## METHOD FOR ESTIMATING THE SRT OF A VEHICLE

Approved by the NZ Transport Agency under clause 3.16(2) of the Land Transport: Vehicle Dimensions and Mass Rule 2016. This methodology meets the requirements of clause 3.16(1) (b) of the Rule.


Figure 1. Vehicle Roll Notation.

## Notation:

$\mathrm{M}_{\mathrm{s}} \quad$ Sprung Mass
$\mathrm{M}_{\mathrm{u}} \quad$ Unsprung Mass
$\mathrm{k}_{\mathrm{t}} \quad$ Tyre stiffness
$\mathrm{k}_{\mathrm{r}} \quad$ Composite suspension roll stiffness
$\mathrm{k}_{\text {aux }} \quad$ Auxiliary roll stiffness
$\mathrm{h}_{\mathrm{a}} \quad$ Axle Cg height from ground
$\mathrm{h}_{\mathrm{b}} \quad$ Roll centre height from ground
$\mathrm{h}_{\mathrm{c}} \quad$ Sprung Mass Cg height from ground
T Wheel track width
$\varphi \quad$ Axle roll angle
$\theta \quad$ Body roll angle relative to the axle
$\alpha \quad$ Static Roll Threshold as a proportion of Mass

Defining new variables M and H as follows

$$
\begin{array}{ll}
\mathrm{M}=\mathrm{M}_{\mathrm{s}}+\mathrm{M}_{\mathrm{u}} & \text { Total Mass } \\
\mathrm{H}=\frac{M_{\mathrm{s}} h_{\mathrm{c}}+M_{u} h_{a}}{M_{s}+M_{u}} & \text { Overall Cg height }
\end{array}
$$

In the general case the suspension has some lash. At some value of the body roll angle, $\theta$, the load on the right hand spring (based on the figure) becomes zero. As the vehicle body rolls through a further angle $\zeta$, which is less than or equal to the lash divided by the spring spacing, no additional roll restoring force from the springs is generated. However, if the suspension has an anti-roll device this will still apply resisting moment. Once the full extent of the lash has been taken up further increments of $\theta$ are possible. At any stage the total body roll angle is $\theta+\zeta$.

Using the graphical approach presented in Winkler et al (2000), consider the rotation of the total mass about R. Assuming small angles, after rotation the co-ordinates of the Cg of sprung mass relative to a roll centre at $R$ are $\left(-h_{c} \varphi-\left(h_{c}-h_{b}\right)(\theta+\zeta), h_{c}\right)$ and the co-ordinates of the Cg of the unsprung mass relative to the same roll centre R are $\left(-\mathrm{h}_{\mathrm{a}} \varphi, \mathrm{h}_{\mathrm{a}}\right)$.

Thus the co-ordinates of the overall Cg are

$$
\left(-\mathrm{H} \varphi-\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)(\theta+\zeta) / \mathrm{M}, \mathrm{H}\right)
$$

and the rotation of the whole mass about R is

$$
\begin{equation*}
\varphi_{\mathrm{T}}=\varphi+\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)(\theta+\zeta) / \mathrm{MH} \tag{1}
\end{equation*}
$$

Consider the sprung mass as a free body and take moments about the roll centre B

$$
\begin{equation*}
\mathrm{k}_{\mathrm{r}} \theta+\mathrm{k}_{\mathrm{aux}} \zeta=\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right) \alpha+\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right) \varphi+\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)(\theta+\zeta) \tag{2}
\end{equation*}
$$

Similarly consider the unsprung mass as a free body and take moments about B

$$
\begin{equation*}
\mathrm{k}_{\mathrm{r}} \theta+\mathrm{k}_{\mathrm{aux}} \zeta=-\mathrm{F}_{2}\left(\frac{\mathrm{~T}}{2}+\mathrm{h}_{\mathrm{b}} \varphi\right)+\mathrm{F}_{1}\left(\frac{\mathrm{~T}}{2}-\mathrm{h}_{\mathrm{b}} \varphi\right)+\mathrm{M}_{\mathrm{u}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{b}}-\mathrm{h}_{\mathrm{a}}\right) \alpha+\mathrm{M}_{\mathrm{u}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{b}}-\mathrm{h}_{\mathrm{a}}\right) \varphi-\mathrm{Mgh}_{\mathrm{b}} \alpha \tag{3}
\end{equation*}
$$

but $\mathrm{F}_{1}+\mathrm{F}_{2}=\mathrm{Mg}$ and $\mathrm{F}_{1}-\mathrm{F}_{2}=\mathrm{k}_{\mathrm{t}} \mathrm{T} \varphi$. Therefore substituting in (3) gives

$$
\begin{equation*}
\mathrm{k}_{\mathrm{r}} \theta+\mathrm{k}_{\mathrm{aux}} \zeta=\mathrm{k}_{\mathrm{t}} \frac{\mathrm{~T}^{2}}{2} \varphi-\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right) \mathrm{g} \alpha-\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right) \mathrm{g} \varphi \tag{4}
\end{equation*}
$$

Equations (2) and (4) can be used to eliminate $\alpha$

$$
\begin{equation*}
\mathrm{k}_{\mathrm{r}} \mathrm{MH} \theta+\mathrm{k}_{\mathrm{aux}} \mathrm{MH} \zeta-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)(\theta+\zeta)=\mathrm{k}_{\mathrm{t}} \frac{\mathrm{~T}^{2}}{2} \mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right) \varphi \tag{5}
\end{equation*}
$$

The moment balance equation Winkler et al (2000) use in their graph is
Moment due to lateral acceleration $=$ Restoring moment from ground - Moment from the offset due to compliance. All moments taken about R.

Rollover occurs when the right hand side of this equation reaches its maximum. From Winkler et al (2000) the Static Roll Threshold (SRT) is given by

$$
\begin{equation*}
\mathrm{SRT}=\frac{\mathrm{T}}{2 \mathrm{H}}-\varphi_{\mathrm{T}} \tag{6}
\end{equation*}
$$

Thus all we need to do is determine $\varphi_{\mathrm{T}}$ at the maximum roll resistance.
Restoring moment from the ground $=\mathrm{k}_{\mathrm{t}} \mathrm{T}^{2} \varphi / 2$ up to a maximum of $\mathrm{MgT} / 2$ when the wheel lifts off. The offset moment $=\mathrm{MgH} \varphi_{\mathrm{T}}$.

Consider now various cases.

## Case 1.

Basic case - suspension has no lash, i.e. $\zeta=0$. The maximum moment occurs when $\mathrm{Mg}=\mathrm{k}_{\mathrm{t}} \mathrm{T} \varphi$, i.e. $\varphi=\mathrm{Mg} / \mathrm{k}_{\mathrm{t}} \mathrm{T}$. Substituting this in equation (5) gives

$$
\theta=\frac{\mathrm{MgT}}{2} \frac{\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)}{\left(\mathrm{k}_{\mathrm{r}} \mathrm{MH}-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)\right)}
$$

Substituting into equation (1) we can evaluate $\varphi_{\mathrm{T}}$ thus from equation (6)

$$
\begin{equation*}
\text { SRT }=\frac{T}{2 H}\left[1-\frac{M_{s}^{2} g\left(h_{c}-h_{b}\right)^{2}}{k_{r} M H-M_{s} g\left(h_{c}-h_{b}\right)\left(M_{s} h_{b}+M_{u} h_{a}\right)}\right]-\frac{M g}{k_{t} T} \tag{7}
\end{equation*}
$$

## Case 2.



Figure 2. Suspension track width.
Suspension has lash. There are two points where the resisting moment versus roll angle curve reduces slope and either of these could be the rollover point depending on which generates the highest resisting moment. The first is when the lash comes into play, i.e. when the right hand side spring (using the convention in the
figure) becomes unloaded. This condition is that $M_{s} g=k_{s} t \theta$, i.e. $\theta=M_{s} g / k_{s} t$ where $k_{s}$ is the spring stiffness and $t$ is the suspension track width as shown in Figure 2. At this point $\zeta=0$.

Substituting in equation (5) gives

$$
\frac{\left(k_{r} M H-M_{s} g\left(h_{c}-h_{b}\right)\left(M_{s} h_{b}+M_{u} h_{a}\right)\right) 2 g}{k_{s} t k_{t} T^{2}\left(h_{c}-h_{b}\right)}=\varphi
$$

and then substituting all the angle values into equation (1) and then (6) gives

$$
\begin{equation*}
\text { SRT }=\frac{T}{2 H}-\frac{M_{s}^{2} g\left(h_{c}-h_{b}\right)}{k_{s} t M H}-\frac{2 g\left(k_{r} M H-M_{s} g\left(h_{c}-h_{b}\right)\left(M_{s} h_{b}+M_{u} h_{a}\right)\right)}{k_{s} k_{t} T^{2}\left(h_{c}-h_{b}\right)} \tag{8}
\end{equation*}
$$

Alternatively the unsprung mass may still be sufficient to resist rollover once the lash has occurred and thus the vehicle can withstand a higher lateral acceleration before rollover. In this case the full extent of the lash is applied, i.e. $\zeta=l / \mathrm{t}$ and wheel lift off occurs i.e. $\varphi=\mathrm{Mg} / \mathrm{k}_{\mathrm{t}} \mathrm{T}$. Substituting these values in equation (5) gives

$$
\theta=\frac{\operatorname{MgTtM}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)-2 l\left(\mathrm{k}_{\mathrm{aux}} \mathrm{MH}-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)\right)}{2 \mathrm{t}\left(\mathrm{k}_{\mathrm{r}} \mathrm{MH}-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)\right)}
$$

As before substituting all the angle values into (1) and then (6) gives

$$
\begin{equation*}
\text { SRT }=\frac{T}{2 H}\left[1-\frac{M_{s}^{2} g\left(h_{c}-h_{b}\right)^{2}}{\left(k_{r} M H-M_{s} g\left(h_{c}-h_{b}\right)\left(M_{s} h_{b}+M_{u} h_{a}\right)\right)}\right]-\frac{M g}{k_{t} T}-\frac{M_{s}\left(h_{c}-h_{b}\right) l\left(\left(k_{r}-k_{\text {aux }}\right)\right.}{t\left(k_{r} M H-M_{s} g\left(h_{c}-h_{b}\right)\left(M_{s} h_{b}+M_{u} h_{a}\right)\right)} \tag{9}
\end{equation*}
$$

If the auxiliary roll stiffness is relatively high compared to the spring stiffness it is possible that rollover will occur after the onset of lash but before the lash is fully completed. In this case $\theta=M_{s} \mathrm{~g} / \mathrm{k}_{\mathrm{s}} \mathrm{t}$ and $\varphi=\mathrm{Mg} / \mathrm{k}_{\mathrm{t}} \mathrm{T}$ but $\zeta$ is unknown. Substituting these values in equation (5) gives

$$
\zeta=\frac{\operatorname{MgTk}_{\mathrm{s}} \mathrm{tM}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)-2 \mathrm{M}_{\mathrm{s}} g\left(\mathrm{k}_{\mathrm{r}} \mathrm{MH}-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)\right)}{2 \mathrm{k}_{\mathrm{s}} \mathrm{t}\left(\mathrm{k}_{\mathrm{aux}} \mathrm{MH}-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)\right)}
$$

As before substituting all the angle values into (1) and then (6) gives

$$
\begin{equation*}
\mathrm{SRT}=\frac{\mathrm{T}}{2 \mathrm{H}}-\frac{\mathrm{Mg}}{\mathrm{k}_{\mathrm{t}} \mathrm{~T}}-\frac{\mathrm{M}_{\mathrm{s}}^{2} g\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{Tk}_{\mathrm{s}} \mathrm{t}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)-2\left(\mathrm{k}_{\mathrm{r}}-\mathrm{k}_{\text {aux }}\right) \mathrm{H}\right)}{2 \mathrm{Hk}_{\mathrm{s}} \mathrm{t}\left(\mathrm{k}_{\mathrm{aux}} \mathrm{MH}-\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)\left(\mathrm{M}_{\mathrm{s}} \mathrm{~h}_{\mathrm{b}}+\mathrm{M}_{\mathrm{u}} \mathrm{~h}_{\mathrm{a}}\right)\right)} \tag{10}
\end{equation*}
$$

The correct SRT value is the one that generates the greater resisting moment to rollover, although, in practice, dynamic effects will tend to favour the value given by equation (8) even if it is slightly lower.

The resisting moment less the overturning moment due to the offset is given by

$$
\begin{equation*}
\text { Moment }=\mathrm{k}_{\mathrm{t}} \frac{\mathrm{~T}^{2}}{2} \varphi-\mathrm{MgH} \varphi_{\mathrm{T}} \tag{11}
\end{equation*}
$$

For each of the three conditions above $\varphi$ and $\varphi_{T}$ are known and so we can substitute in equation (11). The standard sequence of events is that as $\varphi_{\mathrm{T}}$ increases, first there is the onset of lash, the full extent of the lash occurs and finally there is wheel lift off. In this case the two potentially critical events are the onset of lash and wheel lift off and the one with the larger moment determines which situation is critical and gives the SRT value. With a relatively higher auxiliary roll stiffness it is possible for wheel lift off to occur before the onset of lash or alternatively after the onset of lash but before full lash. In the first instance wheel lift off is the critical condition while in the second case the event which generates the maximum moment is critical.

## Case 3.

If the relative roll stiffnesses of the different axles of the vehicle are significantly different an axle may lift off without vehicle rollover or a suspension may take up its lash without rollover. For the purposes of this analysis we consider only the SRT of a single vehicle with a maximum of two axle groups. Winkler et al (2000) consider a tractor-semi trailer as a combination but for the Vehicle Dimensions and Mass Rule each vehicle unit must be assessed in a stand alone mode. For a vehicle unit that is normally part of a rollcoupled combination some assumptions need to be made regarding the characteristics of typical other unit(s) that could be coupled to the vehicle.

The vehicle sprung mass is considered a rigid body. Thus the angle of the sprung mass from the roll centre is the same all the way along the body. The roll centre height is not necessarily a constant along the vehicle. This condition can be written as

$$
\begin{equation*}
\theta_{\text {fiont }}+\zeta_{\text {fiont }}+\varphi_{\text {firont }}=\theta_{\text {rear }}+\zeta_{\text {rear }}+\varphi_{\text {rear }}=\theta_{\text {general }}+\varphi_{\text {general }} \tag{12}
\end{equation*}
$$

Note there is no $\zeta_{\text {general }}$ term in this equation because this angle in included within the $\theta_{\text {general }}$ term. The concept of lash relates to the suspension and has no real meaning in the general position.

Consider the position of the Cg of the sprung mass along the vehicle and using the subscript s to denote this point. At this location $h_{b s}=\left(M_{s_{-} \text {front }} h_{b_{-} \text {front }}+M_{s_{-} \text {rear }} h_{b_{-} \text {rear }}\right) / M_{s}$, and $\varphi_{s}=\left(M_{s_{-} \text {front }} h_{b_{-} \text {front }} \varphi_{\text {front }}+M_{s_{-} \text {rear }} h_{b_{-} \text {rear }}\right.$ $\varphi_{\text {rear }} / / M_{s} h_{\text {bs }}$. These two equations are derived from the rigid body assumption which requires that the roll centre lie along a straight line.

Consider now a moment balance for the sprung mass about the roll centre.

$$
\begin{equation*}
\mathrm{k}_{\mathrm{r}_{-} \text {rear }} \theta_{\text {rear }}+\mathrm{k}_{\text {aux_rear }} \zeta_{\text {rear }}+\mathrm{k}_{\mathrm{r}_{-} \text {front }} \theta_{\text {firont }}+\mathrm{k}_{\text {aux_frion }} \zeta_{\text {firiont }}=\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right) \alpha+\mathrm{M}_{\mathrm{s}} \mathrm{~g}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)(\varphi+\theta) \tag{13}
\end{equation*}
$$

and moment balances for the unsprung masses about centreline on ground

$$
\begin{align*}
& \mathrm{k}_{\mathrm{r}_{-} \text {front }} \theta_{\text {friont }}+\mathrm{k}_{\text {aux_fronts front }}=\mathrm{k}_{\mathrm{t}_{-} \text {front }} \frac{\mathrm{T}_{\text {friont }}{ }^{2}}{2} \varphi_{\text {friont }}-\left(\mathrm{M}_{\mathrm{s}_{-} \text {firont }} \mathrm{h}_{\mathrm{b}_{-} \text {friont }}+\mathrm{M}_{\mathrm{u}_{\text {_front }}} \mathrm{h}_{\mathrm{a}_{-} \text {front }}\right) \mathrm{g}\left(\alpha+\varphi_{\text {front }}\right)  \tag{14}\\
& \mathrm{k}_{\mathrm{r}_{-} \text {rear }} \theta_{\text {rear }}+\mathrm{k}_{\text {aux__rear }} \zeta_{\text {rear }}=\mathrm{k}_{\mathrm{t}_{-} \text {rear }} \frac{\mathrm{T}_{\text {rear }}^{2}}{2} \varphi_{\text {rear }}-\left(\mathrm{M}_{\mathrm{s}_{-} \text {rear }} \mathrm{h}_{\mathrm{b}_{\text {_rear }}}+\mathrm{M}_{\mathrm{u}_{-} \text {rear }} \mathrm{h}_{\text {a_rear }}\right) \mathrm{g}\left(\alpha+\varphi_{\text {rear }}\right) \tag{15}
\end{align*}
$$

Define new coefficients as follows

$$
\begin{aligned}
& M F_{\text {front }}=\frac{\left(\mathrm{M}_{\mathrm{s}_{-} \text {friont }} \mathrm{h}_{\mathrm{b}_{\_} \text {fiont }}+\mathrm{M}_{\mathrm{u}^{\prime} \text { friont }} \mathrm{h}_{\mathrm{a}_{-} \text {friont }}\right)}{\mathrm{MH}} \\
& M F_{\text {rear }}=\frac{\left(\mathrm{M}_{\mathrm{s}_{-} \text {rear }} \mathrm{h}_{\mathrm{b}_{-} \text {rear }}+\mathrm{M}_{\mathrm{u} \text { rear }} \mathrm{h}_{\mathrm{a}_{-} \text {rear }}\right)}{\mathrm{MH}}
\end{aligned}
$$

$(15)+(14)-(13)$ gives

$$
\begin{equation*}
\alpha=\left(\mathrm{k}_{\mathrm{t}-\text { friont }} \frac{\mathrm{T}_{\text {firont }}^{2}}{2 \mathrm{MHg}}-M F_{\text {front }}\right) \varphi_{\text {friont }}+\left(\mathrm{k}_{\mathrm{t}_{-} \text {rear }} \frac{\mathrm{T}_{\text {rear }}^{2}}{2 \mathrm{MHg}}-M F_{\text {rear }}\right) \varphi_{\text {rear }}-\frac{\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)}{\mathrm{MH}}(\varphi+\theta) \tag{16}
\end{equation*}
$$

Using (16) with (14) and (16) with (15) we can eliminate $\alpha$ to give a pair of simultaneous equations

$$
\begin{align*}
& {\left[\left(\mathrm{k}_{\mathrm{t} \text { _front }} \frac{\mathrm{T}_{\text {front }}{ }^{2}}{2 \mathrm{MHg}}-M F_{\text {front }}\right)\left(1-\frac{1}{M F_{\text {front }}}\right)-\frac{\mathrm{k}_{\mathrm{r}_{-} \text {front }}}{M F_{\text {front }} \mathrm{MHg}}\right] \varphi_{\text {front }}+\left(\mathrm{k}_{\mathrm{t}_{-} \text {rear }} \frac{\mathrm{T}_{\text {rear }}{ }^{2}}{2 \mathrm{MHg}}-M F_{\text {rear }}\right) \varphi_{\text {rear }}} \\
& +\left(\frac{\mathrm{k}_{\mathrm{r}_{\text {_fiont }}}}{M F_{\text {front }} \mathrm{MHg}}-\frac{\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)}{\mathrm{MH}}\right)(\varphi+\theta)+\frac{\mathrm{k}_{\text {aux_front }}-\mathrm{k}_{\mathrm{r}_{\mathrm{r}} \text { front }}}{M F_{\text {front }} \mathrm{MHg}} \zeta_{\text {front }}=0  \tag{17}\\
& \left(\mathrm{k}_{\mathrm{t}-\text { front }} \frac{\mathrm{T}_{\text {friont }}{ }^{2}}{2 \mathrm{MHg}}-M F_{\text {front }}\right) \varphi_{\text {front }}+\left[\left(\mathrm{k}_{\mathrm{t}_{-} \text {rear }} \frac{\mathrm{T}_{\text {rear }}{ }^{2}}{2 \mathrm{MHg}}-M F_{\text {rear }}\right)\left(1-\frac{1}{M F_{\text {rear }}}\right)-\frac{\mathrm{k}_{\mathrm{r}_{-} \text {rear }}}{M F_{\text {rear }} \mathrm{MHg}}\right] \varphi_{\text {rear }} \\
& +\left(\frac{\mathrm{k}_{\mathrm{r}_{\text {_rar }}}}{M F_{\text {rear }} \mathrm{MHg}}-\frac{\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)}{\mathrm{MH}}\right)(\varphi+\theta)+\frac{\mathrm{k}_{\text {aux } \_ \text {rear }}-\mathrm{k}_{\mathrm{r}_{\mathrm{r}} \text { rear }}}{M F_{\text {rear }} \mathrm{MHg}} \zeta_{\text {rear }}=0
\end{align*}
$$

 suspension reaches a maximum at wheel lift-off i.e. when $\varphi=\mathrm{Mg} / \mathrm{k}_{\mathrm{t}} \mathrm{T}$ and then stay at this value for all $\varphi$ greater than this critical. This maximum moment is $\mathrm{MgT} / 2$. Equation (17) can be rewritten as follows to take this into account.

$$
\begin{aligned}
& {\left[\mathrm{A}_{\text {front }}\left(1-\frac{1}{M F_{\text {front }}}\right)-\mathrm{B}_{\text {front }}\right] \varphi_{\text {fiont }}+\mathrm{A}_{\text {rear }} \varphi_{\text {rear }}+\left(\mathrm{B}_{\text {front }}-\mathrm{C}_{\mathrm{s}}\right)(\varphi+\theta)+\mathrm{D}_{\text {front }} \zeta_{\text {front }}=-\left(1-\frac{1}{M F_{\text {front }}}\right) \frac{\mathrm{M}_{\text {tyre_friont }}}{\mathrm{MHg}}-\frac{\mathrm{M}_{\text {tyrerear }}}{\mathrm{MHg}}} \\
& \mathrm{~A}_{\text {front }} \varphi_{\text {friont }}+\left[\mathrm{A}_{\text {rear }}\left(1-\frac{1}{M F_{\text {rear }}}\right)-\mathrm{B}_{\text {rear }}\right] \varphi_{\text {rear }}+\left(\mathrm{B}_{\text {rear }}-\mathrm{C}_{\mathrm{s}}\right)(\varphi+\theta)+\mathrm{D}_{\text {rear }} \zeta_{\text {rear }}=-\frac{\mathrm{M}_{\text {tyre_front }}}{\mathrm{MHg}}-\left(1-\frac{1}{M F_{\text {rear }}}\right) \frac{\mathrm{M}_{\text {tyre_rar }}}{\mathrm{MHg}}
\end{aligned}
$$

$$
\begin{align*}
& \mathrm{B}_{\text {front }}=\frac{\mathrm{k}_{\mathrm{r}_{-} \text {front }}}{M F_{\text {front }} \mathrm{MHg}}, \quad \mathrm{~B}_{\text {rear }}=\frac{\mathrm{k}_{\mathrm{r}_{\mathrm{r}} \text { rear }}}{M F_{\text {rear }} \mathrm{MHg}}, \quad \mathrm{C}_{\mathrm{s}}=\frac{\mathrm{M}_{\mathrm{s}}\left(\mathrm{~h}_{\mathrm{c}}-\mathrm{h}_{\mathrm{b}}\right)}{\mathrm{MH}} \\
& \mathrm{D}_{\text {friont }}=\frac{\mathrm{k}_{\text {aux _front }}-\mathrm{k}_{\mathrm{r}_{\_ \text {friont }}}}{M F_{\text {front }} \mathrm{MHg}}, \quad \mathrm{D}_{\text {rear }}=\frac{\mathrm{k}_{\text {aux_rear }}-\mathrm{k}_{\mathrm{r}_{-} \text {rear }}}{M F_{\text {rear }} \mathrm{MHg}} \tag{18}
\end{align*}
$$

At each of the critical points on the solution path, three of the variables can be specified. Using equations (18) and (12) it is then possible to solve for the three remaining variables. In this way all the possible vertices on the solution path can be found. However, it is necessary to check each solution point for validity. At each valid solution point the SRT value can be determined using equation (16). The maximum SRT value calculated in this way is the SRT for the vehicle.

## Vehicle Parameters Required

The equations derived above require the following vehicle parameters:

## Mass:

- Unsprung mass by axle group
- Sprung mass by axle group


## Geometry

- Centre of gravity $(\mathrm{Cg})$ heights of the unsprung masses
- Cg height of the sprung mass
- Wheel track width for each axle group
- Roll centre height for each axle group
- Suspension track width for each axle group


## Stiffness/Compliance

- Tyre stiffnesses
- Suspension spring stiffness for each axle group
- Composite/Auxiliary roll stiffness for each axle group
- Suspension lash for each suspension

Some of these parameters are readily obtained while others require specialised testing, which has probably already been done by the manufacturer. In this latter case manufacturer-supplied data can be used. The approach used in the SRT calculator software is to require a minimal set of user-supplied data which can be readily obtained and to include a conservative set of default values for the data that is more difficult to obtain. Where the parameters for which default values are supplied can vary substantially and have a significant impact on the resulting SRT value, an option for user input of manufacturer-supplied data is offered. A description of the data input process and the default values used follows.

## Mass

The user inputs the vehicle type, the number of axles, the tyre configuration and size and the tare and laden mass for each axle group. The unsprung mass is calculated using default values for the axle and wheel masses. Currently the mass values used are as shown in Table 1 and column 2 of Table 2.

Table 1. Default axle masses without wheels.

| Axle type | Mass (kg) |
| :--- | :---: |
| Truck/ Tractor steer axle | 350 |
| Truck/Tractor drive axle | 700 |
| Trailer axle | 400 |

Table 2. Parameter variations for different tyre size and configuration.

| Tyre size and type | Mass $^{*}(\mathbf{k g})$ | Cg height (m) | Width (m) | Dual spacing <br> $(\mathbf{m})$ | Tyre stiffness <br> $(\mathbf{N} / \mathbf{m})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 17.5 single | 50 | 0.36 | 0.275 | 0.26 | 700508 |
| 17.5 wide single | 70 | 0.36 | 0.365 |  | 980711 |
| 19.5 single | 75 | 0.40 | 0.275 | 0.28 | 700508 |
| 19.5 wide single | 105 | 0.40 | 0.365 |  | 980711 |
| 22.5 single | 100 | 0.49 | 0.275 | 0.30 | 700508 |
| 22.5 wide single | 140 | 0.49 | 0.365 |  | 980711 |

For a dual tyre configuration the mass is double the single tyre mass.
${ }^{\dagger}$ For a dual tyre configuration the tyre stiffness is double the single tyre stiffness
The unsprung mass is equal to the sum of the axle masses and the wheel masses for the group while the sprung mass is equal to the laden mass minus these axle and wheel masses.

## Geometry

The Cg heights for the unsprung masses are default values based on the tyre radius. The values used are shown in Table 2.

Actual variations are relatively small and have only a minor impact because the sprung mass is generally much greater than the unsprung mass.

The Cg height of the sprung mass is calculated from the Cg height of the sprung mass component of the tare and the Cg height of the payload. Default values are used for the Cg height of sprung mass component of the tare. Currently this value is 0.56 m above the unsprung mass Cg for tractors and trucks and 1.25 m above the unsprung mass Cg for all trailers. While this value probably varies significantly from vehicle to vehicle the mass involved is generally relatively small so the impact on the overall Cg is small. The payload Cg height is calculated from the load bed height, the load height and the type of load. At present all load types except general freight and containers are assumed to be uniformly distributed vertically and thus the payload Cg height is midway between the load bed and the maximum load height. For general freight and containers it is assumed that the load is not uniformly distributed and can be regarded as being equivalent to two uniformly distributed layers with the lower layer containing $70 \%$ of the mass and the upper layer $30 \%$.

For the wheel track width, default values are used based on the tyre size and configuration. It is assumed that for all large trucks under consideration the overall width is 2.5 m and hence the width to the outer edges of the tyres is approximately 2.4 m . For single tyres the track width is then set to 2.4 m minus the tyre width (see

Table 2), while for dual tyres the track width is set to 2.4 m minus the tyre width minus the dual spacing. Because of geometric effects the centre for the reaction force on a dual tyre set is not at the midpoint of the tyre contact width but further outboard. The correction factor is $\left(1+(\text { dual spacing/track width })^{2}\right)$.

The roll centre height and suspension track width are functions of the suspension and will be covered in the next section.

## Stiffness/Compliance

Standardised tyre stiffness values are used and these are shown in

Table 2. These values are typical and it is unlikely that a certifier can obtain better data. Furthermore the tyre characteristics will change over time as the tyres wear and there is no requirement for vehicles to be fitted with the same tyres when they are replaced. Thus there is a good case for using standardised realistic values.

The input of correct suspension characteristics is the most difficult aspect of using the SRT calculation algorithm. Details of the performance characteristics of the suspensions do have a significant impact on the resulting SRT value but data on these performance characteristics are not always readily available. The approach used is to incorporate default generic air and steel suspension characteristics into the calculator that are typical of the lower end of the scale so that for most vehicles using the default suspension will predict an SRT at or below its actual SRT. The certifier has the option of not using the default suspensions but instead inputting manufacturer-supplied data for the actual suspensions fitted to the vehicle.

The suspension spring stiffness, the auxiliary roll stiffness and the composite roll stiffness are related and, in theory, any two can be used to calculate the third. In practice we would expect the manufacturer data to give the suspension spring stiffness (for each spring) and either the composite or the auxiliary roll stiffness. The inputs to the SRT calculator software are the suspension spring stiffness and the composite roll stiffness. We will now define these more precisely and show the relationships between them.

Referring back to Figure 1, the sprung mass has rolled through an angle, $\theta$, about the roll centre B. The suspension will generate a resisting moment to this roll, which we will call M . The composite roll stiffness is the $\mathrm{M} / \theta=\mathrm{k}_{\mathrm{r}}$. The value to be input to the software is per axle and units used are $\mathrm{Nm} / \mathrm{radian}$. If the values provided by the manufacturer are in any other units a conversion is required.

Part of this roll stiffness is generated by the vertical compression of the suspension. The spring stiffness value, $\mathrm{k}_{\mathrm{s}}$, required for input to the software is the value for one side of the axle (i.e. per spring) and is in $\mathrm{N} / \mathrm{m}$. Any other units require conversion. If the suspension track width is $t$, then the restoring moment generated by vertical compression of the suspension is $\mathrm{k}_{\mathrm{s}} \mathrm{t}^{2} \theta / 2$ and hence the roll stiffness associated with vertical compression of the suspension is $\mathrm{k}_{\mathrm{s}} \mathrm{t}^{2} / 2$. All other roll stiffness generated by the suspension through anti-roll bars and other mechanisms is called the auxiliary roll stiffness, $\mathrm{k}_{\text {aux }}$. Thus,

$$
\begin{equation*}
k_{r}=k_{a u x}+\frac{k_{s} t^{2}}{2} \tag{19}
\end{equation*}
$$

Given the spring stiffness, suspension track and auxiliary roll stiffness we can calculate the composite roll stiffness.

The suspension track width is measured in metres and is the centre-to-centre distance between the connections of the suspension to the axle.

Steel suspensions typically have some "lash" in the suspension where, when the spring load changes from compression to tension, the axle moves through a small deflection with minimal resisting force. For the input to the software this lash is the distance moved by the axle measured in millimetres, which, often, will be smaller than the lash at the spring slipper. Figure 3 illustrates a steel leaf spring configuration with suspension lash at both ends of the spring. Other designs have one end of the spring attached with a pin joint and only have lash at the other end. To determine the lash it will usually be simpler to measure the free play of the spring at the slipper mount(s) and then to calculate the lash at the axle from the geometry of the suspension. For example, if the axle is mounted halfway between the spring hanger and the slipper, the lash
at the axle will be the average of the suspension lash at the two ends of the spring. If one end of the spring has no lash the axle lash would be half the suspension lash. For an air suspension the suspension generates very little restoring force in tension and so the system can be regarded as having a large lash value. Air suspensions always have substantial auxiliary roll stiffness so this does not mean that there is no additional resistance to roll once the suspension becomes unloaded.


Figure 3. Illustration of suspension and axle lash

The other suspension parameter needed for the calculator is the roll centre height. For the purposes of the calculator this is measured in metres upwards from the axle centre. Thus if the roll centre is below the axle the value will be negative. Note that in the derivation of the equations the roll centre height was measured from the ground.

Generic steel and generic air suspensions with suitable parameters are included in the calculator. The values of those parameters are shown in Table 3.
Table 3. Suspension parameters used by SRT calculator.

| Suspension <br> Name and <br> Model <br> Number | Suspension <br> spring <br> stiffness <br> $(\mathbf{N} / \mathbf{m})$ | Suspension <br> track width <br> $(\mathbf{m})$ | Auxiliary roll <br> stiffness per <br> axle <br> (Nm/radian) | Composite <br> roll stiffness <br> per axle <br> (Nm/radian) | Axle lash <br> $(\mathbf{m m})$ | Roll centre <br> height <br> from axle <br> $(\mathbf{m})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Generic - <br> steer axle | 185000 | 0.8 |  | 130000 | 15 | -0.02 |
| Generic steel | 1000000 | 0.97 |  | 520000 | 30 | 0.2 |
| Generic air | 350000 | 0.97 |  | 780000 | 300 | 0.2 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

*Not needed if composite roll stiffness is known
To use a user-specified suspension, values for each of the parameter columns in Table 3 except auxiliary roll stiffness need to be provided by the manufacturer. If auxiliary stiffness is given instead of composite roll stiffness, equation (19) can be used to calculate the composite roll stiffness.

