M - T \mid S_I = i, S_R = j, B_R = u, B_I = w, N = N_0)}{\mathbb{E}(A - M - T \mid B_R = u, B_I = w, N = N_0)} \frac{\mathbb{E}(A - M - T \mid S_I = w, S_R = u, B_I = j, B_R = s, N = N_0)}{\mathbb{E}(A - M - T \mid S_I = w, S_R = u, N = N_0)} \quad \text{(Equation 8.1)}$$

With this, if the transaction between (w, u) and industry-region (i, r) has input value $A - M - T$ then we can replace this transaction with NR transactions, one for each region-industry pair (j, s). The amount of the transaction is $(A - M - T)D_{js}^{ir}$, and it has zero transport margin and zero retail/wholesale margin. If j is another wholesale or retail industry, then D_{js}^{ir} will need to be replaced with the vector $D_{kp}^{js}D_{js}^{ir}$ and the entries added to the appropriate entries of D, repeating as needed for chains of wholesale transactions.

Similarly, we create virtual transactions for each transaction in which there is a transport margin as follows.

Transportation margins in transactions represent inputs by the appropriate transportation industry into the industry of the **buyer.** The region of operation for the transportation input is information the supplier may be privy to, but generally not the buyer. For each industry and pair of regions, we need to estimate the mode shares of transport use by suppliers from one industry and region and buyers in another region – this is a parameter that specifies the distribution of the variable 'transport mode'. Lacking information to the contrary, as a prior for this parameter we assume the transport mode is independent of the buyer's region – the posteriori distribution may not display this independence, of course. That is, we set a prior probability (mean proportion of transport cost for the given mode) for the transport mode V_T based in region u as:

$$p(V_T, L_T = u \mid B_R = s, S_R = r, S_I = i) = \frac{p(S_I = N_{V_T}, S_R = u \mid B_I = i, B_R = r)}{p(S_I \in N_T \mid B_I = i, B_R = r)} \frac{\mathbb{E}(A \mid B_I = i, B_R = r, S_I = N_{V_T}, S_R = u)}{\mathbb{E}(A \mid B_I = i, B_R = r, S_I \in N_T)} \quad \text{(Equation 8.2)}$$

where N_{V_T} is the transportation industry that provides mode V_T, L_T is the region in which the transport industry is based and N_T is the set of all transportation (cargo and courier) industries. Again, the probabilities in the right-hand-side are in terms of the relative frequency of transactions.

Thus for a transaction where the buyer is industry-region (i, r) and the supplier is industry region (j, s) we generate a transaction for each pair (k, u) where u is a region and k is a transportation industry, with say mode V_T. This transaction has supplier (k, u) and buyer (i, r); the transaction amount is:

$$p(V_T, L_T = u \mid B_R = r, B_I = i, S_I = j, S_R = s)T \quad \text{(Equation 8.3)}$$

The transport and retail/wholesale margins for this virtual transaction will be zero by construction.

8.1.3 Incorporating the trade matrices

The main intention of describing the model of the Xero data in such detail is to show that the trade and technical coefficients can be naturally seen as parameters of the model, so that fitting the model allows the hyper-parameters defining the distributions of trade and technical coefficients to be updated.

If the transactions in the Xero data are fully representative, then the trade coefficients T_{ij}^{rs} are related to some of these quantities as:

$$T_{ij}^{rs} = p(\, S_R = r \mid S_I = i, B_I = j, B_R = s, N = N_0) \, \frac{\mathbb{E}(A - M - T \mid S_I = i, S_R = r, B_I = j, B_R = s, N = N_0)}{\mathbb{E}(A - M - T \mid B_I = j, B_R = s, S_I = i, N = N_0)} \quad \text{(Equation 8.4)}$$

where N_0 is 'intermediate inputs'.

Note with our use of virtual transactions, this is true for transportation, retail and wholesale industries as well.

The probability is the first factor on the right-hand side is in terms of the frequency of transactions, not the amount of the transaction. If there is bias in regional representation or where the relative average transaction amounts differ greatly from the true average, the trade coefficients will be poorly estimated. But if the bias in these senses can be estimated then the transaction coefficients can be adjusted for this bias.

8.1.4 Incorporating the technical coefficients

Technical coefficients are related to the model components as:

$$A_{ij}^{r} = p(S_I = i \mid B_I = j, B_R = r) \, \frac{\mathbb{E}(A - M - T \mid B_I = j, S_I = i, B_R = r)}{\mathbb{E}(A - M - T \mid B_I = j, B_R = r)} \quad \text{(Equation 8.5)}$$

If the technical coefficients do not change with the size of the business and we have full visibility of all the purchases a business makes, the Xero data should conform to the use of inputs as described by the technical matrices.

8.2 Electronic road user charge data

Taxes to support roads are levied on petrol at the point of sale, but not for diesel as some consumption of diesel occurs on private roads, such as farms. Instead of a sales tax on diesel, the road user charge system has been adopted, in which operators of diesel-fuelled vehicles buy licences to use their vehicle on public roads – these licences cover certain amounts of distance travelled before expiring, and there are different licensing costs for different classes of vehicles.

Several private companies have been given the delegated authority to assess and collect road user charges on behalf of the Transport Agency using a system by which a vehicle's travel is tracked by GPS and the appropriate levy automatically calculated. This is the electronic road user charges (eRUC) system, and the data collected in the course of operating this system is called the eRUC data.

It is an 'opt in' system – no vehicle operator is obliged to use the eRUC system. The users tend to be large fleet operators rather than owner-operators or operators of small fleets. At the time of writing there were around 18,000 heavy vehicles taking part in the system, mostly in the North Island. The dataset is growing quickly, to the order of millions of new data points every day.

The system has been in place since 2011, so there are currently five years of data. The eRUC operators are obliged to keep their data for seven years. The data is 'owned' by the eRUC operators, and it is not known what they will do with the data older than seven years.

Vehicles and trailers in the system are fitted with devices attached to their wheel hubs. These devices comprise an odometer, a GPS unit and a cellular telecommunications connection to transmit data to a central repository.

The frequency at which data is transmitted depends on the vehicle's movement, capturing when the vehicle is at rest and in motion, sufficiently granular to be able to reasonably accurately assess driving speeds and the length of periods of inactivity.

Part of the agreement between the eRUC system operators and the Transport Agency is that the latter has agreed to use only aggregated data to preserve the privacy of the freight operators. To this end, the Transport Agency has engaged Beca to create tables of data from the raw eRUC data which would be used for reporting and analysis. Beca currently owns these tables and is likely to want to be compensated for any further use.

Each datum consists of a time-date stamp, location data and the identification code of the trailer or vehicle. The identification code is linked to static information about the trailer or vehicle: the road-user-charge vehicle class (numbers of axles, weight bearing capacity, whether or not self-propelled, slightly different from the standard vehicle classification in the NZ Transport Agency (2013) *Economic evaluation manual*.

If a vehicle is towing one or more trailers, those trailers may also have eRUC devices and hence will also be generating eRUC data.

To use eRUC data for developing RIOTs it is necessary to identify origins and destinations of vehicle trips, volumes or values of cargo being carried, the industries producing the product being carried, and the industries owning the destination of cargo. In addition, the bias present in the data due to the eRUC system being opt-in needs to be assessed, so adjustments might be made for regions and industries whose freight transport needs are more frequently being met by operators not using the eRUC system.

Making use of the eRUC data will require a deal of data integration and data preparation. The 'ticker-tape' of vehicle/trailer location data will need to be organised into a form that can be analysed. This might be similar to what has been done by Beca. Moreover, the data needs to be integrated with Land Information NZ (LINZ) or the regional council rates or the Companies Office data to turn location data into information about the industries owning or operating at the locations. Before any modelling can occur, there is significant data preparation to be done.

We take a conceptual view that a powered vehicle in the eRUC system is moving through a sequence of distinct 'states'. These states relate to the vehicle being at or travelling towards a location and what the operator is doing while at that location. These states would be:

1. Travelling between locations
2. Refuelling or resting
3. Dropping one or more trailers
4. Acquiring one or more trailers
5. Swapping one or more trailers
6. Unloading a truck or trailer
7. Loading a truck or trailer.

The state the vehicle is in depends on the location they are at. Altering a B-train's configuration of trailers would more likely happen at a depot than at the location of a pick-up, for example. We will therefore need a probabilistic classification of locations, say into depots, production origins, production destinations, refuelling, service or rest areas, domestic air terminals, export air terminals, rail terminals, export shipping terminals and domestic shipping terminals. Terminals and depots could be identified directly from a list of addresses of these, or inferred from the number of vehicles present at any time, their behaviour and proximity to sources of production or transport infrastructure.

The state also determines what might happen at the location. If swapping one or more trailers, then the vehicle will move serially around the depot with pauses to unhitch and then re-hitch trailers – and the identification codes of the trailers travelling proximally to them will change. The state is related to time spent at the location, measures of short distances travelled while there, changes in 'fellow traveller' eRUC identification codes, and the like.

At any given time, every vehicle and every depot has a mix of cargo in various quantities – with average price/volume estimates we can estimates this as values rather than volumes. When a vehicle drops, loads, or swaps a trailer at a depot the mix for the vehicle and the depot changes. How it changes is unknown, but is informed by the origin and destination of that vehicle and other vehicles using the same depot. When the region and industry of destination is known, the regional input coefficients, together with national supply and use tables, will provide information about what cargo a vehicle unloading at that destination might be carrying.

To identify what industry produced the cargo being carried we need to deduce what industry is operating at the location of a pick-up. This could be done by using the LINZ data or regional council rate data to match addresses to find property ownership or property-usage information.

Over time a vehicle's location, location type and state change. What region the location is in, when the location type is a production destination, depends on the regional input coefficients, the location of the production origin, the industries present in the cargo and the industry of final destination. The challenge is that the regional input coefficients are assessed annually and there may be seasonality in the state transitions. There is sufficient data to include seasonality into the model, so by averaging over a year we could obtain measurements – sets of regional input coefficients – that could be used to improve the regional input-output models. Moreover, this seasonality could then be integrated into the regional input-output models to provide seasonal regional input-output models.

Parameters to the model will include transition probabilities between location types conditional on qualities about the vehicle, its cargo, and qualities about the previous location type such as its region; and transition probabilities between states, conditional on location types and the activity of the vehicle at each location. These transition probabilities, once the model has been fitted, can combine with other parameter estimates and estimates of cargo value by industry, to give probabilities of instances of road freight trade between regional industries. After adjusting for bias, and other freight transport modes, this could improve the estimation of the region-input output model parameters.

8.3 Qrious data

Qrious is an entity within Spark Ventures that produces information products based on Spark's cellular telecommunications data.

Qrious generally makes products – often web-based dashboards – for customers rather than providing data. They have a capability to deploy through APIs, but have not done so yet. Some products are continually updated as data is collected.

The data is about the location of cellular devices that are part of the Spark network – this includes foreign, roaming users, who can be identified and removed as necessary. Spark has a significant market share – numbers are commercially sensitive, but in the order of 45% of the market (biased between household and business/corporate use). Over two billion new records are collected every day.

Similar to eRUC data, a stream of location data is logged for each device. A datum consists of:

1 A device id unique to that device, but not identifying the owner

2 A date-time stamp and location data (coordinates triangulated from the nearest cell-phone towers).

Qrious will not release information if it would allow the breach of an entity's privacy. In practice this means that the smallest granularity at which they will release data is at the meshblock level.

Cell phones and other cellular devices tend to be carried by people. Accordingly, the Qrious data is about the movement of people, regardless of the transportation mode, in contrast to the eRUC data where the data is about the movement of road freight vehicles. But as with the eRUC data the challenge in making use of it in an input-output context is to convert these flows into information about trade, consumption and intermediate inputs. There is nothing monetary about the Qrious data, and unlike the eRUC data there are no simple proxies one can construct from volumes and average price per volume estimates.

The system is a complex one as it describes people movements in general and not just for a specific purpose, such as travelling to provide services. This contrasts with the eRUC data where the purpose is to move cargo and seldom would describe a driver's movements while idly shopping to fill a rainy Saturday afternoon.

There is information about the provision of services in the data, but it would be a significant challenge to extract it. Even if a device could be associated to a place of business, and the industry of that business identified, there is nothing but circumstantial evidence to link movements of that device to other places of business, to the provision of service or the monetary value of that service. And private devices will change business associations over time, or people will obtain new devices without concurrent ownership of the previous device to allow 'hand over' of the data stream. The reader will have no difficulty identifying a large number of perfectly normal human behaviours that will make the association of cellular devices to the provision of industrial inputs extremely challenging.

Having said that, it is possible the Qrious data could improve understanding the regional distribution of air travel expenditure to industries, to domestic tourism and to household consumption on the borders between regions. We note this use of data is different from that of the previous datasets in that we simply would try to use Qrious data to estimate regional apportionment of certain inputs, rather than building a model that can be related to the regional input-output model.

To describe this use, we point out some inferences that can be made on the Qrious data; these could be used to address some of the listed uses.

From the basic data one can attempt to infer other information:

- Devices in common ownership – though this can be problematic in multi-story buildings with multiple residents due to an inability to distinguish vertically or due to errors in measuring distances using GPS data. Moreover, only likely to be able to detect household ownership, not individual ownership.
- Frequent location at night – as a proxy for home residence. Again, there are accuracy issues when the address is multi-story as the residence within the multi-residence dwelling might not be identifiable.
- Frequent location during the day – as a proxy for office location. Same issues as for identifying home residence.
- Detection of instances of air travel, identifying origin and destination airports, and surface travel at either end of the plane journey.

- Detection of frequent air travellers, identifying origin and destination airports, average demographics of destination meshblocks, and meshblocks of the traveller's most common locations in each destination and at origin.
- Assess a meshbock's 'roaming radius': average distance from home residence travelled in any given day, by meshblock.
- Identify whether a business or a personal user of air travel by comparing travel patterns.

If frequent business travellers can be identified, and their place of business linked to an industry or industries (say through a Companies Office API or the Google Maps API), then it would be possible to count the number of instances of air flights – ideally the ones estimated as being for business. This could be used to estimate the probability of inter-regional service trades, further broken down by the industries being serviced as the region of delivery, if we can identify the industries of the businesses that travellers spend significant time at. (Note this is problematic in that sales visits will need to be separated from service provision visits). This might inform the aspects of the trade matrices – those related to the inter-regional trade in services, mostly.

Transport margins are apportioned to the buyer of services, so after making some assumption about air transport margins, we might apportion inputs from air passenger travel to regions or regional industries.

The identification of the 'roaming neighbourhood' of device owners at locations near regional borders could be used to improve the estimates of region sources of household consumption, by measuring what proportion of the roaming neighbourhood's retail stores are in the contiguous regions. This is still problematic as it does not take into account internet purchases, but that might be addressed to a degree by estimating the number of deliveries from courier depots to households and employing an estimate for average value by delivery.

Even if we are able to infer business location, business travel and such, based entirely on circumstantial evidence, the problem remains of understanding the bias in the data – is the data more frequently about some types of businesses or entities that is warranted as a proportion of the population? Spark may be willing to release high-level statistics that show certain demographics of their device holders – such as the proportion of accounts owned by businesses, broken down into bands according to the number of phone numbers or accounts or devices, which might be a proxy for business size by employee number This is information Statistics NZ publishes at a regional level.

8.4 Marketview data

Marketview, a subsidiary of the Bank of New Zealand (BNZ), provides information and data about electronic card (credit card or EFTPOS) spending. It has been in existence for over 15 years and in that time has expanded its data to cover more than just the transactions of BNZ customers: through an association with Paymark, a card transaction clearing-house, Marketview has access to 75% of New Zealand card transactions covering 80,000 vendors.

Marketview data is currently used by MBIE and Statistics NZ to measure tourism outputs.

The BNZ has a 20% share of the credit card market, roughly. It has a similar share of main bank customers – the definition of main bank can vary from bank to bank, but generally it is where income is deposited. The Paymark data allows the BNZ customer data to be assessed for bias in card spending activity (both amounts and vendor categories). Census data is used with BNZ address and date-of-birth data to assess bias in spatial location and demographics. Marketview notes on their website that they are under-represented in the under 15-year-old demographic; the BNZ requires credit card holders to be at least 18 and debit card holders to be at least 13.

The use of Paymark data began in 2007, giving time-series data from 2008. Data is updated daily, so if a RIOT was built and improved using Marketview data, it might be possible to see season effects of household consumption on regional trade and production.

Cards are classified as personal credit cards, debit cards, business credit cards and fuel cards. Retail spending is that which is done by personal credit and debit cards. Marketview is considering whether to develop a statistical model to identify business spending on personal credit cards or debit cards (where presumably the person would be subsequently reimbursed by their business).

Marketview has categorised the Paymark vendors by ANZSIC code – this is primarily focused on retail vendors. This coding has not been cross checked with ACC data or other primary sources of ANZSIC data. In addition, the vendors' locations are known and can be reported at the meshblock level.

Transactions that are reversed are not removed from datasets, an issue Marketview thinks would affect only a very small portion of transactions.

Marketview data does not include direct deposits, hire-purchase, or cash transactions. Hence the value of the data is more in household spending than in business spending, and as there is thought to be a bias towards cash spending for some demographic groups, some care needs to be taken when using the data to estimate regional household consumption.

Some internet commerce will be captured, provided payment is not made by direct deposit or via PayPal or similar credit-card proxy. Credit card payments to PayPal can be identified, and so it is possible to estimate the total amounts of internet commerce, but not the location nor industry of the vendor.

8.4.1 Using Marketview data to estimate regional household consumption

On the face of it, Marketview data provides estimates at the meshblock level of retail spending by households. However, there are gaps in the data: non-card payment transactions are missing; the industry and region of the products purchased are unknown and the retail margin is unknown.

Demographic information about consumer preferences for cash, hire-purchase, or direct deposit against card payment methods, needs to be collected so demographic biases to non-card payment methods can be accounted for. Once this is done, total regional spending by retail industry category can be estimated. This can be used as a set of constraints when building the RIOT – note that it will miss significant categories of household consumption such as electricity generation. It also could be used to build a prior on regional household consumption of industry output, requiring some assumptions about the relative proportions of industry output sold through each retail category.

As evidenced by the existing application to tourism, the strength of Marketview data is in the consumption of accommodation, in that the product is retailed by the producer and generally paid for by credit card. Thus, a prior estimated using the Marketview data for the proportion of accommodation supplied by one region to another should be quite robust; it could even be treated as a constraint, rather than a prior.

The data generation process for card transactions data arguably has little to do with trade or production technology (with some exceptions, such as trade in retail services). Consumers might prefer some regional production over others (such as wine or seafood), but generally products will not be regionally labelled or similar enough that any preference would be for parochial reasons and not due to product heterogeneity. Thus, a model of the data generation process for Marketview data would not need trade coefficients or technical coefficients as parameters.

9 Assessing the economic impact of transport

Transport can affect the economy in a number of ways. There can be a direct impact through capital investment in transport infrastructure or through value added by transportation activities; or there can be indirect impact through the transportation system facilitating trade, which might affect the efficiency of businesses or the competitiveness of markets.

9.1 The direct impact of the transport industry

It is a challenge to interpret value added by transportation activity because transportation is generally a cost to business. Having a large amount of value added by the transport industry could be an indication that the transport industry is inefficient and uncompetitive; or it could be an indication that businesses have structured their supply chain so efficiently that transport is an integral part of their operations. As manufacturers with complex supply chains increase their production it is conceivable their transport costs increase more than other types of inputs, as the costs of other inputs may demonstrate an economy of scale, while transport costs scale with volume and distance. In order to evaluate the impact that transportation services have on the economy we would need to understand the relationship between production efficiency (neglecting transport and retail/wholesale services), the scale of an industry's outputs, and transport costs – thus if the transport expenditure was accountable for a degree of efficiency (measured in units of production output per non-transport input) the impact of transport would be the difference between the actual production and the expected production without transport input, and less the cost of transport in production.

However, this econometric approach is problematic. Each industry is likely to have its own production function and the number of regions is unlikely to give sufficient data points, unless data is available over a number of years; New Zealand's 20 regions are definitely not sufficient. Non-parametric methods such as data envelopment analysis could be employed, but are still problematic: in its basic form data envelopment analysis is sensitive to outliers and always identifies firms with perfect efficiency, possibly reducing the overall distance to the production frontier and thereby reducing the measured impact of transport.

Moreover, the input-output framework does not report on capital. The standard production economics view is to consider output as a stochastic function of labour, inputs and capital. Labour costs may vary widely or be unreported – this is the case of the WIOD (Dietzenbacher et al 2013).

We argue that a RIOT is not particularly useful for measuring the impact transport has on the economy as a whole, at least not in terms of how much net value added can be attributed to transport. It is possible to understand the dynamic of supply and use within the transport industry and the role transport has both 'upstream' and 'downstream' in the value-chain.

Every unit of output by a regional industry is composed of amounts of value added by contributing regional industries. Some of these regional industries provide intermediate inputs to the producing regional industry, but they need not, instead proving inputs to eventual suppliers to the producer. We say that the *downstream* of a regional industry is all the regional industries that provide value added. Similarly, the *upstream* of a regional industry is the regional industries that receive value added by the regional industry in question. It is possible for the total upstream value added per unit of output to be greater than one, but the total downstream value added per unit of output is always one (or near to one, allowing for the vagaries of data, and including imports).

If we interpret a technical matrix as a set of production functions, then downstream describes the creation of value added when one more unit of production is consumed, a demand-driven view. The upstream describes the increase in value added attributed to an industry when one more unit of production is produced by every eventual buyer, a supply-driven view.

When a regional industry upstream from a given regional industry increases its output, there is a corresponding increase in value added attributed to the regional industry in question. In order to produce that value added, the regional industry increases its production with a downstream distribution of value added. That downstream distribution is accounted for in the downstream value added of the regional industry that increased its output. The difference provides a measure of how much value added the given regional industry 'absorbs' from the upstream regional industry when production is increased. For transport industries the smaller this proportion the better.

In order to calculate this absorption rate we need to decompose output into its components of value added, which we do in section 9.1.1.

9.1.1 Decomposing a unit of output into value added

We want to consider the value chain for each industry: in production what value is added by which industries? Let V_{ij}^{rs} denote the proportion of output by industry j in region s that is value added by industry i from region r. We develop an expression for V_{ij}^{rs} in terms of the regional inputs matrix and the proportion of output that is value added.

Observe that output x_j^s can be written as the sum of purchases, imports and value added (including taxes and subsidies):

$$\begin{aligned} x_j^s &= v_j^s + Z_{\cdot j}^{\cdot s} + m_j^s \\ &= v_j^s + m_j^s + \sum_{ir} Z_{ij}^{rs} \\ &= v_j^s + m_j^s + \sum_{ri} A_{ij}^{rs} x_j^s \end{aligned} \qquad \text{(Equation 9.1)}$$

so a unit of output satisfies:

$$1 = \frac{v_j^s}{x_j^s} + \frac{m_j^s}{x_j^s} + \sum_{ir} A_{ij}^{rs} \qquad \text{(Equation 9.2)}$$

Iterating this, we obtain:

$$\begin{aligned} 1 &= \frac{v_j^s}{x_j^s} + \frac{m_j^s}{x_j^s} + \sum_{ir} A_{ij}^{rs} \left(\frac{v_i^r}{x_i^r} + \frac{m_i^r}{x_i^r} + \sum_{ku} A_{ki}^{ur}\right) \\ &= \frac{v_j^s}{x_j^s} + \frac{m_j^s}{x_j^s} + \sum_{ir} A_{ij}^{rs} \left(\frac{v_i^r}{x_i^r} + \frac{m_i^r}{x_i^r}\right) + \sum_{ikru} A_{ij}^{rs} A_{ki}^{ur} \\ &= \frac{v_j^s}{x_j^s} + \frac{m_j^s}{x_j^s} + \left(A^t(\bar{v} + \bar{m})\right)_j^s + \left((A^2)^t(\bar{v} + \bar{m})\right)_j^s + \cdots \\ &= \left((I + A + A^2 + \cdots)^t(\bar{v} + \bar{m})\right)_j^s \\ &= (I - A)^{-t}(\bar{v} + \bar{m})_j^s \end{aligned} \qquad \text{(Equation 9.3)}$$

provided the inverse exists.

We denote the quotient of value added by outputs as the vector $\bar{v}$, and the quotient of imports by output as $\bar{m}$, and we consider the regional inputs coefficients matrix A to be a square matrix indexed by the pairs of regions and industries. The inverse exists, by the same argument we presented for the Leontief inverse.

The entry V_{ij}^{rs} accounts for how much value added regional industry pair (i,r) provides to the production of (j,s). This will be through all the instances in the value-chain for (j,s) where (i,r) provides inputs – either directly or through intermediaries of varying remove. Thus:

$$\begin{aligned} V_{ij}^{rs} &= \bar{v}_i^r \delta_{ij}\delta_{rs} + A_{ij}^{rs}\bar{v}_i^r + \sum_{uk} A_{ik}^{ru}A_{kj}^{us}\bar{v}_i^r + \cdots \\ &= \left(\delta_{ij}\delta_{rs} + A_{ij}^{rs} + (A^2)_{ij}^{rs} + \cdots\right)\bar{v}_i^r \\ &= \bar{v}_i^r (I-A)^{-1}{}_{ij}^{rs} \end{aligned} \qquad \text{(Equation 9.4)}$$

Hence $V = \Delta(\bar{v})(I-A)^{-1}$. From the considerations in the previous paragraphs, we see the column sums of V are the difference between 1 and the total value added by imports to an regional industry.

Let ind_T denote the industries directly associated with transport: land transport, water transport, air transport, accommodation and hotels, auxiliary transport activities (such as travel agents).

Fix an industry-region (i,r) with $i \in ind_T$. For an industry-region pair (j,s) we can consider the value added by (i,r) when (j,s) produces Δx more output: $V_{ij}^{rs}\Delta x$. In order for (i,r) to produce this value added it needs to produce $\frac{V_{ij}^{rs}\Delta x}{V_{ii}^{rr}}$ more output, resulting in $\frac{(1-V_{ii}^{rr})}{V_{ii}^{rr}}V_{ij}^{rs}\Delta x$ value added attributed to the downstream of (i,r). Hence the amount of additional value added not absorbed by (i,r) is (perhaps unsurprisingly):

$$\left(1 - V_{ij}^{rs}\right)\Delta x - \frac{1 - V_{ii}^{rr}}{V_{ii}^{rr}}V_{ij}^{rs}\Delta x = \left(1 - \frac{V_{ij}^{rs}}{V_{ii}^{rr}}\right)\Delta x \qquad \text{(Equation 9.5)}$$

With this we develop an index of regional transport industry efficiency as:

$$R_i^r = \frac{\sum_{sj}\left(1 - \frac{V_{ij}^{rs}}{V_{ii}^{rr}}\right)x_j^s}{\sum_{uk} x_k^u} \qquad \text{(Equation 9.6)}$$

When R_i^r is close to 0 (or even negative), there is at least one large regional-industry where (i,r) is absorbing a significant proportion of value added in production – negative would imply (i,r) has significant input costs. When it is close to 1, for all the large regional-industries (i,r) absorbs very little of the value added.

9.1.2 Example: World Input-Output Database

We performed the analysis above for the 2013 release of the 2000 WIOD (Dietzenbacher et al 2013) available for download at www.wiod.org/database/wiots13. This IRIOT is for 40 countries plus the 'rest of the world' and has three transport industries: land transport, water transport, and air transport.

Table 9.1 gives the results of the analysis described in the previous section applied to the WIOD 2000. Note that the score is the values of R_i^r and the 'index' is $1 - R_i^r$ expressed as 10,000[th]s and rounded.

Table 9.1 Indices of transport industry efficiency (land, water and air transport) for various countries

Country	Land transport (score/index)	Water transport (score/index)	Air transport (score/index)
United States	0.9896/104	0.9990/11	0.9964/36
Rest of the world	0.9919/82	0.9984/17	0.9990/11

Country	Land transport (score/index)	Water transport (score/index)	Air transport (score/index)
Japan	0.9929/71	0.9984/16	0.9991/9
China	0.9962/38	0.9984/16	0.9994/6
Italy	0.9973/28	0.9998/3	0.9995/5
Germany	0.9976/24	0.9996/5	0.9993/7
Great Britain	0.9976/24	0.9998/3	0.9995/5
India	0.9982/19	0.9999/1	0.9999/1
Mexico	0.9984/16	0.9999/1	0.9999/2
France	0.9985/16	0.9998/2	0.9997/4
Spain	0.9988/12	0.9999/1	0.9998/3
Canada	0.9988/12	0.9999/1	0.9997/4
Turkey	0.9990/11	0.9999/2	0.9999/1
Brazil	0.9990/11	0.9999/1	0.9999/2
Russia	0.9990/10	0.9999/1	0.9999/2
Australia	0.9993/8	0.9999/1	0.9997/3
South Korea	0.9993/7	0.9994/7	0.9997/3
Sweden	0.9994/6	0.9999/2	0.9999/2
Netherlands	0.9995/6	0.9998/2	0.9997/3
Denmark	0.9997/3	0.9995/6	0.9999/1
Belgium	0.9995/6	0.9999/1	0.9998/3
Poland	0.9996/4	0.9999/1	0.9999/1
Austria	0.9996/4	0.9999/1	0.9999/1
Taiwan	0.9997/3	0.9998/3	0.9998/2
Greece	0.9999/1	0.9997/3	0.9999/1
Finland	0.9997/3	0.9999/1	0.9999/1
Slovakia	0.9999/2	0.9999/1	0.9999/1
Romania	0.9999/2	0.9999/1	0.9999/1
Portugal	0.9999/2	0.9999/1	0.9999/1
Ireland	0.9999/2	0.9999/1	0.9999/2
Indonesia	0.9999/2	0.9999/2	0.9999/1
Hungary	0.9999/2	0.9999/1	0.9999/1
Czech Republic	0.9999/2	0.9999/1	0.9999/1
Slovenia	0.9999/1	0.9999/1	0.9999/1
Malta	0.9999/1	0.9999/1	0.9999/1
Latvia	0.9999/1	0.9999/1	0.9999/1
Luxemburg	0.9999/1	0.9999/1	0.9999/1
Estonia	0.9999/1	0.9999/1	0.9999/1
Cyprus	0.9999/1	0.9999/1	0.9999/1
Bulgaria	0.9999/1	0.9999/1	0.9999/1

We observe that land transportation absorbs the most value added under a uniform expansion of global GDP and, in particular, United States land transportation absorbs 1.04% of global value added (US$328b of US$31.6t reported in the WIOD 2000 table), far and away the largest of all transportation industries. Most transportation industries provide value added to their own country's industries, so this degree of impact is likely due to the size of US production as a proportion of global production.

India and Italy have index values that are not commensurate with the amount of output they produced in 2000. We conjecture that industries in India and Italy would be more competitive as suppliers (both domestically and abroad) if they improved the efficiency of road transportation.

Finally, we observe that with the exception of the United States, air transport absorbs very little value added. This is due to the very large size of the United States' financial services and 'rental of materials and equipment and other business activities' industries, which both use a large amount of air transport inputs. This latter industry includes consulting services, software licensing, intellectual property services and other professional services.

9.2 Transport industry inter-dependencies

Transport industries do not exist in isolation from other transport industries. Land, water and air transport are inter-dependent (and depend on other industries such as transport services, fuel, transport equipment). How this is expressed in terms of regional dependencies is the subject of this section.

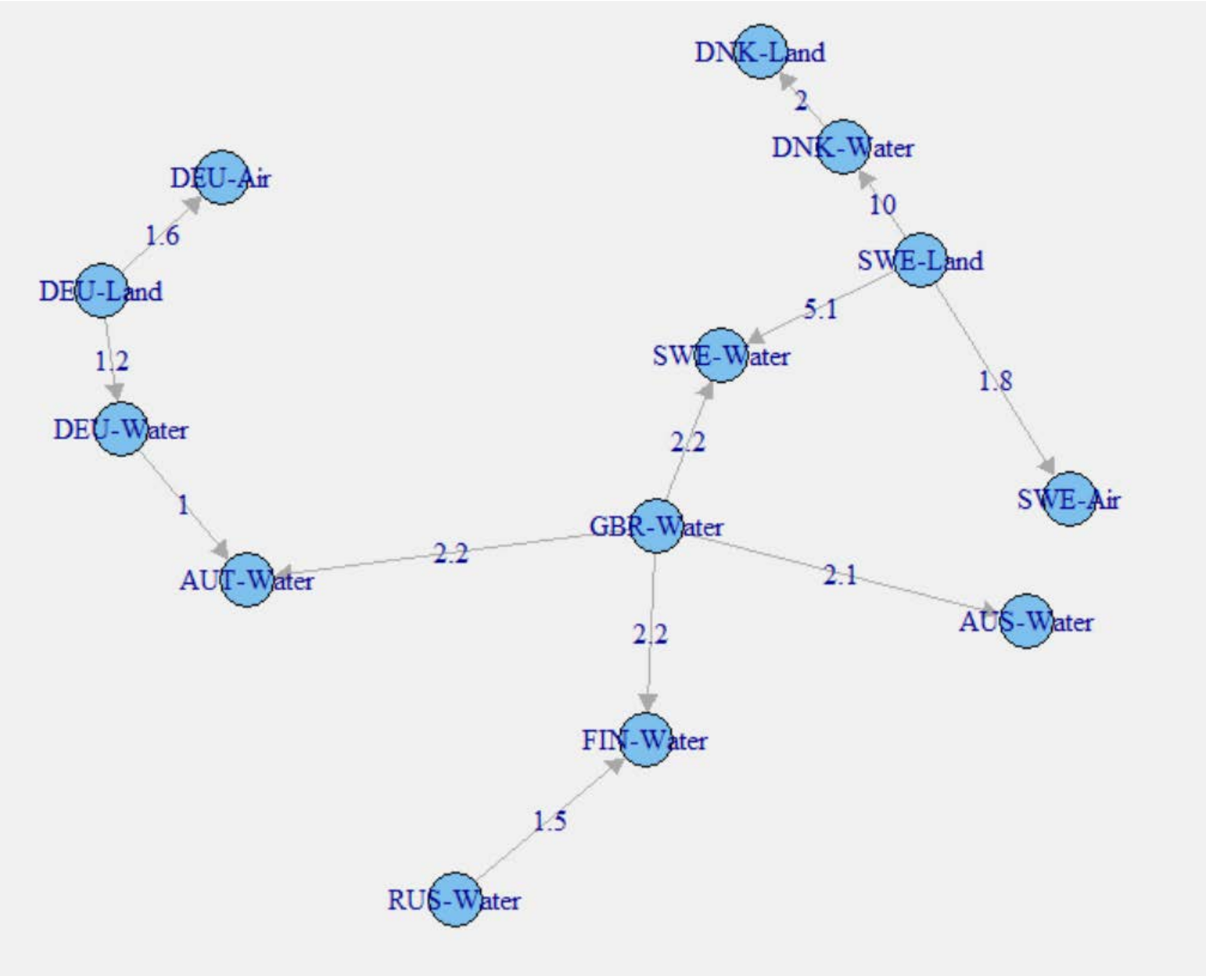
Having decomposed value added as above, we restrict our attention to value added provided by transport industries to other transport industries, by region. Many regions and transport industries may be essentially independent of other transport industries in other regions, but the business model for others may be different – they may tranship cargo to other regions and pay the transport industries in those regions directly for this; they may hire transport equipment from other regions to suit their needs; they may outsource their transport needs by contracting to other regions' transport businesses. (These transport industries are distinct from 'auxiliary transport services', transport equipment manufacturing, transport equipment repairs, or fuel and energy for transport.) Thus, if a region wishes to expand its transportation industry production while maintaining its current business model, it is useful to know which contributing regions and industries would also need to be developed, either through investments in capital or through bolstering labour inputs.

Figure 9.1 shows the situation for a number of countries' transport industries from the 2000 WIOD. This is a directed graph whose vertices are some transport industries in a number of countries. There is an edge beginning at a country's industry (the 'origin' of the edge) and ending at another country's industry (the 'destination' of the edge); the origin must provide at least 1% of the value added in a unit of production by the destination. The numbers labelling the edges are the percentages of value added the origin country's industry is contributing to the production by the destination. We have suppressed the percentage of value added stemming from each industry's own production.

The label 'land' means land transportation; the label 'water' means water transport; and 'air' means air transport.

From this graph, we can conclude water transport in Denmark is very enmeshed with Swedish land transport (10% of value added); Swedish water transport is dependent on Swedish land transport (5%) and British water transport (2.2%); etc.

Figure 9.1 Flows of value added between some transport industries in various European countries



9.3 Indirect impacts of transport: regional importance

Inputs into industries are combinations of goods and services sourced from different regions and industries. A region could be considered important to a regional industry if much of its downstream value added is routed through the region; policy targeted at regional development would be advised to ensure transportation industries in regions of importance are capable of supporting increased trade. This section looks at how RIOTs can be used to measure such regional importance.

The amount of eventual input from industry-region (i,r) into industry-region (j,s) that routed directly through a region u is $\sum_k A_{ik}^{ru} A_{kj}^{us}$ (assuming u is not r or s). Allowing more indirect routes, we introduce the notation:

$$T_{ij}^{rs}(u;n,m) = \sum_k (A^n)_{ik}^{ru} (A^m)_{kj}^{us} \quad \text{(Equation 9.7)}$$

for non-negative integers n and m so $n+m>0$. Thus, the amount of value added contributed to a unit of production by (j,s) via a region u is the sum:

$$I_j^s(u) = \sum_{n,m,r,i} \bar{v}_i^r \, T_{ij}^{rs}(u;n,m) \quad \text{(Equation 9.8)}$$

The value added flowing through one region can also flow through another, so this measure does not provide a decomposition of value added.

This measure can be used to assess the importance of one region to another:

$$I^s(u) = \frac{\sum_j I_j^s(u) x_j^s}{\sum_{ju} I_j^s(u)\, x_j^s} \qquad \text{(Equation 9.9)}$$

We weighted the importance of the region to each regional industry by the amount of output by the regional industry, so the measure of importance of one region to another is affected more by high output industries in the purchasing region. Price changes can affect this measure of importance, so the importance is sensitive to relative volumes and relative prices.

If trade volumes are of interest, then we have a symmetric measure of importance as:

$$I(r,s) = I^r(s) + I^s(r) \qquad \text{(Equation 9.10)}$$

In practice, the most important region for a region is itself, but that will fluctuate greatly according to how able a region is to meet its own input needs; smaller regions or regions that specialise in a few industries see a greater reliance on imports from other regions.

This set of pair-wise estimates of inter-regional flows can also be used to identify trading blocs by constructing a directed (network) graph and employing community detection (vertex clustering) techniques.

9.3.1 Example: Importance between trading nations, using the World Input-Output Database.

In tables 9.2 to 9.5 we present for several countries represented in the 2000 WIOD, all the countries through which at least 1% of value added flows. The countries are Australia, Cyprus, the USA and China.

In interpreting these tables it is important to remember these assessments of mutual importance are made in respect of intermediate inputs, not in terms of final consumption. Therefore, these links are about the infrastructure of business-to-business trade: transport infrastructure, shared standards, audit practice, customs clearance, tort and contract law, etc.

Table 9.2 Estimates of value-added flow volumes between various economies and Australia – those economies that have value-added flow volume at least 1%

Value-added flow volume between…		**and…**
China	1.5%	Australia
Great Britain	1.2%	
Indonesia	2.2%	
Japan	1.6%	
Korea	1.8%	
Rest of world	4.4%	
Taiwan	1.8%	
USA	2.7%	

Thus, Australia would benefit (assuming efficiency savings were passed on) if the USA were to improve the efficiency of its land transport network.

Cyprus illustrates how a small region will have more significant trading conduits.

Table 9.3 Estimates of value-added flow volumes between various countries and Cyprus – those economies that have value-added flow volume at least 1%

Value-added flow volume between…		**and…**
Belgium	1.1%	Cyprus
China	1%	
Germany	2.9%	
France	1.7%	
Great Britain	2.5%	
Greece	2.5%	
Italy	3%	
Japan	1.3%	
Netherlands	1.2%	
Rest of world	6.8%	
Russia	4.8%	
USA	3.2%	

The USA has relatively few suppliers of value added, but supplies value added to a large number of countries. Note that when the value-added flow is high it is generally because the USA is providing a lot of value added to the production of the trading partner. In the case of Ireland, it would be interesting to understand the nature of the high flow-volume – it might be attributable to transfer payments by US technology firms to Irish subsidiaries as part of tax planning.

Table 9.4 Value-added flow volumes between selected economies and the United States – those economies with value-added flow volumes at least 1%

Value-added flow volume between…		**and…**
Australia	2.7%	United States
Austria	1.9%	
Belgium	4.0%	
Brazil	2.4%	
Canada	16.6%	
China	1.9%	
Cyprus	3.2%	
Czech Republic	2.8%	
Germany	2.8%	
Denmark	2.1%	
Spain	1.6%	
Estonia	2.3%	
Finland	3.1%	
France	2.8%	
Great Britain	3.7%	
Greece	3.9%	

Value- added flow volume between...		and...
Hungary	3.3%	
Indonesia	2.3%	
India	1.4%	
Ireland	10.8%	
Italy	2.1%	
Japan	2.4%	
South Korea	4.5%	
Lithuania	1.5%	
Luxemburg	3.6%	
Latvia	1.5%	
Mexico	18%	
Malta	6.7%	
Netherlands	4.3%	
Poland	1.4%	
Portugal	1.5%	
Romania	1.4%	
Rest of world	9.9%	
Russia	1.5%	
Slovakia	1.4%	
Slovenia	1.5%	
Sweden	3.8%	
Turkey	1.5%	
Taiwan	5%	

In 2000, China was not nearly as active in global value-chains, focusing on its neighbours in South-East Asia.

Table 9.5 Value- added flow volumes between various countries and China – those economies that have value- added flow volume at least 1%

Value- added flow volume between...		and...
Australia	1.5%	China
Canada	1.2%	
Cyprus	1%	
Germany	1.2%	
Estonia	1.1%	
Great Britain	1.1%	
Hungary	1.1%	
Indonesia	2.2%	
India	1.1%	
Ireland	1.1%	
Japan	3.1%	

Value- added flow volume between…		**and…**
Korea	3.7%	
Malta	1.2%	
Netherlands	1.9%	
Rest of world	5%	
Taiwan	3.5%	
USA	1.9%	

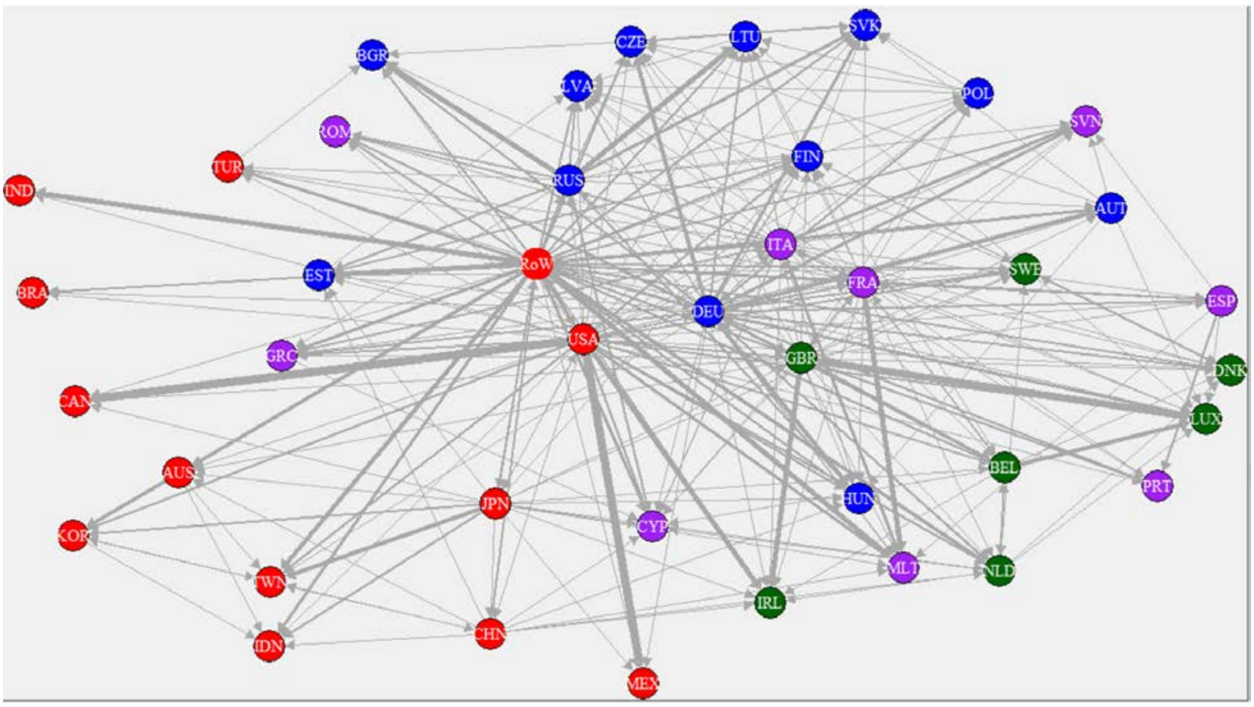
9.3.2 Global value-chains in 2000

Form a graph which has the countries in the WIOD as vertices, and where there is an edge between country r and country s provided $I^s(r)$ is greater than 0.01 – this is an arbitrary cut-off, of course. This is a directed graph that illustrates the importance of transport and trade facilitation in value chains. Transfers between countries to meet final consumption demand is not present in this graph – this is just business-to-business transfers. Thus a country that imports little for its production and whose exports are primarily for consumption will only have a few incoming edges and no outgoing edges.

A community in a graph is a clustering of vertices, otherwise called a vertex labelling. Heuristically, a vertex is assigned to a community when it is more connected to fellow community members than to vertices in different communities. The modularity of a vertex labelled graph provides a measure of that heuristic. Too many communities and vertices and the modularity will penalise a vertex labelling for not placing highly connected vertices in the same community; too few communities and the modularity will penalise a vertex labelling by not separating two vertices that have few connections in common. By optimising the modularity it is possible to detect a community structure, though this is not the only manner in which communities can be algorithmically created.

The graph of countries from the 2000 WIOD is small and it is quite feasible to directly optimise the modularity, producing figure 9.2. There are four communities.

Figure 9.2 Depicting the importance of one economy to another in supporting production



The reader should not read too much into the placement of the vertices – there is no vertical or horizontal scale, but simply points in space. The thickness of the lines between two countries increases with the proportion of value added transferred from the source to the target.

10 Conclusions

The purpose of this work was to create a methodology for developing regional input-output tables that made best use of the data available, both official statistical data and third-party data; and to investigate how a regional input-output table could be used to understand the economic impact of transport. The work achieved all these objectives.

The methodology we developed was largely an extension of the Bayesian approach of Rodrigues, but with some novelty. Rodrigues demonstrated how to reformulate the various matrix update methods (RAS, KRAS and the like) as a Bayesian optimisation problem. In such a problem, the unknown quantity (in this case a matrix) is considered to be a random vector whose distribution of values is affected by a set of constraints and knowledge of the uncertainty in the measurement of these constraints. We extended Rodrigues's approach to regional input-output tables and incorporated not just the constraints that produce the matrix update methods but also:

- constraints on regional trade (through an extension of the CHARM method)
- information such as location quotients or subject-matter expertise to inform priors and allowing the uncertainty of such ad hoc methods to be included in the uncertainty of the table's values.

This approach integrates all the known approaches into a single framework, and the assessments of table uncertainty provide intelligence on how additional data might best improve a table for a particular purpose.

The main components of a regional input-output table describe a dynamic of monetary transfer between regional industries and other regional industries or final use. Real-life data (as opposed to statistical data obtained from surveys) can be modelled in terms of a data-generation process. Such a model will have parameters that are fitted to the data as part of building the model; in a Bayesian setting you would start with priors on the parameters and use the data to update the priors. When you can include regional input-output table data as parameters in a data-generation model you can use the real-life data to update the regional input-output table.

We evaluated a number of sources of data for this purpose: eRUC, Marketview, Qrious, and Xero, Only Xero and Marketview are likely to be of any use for the data-generation model approach to informing regional input-output models, as they are the only data sources that are inherently monetary in nature (eRUC and Qrious are essentially location data); of these two, Xero shows the most promise for updating the dynamics of regional input-output, though Marketview would be extremely useful for establishing priors for final use or constraints on regional tourism (as is already done).

In input-output accounting transport is a cost to the purchaser. As such, it is difficult to use regional input-output tables to evaluate the impact of transport on the economy – perhaps with a time series of tables one could employ production efficiency techniques to such an end, but we considered only what can be measured from a single table. We developed three measures of impact:

1. The efficiency of a regional transport industry can be measured in terms of how much value added it provides as a proportion of the output of the purchasing industries. In this measure, large amounts would be interpreted as inefficiency, though it is likely this measure is very coarse.
2. Transport industry inter-dependencies can be quantified using regional input-output tables to understand how air, sea and road transport cooperates to deliver regional transport. This is not an absolute measure of impact, but a measure of the impact one regional transport industry has on another regional transport industry.

3 The use of transport is in the movement of people and things. Thus an indirect impact is in how transport facilitates trade. We measure the importance of a region to another region in terms of how much value added flows through a region into another region's production. Transport or value-chain links between a region and another regional of high importance should be prioritised, so this measure provides a quantification of importance that informs resource allocation.

We generally conclude it is not feasible to measure the direct economic impact of transport using a single regional input-output table, though indirect effects can be measured and would provide useful input into transport planning at a regional level.

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Appendix A: Glossary

ACC	Accident Compensation Corporation
API	application programming interface
BNZ	Bank of New Zealand
CHARM	cross-hauling adjusted regionalisation method
CPI	consumer price index
FTE	full-time equivalent employee
GDP	gross domestic product
GDP(E)	gross domestic product using the expenditure method
GRIT (method)	generation of regional input-output tables
IOT	input-output table
LBD	Longitudinal Business Database
LEED	Linked Employer Employee Database
LINZ	Land Information New Zealand
LQ	location quotient
MBIE	Ministry of Business Innovation and Employment
MRIOT	multi-region input-output table
NZBN	New Zealand Business Number
NZIER	New Zealand Institute of Economic Research
Prior	Used in Bayesian statistics. A probability distribution or function that is updated to produce a posterior using Bayes' theorem and data. Heuristically, a prior can be thought as a 'best guess' for how a random variable should be distributed prior to having data.
RIOT	regional input-output table
Transport Agency	New Zealand Transport Agency
WIOD	World Input-Output Database