Dynamic Economic Model

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Report authors: Nicola Smith, Garry McDonald, Emily Harvey

www.me.co.nz

Market Economics Limited
Level 5, 507 Lake Road
PO Box 331 297, Takapuna
Auckland 0740, NZ
+64 9 915 5510

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Glossary

CES Constant Elasticity of Substitution.

CET Constant Elasticity of Transformation.

CGE (Computable General Equilibrium) A class of applied economic models often used to illustrate an economy's responses to changes in policy, technology or other external shocks. Typically CGE models recognise a number of different types of economic agents (usually different types of industries, households and government), conceptualised as either profit or utility maximisers. Optimisation algorithms are employed to determine the set of prices for all commodities and factors of production that would prevail subject to selected constraints (e.g. all commodity and factor markets clear, and total income equals total expenditure for all agents).

CPI Consumer Price Index.

Enterprise In this model, following Statistics New Zealand's definition, an Enterprise is an institutional unit operating in New Zealand, and can be a company, partnership, trust, estate, incorporated society, producer board, local or central government organisation, voluntary organisation, or self-employed individual.

ERI (Economics of Resilient Infrastructure) A four year research programme funded by the Ministry of Business, Innovation and Employment.

GDP (Gross Domestic Product) The total market value of goods and services produced in an economy after deducting the cost of goods and services utilised in the process of production, but before deducting allowances for the consumption of fixed capital.

Household In this model Households include New Zealand resident individuals and families, and Private Non-Profit Organisation (PNPO) serving households.

Industry Value Added Value added summed according to aggregated industry groupings.

Input-Output analysis A quantitative economic technique that represents the interdependencies between different branches (industries or sectors) of a national economy or different regional economies. The technique depends on a matrix of raw economic data collected by companies and governments to study the relationship between suppliers and producers within an economy. Of particular interest is the extent that the outputs of one industry become the inputs to another.

MERIT (Modelling the Economics of Resilient Infrastructure Tool) A new analytical tool developed in the four-year Economics of Resilient Infrastructure (ERI) research programme that enables quantification of the economic impacts of infrastructure failure, and evaluation of the consequences of resilience-enhancing strategies.

RoNZ (Rest of New Zealand) Once the study region or region of interest is chosen, all other regions from the multi-regional Social Accounting Matrix (SAM) are aggregated to form a single 'rest of NZ' region.

SAM Social Accounting Matrix.

System Dynamics A methodology for understanding certain kinds of dynamic systems. The methodology concentrates on mapping the feedback relationships between different components or relationships within a system, and simulating changes in systems over time.

Value Added The value added to goods and services by the contributions of capital and labour, i.e. the value of output after the cost of bought-in materials and services has been deducted. It includes the national accounts categories 'gross operating surplus', 'compensation of employees', 'other taxes on productions' and 'subsidies'. The sum of all value added is equal to Gross Domestic Product (GDP), excluding taxes on products and import taxes net of subsidies. Thus in New Zealand, total value added is equal to approximately 88% of GDP.

1 Introduction

1.1 Background

The Economics of Resilient Infrastructure (ERI) programme is a four-year research programme funded by the Ministry of Business, Innovation and Employment. An important output of this programme is the development of a new analytical tool that will enable researchers and stakeholders to quantify the economic impacts of infrastructure failure, and evaluate the consequences of post-disaster recovery strategies. The research team has named the economic tool 'MERIT' (Modelling the Economics of Resilient Infrastructure Tool).

The Dynamic Economic Model (referred to often in this report simply as 'the model') is a multisectoral and multi-regional dynamic economic model that constitutes a principal component of MERIT. The model has been intentionally designed to imitate the core features of a Computable General Equilibrium (CGE) model. CGE models tend to be the favoured approach and 'state-of-art' in the modelling of regional- and national-level economic impacts. Among the advantages of these types of models are the whole-of-economy coverage, the capture of not only indirect (i.e. the so-called upstream and downstream multiplier effects generated through supply chains) and induced (i.e. as generated through household consumption) economic consequences, but also of the 'general equilibrium' (pricing) impacts.

While the model incorporates core features of a CGE model, it is important to note that it differs from a 'standard' CGE model in that it is a System Dynamics model formulated using differential equations that describe how the elements in the system change with time. This is an innovative extension to economic modelling undertaken in part to improve our ability to capture the impacts of infrastructure outages that push a system away from equilibrium.

It is also worth noting that building an economic model such as described in this report is an important, but not sufficient step required to evaluate the economic consequences of system shocks, such as road or other infrastructure outage. Also included within MERIT is the Business Behaviours Module (Brown *et al.* 2015), developed using empirical data gathered following the 2010/2011 Canterbury Earthquakes. It seeks to describe the effect of disruptions on an organisation's operability or ability to meet demand. The development of plausible and useful disruption scenarios, including costing the impact of transport outages, is also another significant part of the process of modelling economic disruption scenarios.

Once information is transformed into appropriate inputs and the model run, the Dynamic Economic Model is able to produce a variety of indicators to help us evaluate the impacts in aggregate and by industry of an infrastructure outage, including Gross Domestic Product (GDP), regional Value Added (similar to a regional equivalent of GDP), value of exports and imports, and household utility. It is worthwhile noting that the latter indicator is conceptually consistent with measurements that are sought to be calculated in a cost-benefit analysis. The model thus has the potential to be used for cost-benefit analysis as well as economic impact analysis.

1.2 Objectives of this report

This report is intended to be used as a general resource for any person seeking either to apply MERIT directly, or gain a better understanding of outputs that have been generated from MERIT. We thus seek to comprehensively record all of the equations contained within the model, outline key assumptions, and specify sources of data used in the model. The report should be read in conjunction with the earlier research report which outlines the methods used to create the regional Social Accounting Matrices (SAMs) that form the base economic accounts used in this model (Smith *et al.*, 2015).

2 The Dynamic Economic Model

2.1 Economic modelling context

2.1.1 Underlying theoretical basis

In economics, the general equilibrium theory of market behaviour and the extension of the theory into Computable General Equilibrium (CGE) with work by Johansen (1960) means that CGE models are now a well-established technique for describing economic behaviour. Despite its widespread applicability and use, however, it is often criticised for an inability to properly deal with such things as time-path trajectories and out-of-equilibrium dynamics (Barker, 2004; Grassini, 2004; Scrieciu, 2007). The problem derives from the fact that the CGE model is concerned purely with the identification of steady states of economic equilibrium and has little or no functionality when tasked with establishing the time paths between steady states or the dynamics of non-equilibrium economic systems. In the real world, economies do not tend to have steady states of equilibrium but are constantly changing due to the influence of complex sets of destabilising forces. The Dynamic Economic Model incorporates elements of CGE modelling as an approach but uses them in a systems dynamics context which is a modelling framework used for analysing and simulating complex dynamic systems. One of the key aspects of turning a standard CGE model into a dynamic model is to explicitly model supply and demand relationships. The systems models can be viewed in terms of causal diagrams which incorporate feedback loops that tend to cause the system to naturally gravitate towards some equilibrium point. The establishment of these price-related balancing feedback loops is an essential component of this type of model.

Static CGE models have been converted to dynamic models in some applications by allowing key stocks, usually related to labour and capital resources, to be varied over time. The System Dynamics approach adopted here allows a similar extension. The model also incorporates a system of *information delays*, a concept from the System Dynamics approach, that serve to imitate the action of decisions made in the feedback process. In particular the information delay seeks to incorporate gradual adjustments of beliefs that happen as a result of past experiences making similar decisions. This is done by incorporating a smoothing function that causes variables in the model to adjust gradually to current information, which means that recent information strongly influences their value, with the impact decreasing as time passes.

2.1.2 System Dynamics

Jay Forrester, at the Massachusetts Institute of Technology, developed System Dynamics during the mid-1950s (Forrester, 1961, 1969, 1971). System Dynamics is often described as a computer-aided modelling approach to policy analysis and design (e.g. Richardson, 2011). However, models constructed within System Dynamics programming languages are also frequently employed in problems that are not of a strict policy-orientation, for example design and engineering applications.

The System Dynamics approach relies specifically on using numerical methods (involving finite differential equations) to approximate solutions for ordinary differential equations along a path of successive 'time-steps'. Although these numerical approximation necessarily introduce some questions of accuracy, they are necessary in most cases, as the nonlinearity of the equations involves makes obtaining analytic solutions impossible. Furthermore, it significantly widens the scope of modelling exercises, enabling very complex systems to be represented within a computer simulation model, even by practitioners with no advanced mathematical training. Two popular graphical programming languages are now available for facilitating the construction of System Dynamics models, STELLA®and Vensim®. Both contain visual display and input and output features that enable users to easily grasp model structures, interactively run models and review results.

A core set of concepts employed in the development of this model is the distinction between *endogenous* variables: *stocks* or *auxiliaries*, and *exogenous* inputs: fixed model parameters (constants) or pre-determined time-varying inputs. In short, stocks are the independent variables within a simulation model that determine the condition of a system. These stocks accumulate (or dissipate) over time, and would continue to exist even if all relevant inflows and outflows (changes) to that stock ceased to exist. Stocks are endogenous as they are calculated within the model by solving the differential equations that describe the rates of change of the stocks. Note that the initial conditions for stocks are determined by the modeller, and are a special kind of exogenous input. By contrast, auxiliaries (sometimes also termed converters) can be thought of as 'intermediate steps' in the often complex functions defining rates of change of stocks. Auxiliaries are used to provide clarity to the modelling process by explicitly showing the steps required in the calculation of the rates of change or other output variables of interest. They also prevent the need for repetition in cases where the same calculations influence the rates of change of more than one stock. Auxiliaries are endogenous, as they are calculated explicitly at each time step from the values of the stocks and exogenous inputs in the model. The other model components are the fixed parameters (constants), and the time-varying inputs that are specified in advance (before the model is run). These are defined exogenous to the model, and can be varied by the modeller to investigate different scenarios.

2.2 The Dynamic Economic Model

2.2.1 Model structure

The basic structure of the Dynamic Economic Model is determined by the underlying regional Social Accounting Matrix (SAM) at its core (Smith *et al.*, 2015). The model considers two regions: the region of interest (selected from 15 possible regions) and the rest of New Zealand (RoNZ). For each region, the model describes the behaviour of representative agents (41 industry categories, 1 Household, 1 Enterprise, local government within each region, and central government). Each industry agent chooses the quantity and type of commodities (aggregated to 54 commodity categories) to produce, based on the prices of those commodities relative to the costs of production. Household, Enterprise, and government agents receive income from a variety of sources (e.g. wages and salaries, business profits, dividends, taxes, and transfers from other agents), and then allocate this income towards a variety of expenditure options (e.g. purchases of goods and services, savings, taxes, and transfers to other agents).

The model incorporates 'price' variables for all commodities and factors of production (i.e. types of labour and capital). These prices change in response to imbalances between supply and demand, and then 'nested' production functions allow the economy to react to these imbalances through substitution of demands and/or production between different types of commodities or factors. For example, if the demand for NZ-manufactured goods exceeds the supply, then the price of domestic goods will increase. This price increase (relative to foreign goods prices) will then lead to NZ-manufactured goods being substituted for goods produced overseas, thus reducing domestic demand and reducing prices. Similar substitution occurs in the factors and commodities used in production, and the region (within NZ) that the goods are demanded from. On the supply side, the relative prices determine how the supply of commodities and factors are split. For example the supply of goods manufactured in NZ is split between the NZ and export markets depending on the relative prices in each market, so if domestic goods prices increase more of the goods produced will be allocated to the NZ market, which will increase domestic supply, thus decreasing prices.

The model incorporates the dynamics of economic growth by keeping track of stocks of capital held by each industry. Capital stocks accumulate via investments in new capital and are diminished via the ongoing process of depreciation.

The model also includes accounts that keep track of financial flows between NZ and the rest of the world (i.e. balance of payments). When the demand for NZ currency starts to outstrip supply this causes the exchange rate to rise. Changes in the exchange rate change the price of NZ goods relative to overseas goods, thus influencing demand and supply relationships. The model uses the NZ commodity prices along with exogenously specified world commodity prices to determine the supply and demand of exports and imports, where NZ is assumed to be a 'price-taker' so NZ production does not affect the world prices but world prices can affect NZ prices.

2.2.2 Model specifics

The model is divided into eleven modules: households, governments, enterprises, industries, commodities, factors, labour, capital, savings & investment, the rest of the world, and a final module to calculate output variables of interest. Each module is described in detail in Section 3.

Much of the information for the model originates from a set of regional SAMs constructed for the financial year ending March 2007. The base year of the model, the regions available to be considered, and the minimum aggregation of industries and commodities are all determined by this process. A detailed report outlining the construction of a national and set of regional SAMs is also available (Smith *et al.*, 2015).

The SAMs were constructed for the 2006-07 financial year as this was the latest year for which a comprehensive set of national accounts, including in particular the national Supply and Use Tables, was available from Statistics New Zealand.[1] Based on the timing of the multi-regional SAMs, we can interpret the time at which model simulations commence (i.e. $t = 0$) as being half way through that financial year, or 1 October 2006.

The study region or the region of interest must be selected from the 15 NZ regions in the regional SAMs. The available regions are defined consistently with regional council boundaries, except in

[1]Statistics New Zealand has recently released a new set of Supply Use Tables, for the year ending March 2013. Accordingly the research team at M.E has commenced updating the multi-regional SAMs. This will allow the Dynamic Economic Model to be updated with more recent economic information, and provide improved capabilities for calibration.

the case of Nelson and Tasman which are aggregated to form a single region. Once the region of interest is selected, all other regions are aggregated to create the inputs for the model in terms of the region of interest and the RoNZ. This approach provides consistency with the definitions and structures of the regional SAMs (Smith *et al.*, 2015).

The 'default' version of the model contains 41 different industry types and 54 commodity types that are aggregated from the SAMs which have 106 industries and 205 commodities. The model and input files are set up so that the model can easily be further aggregated from the 41 industry and 54 commodity types if the application warrants such an aggregation. When selecting appropriate definitions we must often make trade-offs between providing detailed information and computational processing times and resources[2].

The Consumer Price Index (CPI) and Gross Domestic Product (GDP) indices are set to 1000 at $t = 0$. Following the convention in many CGE models, all prices are set equal to one for the reference year economic accounts. For our model this means that prices are all relative to the prices of the base year in 2007 NZ dollars ($\$_{2007}$). For example, if 20 kg of raw milk solids could be purchased for $\$_{2007}100$ during the base year, and the model shows the price increases from 1 at $t = 0$ to 1.5 at $t = 10$, then only 13.3kg of raw milk solids can be purchased for $\$_{2007}100$ at $t = 10$ or, put another way, that 20kg of raw milk solids would cost $\$_{2007}150$ at $t = 10$.

2.3 Underlying assumptions

Some of the key assumptions that underpin the model's structure are as follows:

Agent Behaviours For each economic region, the economy can be described by the behaviour of a group of representative agents (industries, households, enterprises, local government, and central government). Industries are assumed to make choices about production and consumption solely based on the relative costs of inputs and values of production. Household, Enterprise, and Government agents receive income from a variety of sources (including from wages and salaries, business profits, dividends, taxes and transfers from other agents) and, in turn, allocate this income to a variety of expenditure options (purchases of goods and services, savings, taxes, and transfers to other agents).

Base Price Adjustment Time Lags As already explained, the model is a dynamic model able to describe not only the distribution of economic impacts across different sectors, but also the distribution of impacts through time. This extension to CGE modelling is achieved essentially by creating price levels for all base commodities and factors of production (i.e. labour and capital). A key assumption is that all prices adjust upwards when supply is less than demand, and downwards when supply is less than demand. The parameters that determine the how far and how fast prices move in response to imbalances between supply and demand (α) are set via model calibration. The model does not at any stage attempt to compute the prices necessary to reach equilibrium (supply = demand) at any given time, instead the model calculates the changes in the base prices at each iteration (time step) and the new prices serve as inputs to the next iteration (time step). This creates time lags in base price adjustments in response to changes in supply or demand that depend on the α parameters. The way in which the time lags in base prices all interact over time contribute strongly to the dynamic behaviours captured by the model.

[2]Presently the online version of the model provides for 14 industries and 15 commodities, due to browser memory constraints.

Other Adjustment Times in the Model There are many other variables of the model that do not adjust instantaneously. We can break these down into two subsets: variables that we believe should adjust almost instantaneously but that cannot be set as such within the model, and variables that we believe should adjust over a longer time. The price of 'composite' commodities or factors, i.e. commodities or factors that are made up of base commodities and factors, fits into the first category. If we were to calculate these prices instantaneously (at the same time step as the base prices and other variables in the model) this would create simultaneous equation loops. Solving these at each time step would be computationally very difficult, and furthermore it would be assuming that all agents have perfect information about the simultaneous actions of all other agents in the system. In this model we instead allow a delay of $\tau_{prices} = \Delta t$, so that prices update using the information from the previous iteration (time step) to determine current composite prices. Some variables in the model we believe should use information from the past over a longer range than one time step. For example, rather than using the instantaneous income to calculate expenditure, or even the income from the previous time step, this model smooths the income over a longer time, $\tau_{income} > \Delta t$. Other examples in the model include the industry production which is determined by considering the demand over the past $\tau_{industry} \approx 3$ months, and the Interest rate and Cash surplus, which are smoothed over the times $\tau_{interest}$ and $\tau_{casurplus}$, respectively.

Input Parameter Estimation The model incorporates a large number of other input parameters. Due to limitations in the availability of official statistics, and the significant resource required to develop alternative datasets, we have developed a full set of economic accounts only for a single year. As already explained these accounts, termed SAMs, are based on the 2006-07 financial year in accordance with the national supply and use tables released by Statistics New Zealand. Many of the input parameters are derived from this SAM (e.g. Constant Elasticity of Substitution (CES) and Constant Elasticity of Transformation (CET) share and scale parameters, proportion of income transferred overseas, commodity inputs required per unit of production), and are set as constant over a model run. It is thus assumed that relationships and behaviours exhibited during the 2006-07 financial year are a good approximation of relationships/behaviours in future modelled years.

Resolution A particular scenario under investigation may impact on part of a region more intensely, or in other words involve an uneven spatial distribution of impacts within the region. Although the model could in theory be designed to receive economic accounts for varying spatial boundaries, it is important to note that the establishment of these accounts (SAMs) is a very laborious process. Related to model resolution, we have a set number of 'average commodities' traded by 'representative agents'. It is not possible for the model to capture changes that effect only a subclass of the selected commodities or representative agents so all changes modelled must generally be 'averages' that apply across the whole of the relative class.

Functional Forms Like many CGE models, the model repeatedly relies on the CES and CET functional forms to represent alternative demand (input) and supply (production) choices. 'Nested' CES and CET production functions allow the economy to react to imbalances between supply and demand in commodities/factors, through substitution of demand and/or production. These substitution possibilities occur in response to changes in relative prices. For example, a CES function describes the way in which demand for NZ-manufactured goods can be substituted for demand for goods produced overseas, if the price of domestic goods becomes too expensive relative to foreign goods. A separate CES function also describes the substitution between local-manufactured goods (i.e. produced within the same region) and the goods produced in the RoNZ. A CET function describes how the supply of goods produced in a region are split between the domestic and overseas markets, based on the relative prices, to maximise profit. While a separate CET function determines how the supply of goods to the domestic market is split between the local region and the RoNZ.

2.4 Model advantages and limitations

2.4.1 Key advantages

It is important to note that while the Dynamic Economic Model incorporates core features of a CGE model, it differs from a 'standard' CGE model in that it is a formulated based on System Dynamics, which considers how *stocks* change over time based on rates of change of the stocks with time. This is an innovative extension to economic modelling undertaken to improve our ability to capture the impacts of infrastructure outages. Standard economic models are 'equilibrium' models that describe conditions existing in an economy when a set of pre-determined conditions are met (e.g. supply = demand for all commodities and factors, and income = expenditure for all economic agents). For the analysis of infrastructure outages, however, an equilibrium-based analysis may not be helpful, as the time to reach equilibrium will often be longer than the actual length of the infrastructure outage, and during the period of disruption the economy is likely to be exhibiting non-equilibrium behaviour e.g. industries may be operating at a loss. The Dynamic Economic Model is a *simulation* model that shows a *transition pathway* towards the equilibrium determined by the underlying CGE model. However, due to changing external conditions the equilibrium towards which the economic system is moving will continue to change over time and it is not necessary that the equilibrium is actually achieved.

Some of the other key advantages of the Dynamic Economic Model compared to alternative approaches that may be employed in the analysis of impacts of an infrastructure outage:

- Once direct impacts are estimated, the Dynamic Economic Model simulates all of the flow-on impacts through the rest of the economy, sometimes referred to as 'cascading' or 'higher-order' impacts. This includes successive rounds of changes in demand for goods and services as a result of production supply chains (i.e. 'indirect' effects) and changes in consumer spending as a result of changes in household income (i.e. 'induced' effects) as typically captured by an Input-Output analysis. However, because the Dynamic Economic Model also captures price changes and substitution, it is not subject to the problems of impact overestimation that are typically encountered in an Input-Output analysis.

- The Dynamic Economic Model is able to produce a variety of indicators to help us evaluate the impacts of an infrastructure outage, including GDP, regional value-added (similar to a regional equivalent of GDP), value of exports and imports, and household utility. It is worthwhile noting that the latter indicator is conceptually consistent with measurements that are sought to be calculated in a cost-benefit analysis. The Dynamic Economic Model thus has the potential to be used for cost-benefit analysis as well as economic impact simulation.

- Properly accounting for distributional impacts is a long-standing issue for economics. When undertaking cost-benefit studies it is often simply assumed (improperly), either implicitly or explicitly, that if benefits are greater than costs this is overall good for society. However, if there are distributional impacts (i.e. some people benefit while others experience costs) this is not necessarily justified in economics[3]. While there is still significant work to be undertaken in the analysis of distributional impacts, the Dynamic Economic Model does provide some information on the distribution of economic impacts, i.e. across different regions of interest and across different economic industries. A future extension of the model could involve disaggregating the single representative household agent into different types (e.g. by income, rural and urban and/or Māori and non-Māori categories). This would allow for more in-depth consideration, under alternative event/policy scenarios, of the distribution of impacts across different household types.

- When we apply the Dynamic Economic Model we are required to undertake a series of simulations looking across the entire economy. The nature of this process helps to ensure that some cross checking of results occurs. It is our professional opinion that compared with more *ad hoc* methods (particularly cost-benefit analysis which typically involves a series of separate analyses for different benefits and costs), applications of the Dynamic Economic Model are less-likely to result in the overestimation of impacts through double-counting[4], and are more likely to encourage consideration of the co-generation of costs and benefits from any economic change (e.g. while additional freight charges are a disadvantage for those purchasing commodities, they are a benefit for the road transport sector)[5].

2.4.2 Limitations and caveats

The Dynamic Economic Model, and indeed the 'MERIT' (Modelling the Economics of Resilient Infrastructure Tool) as a whole, is a new and innovative approach to the analysis of economic disruptions. The novelty of the model is a strength, enabling behaviours and dynamics to be captured and communicated in a thus far not achieved by other approaches. However, it is also important to keep in mind that because of its novelty and recent development, the Dynamic Economic Model (and MERIT) has not had the benefit of years of reflection and refinement. There are still many avenues to be explored in the further development and refinement of the model. Some of these are discussed below:

Further calibration The research team is in the process of undertaking further calibration of the model. Ideally this will be an ongoing process, extending well beyond the end of the Economics of Resilient Infrastructure (ERI) research programme, as significant further opportunities exist to test components of the model against real data where available. For example, the research team would like to try and source further data on the response of export demands to changes in price to help further calibrate the export elasticity of demand parameters used within the model. We are also keen to obtain examples of infrastructure outages, where accompanying economic information is also available, to help improve the calibration of the model. Furthermore, now that Statistics New Zealand has released an update of the National Supply and Use Tables, the research team has commenced the development of a new set of regional SAMS. Once completed these tables will provide comprehensive economic accounts for two years and improved

[3] For example, if a scenario/policy under investigation results in a gain of $100 to individual A and a loss of $50 to individual B, we cannot assume that social welfare has increased; for if A is rich and B is poor, it may be that the loss of satisfaction to B of $50 is far greater than the gain of $100 for A.

[4] When undertaking a welfare or cost-benefit analysis of a road outage it would, for example, be easy to double-count the costs of additional travel to work, both as a loss of leisure time and reduction in labour.

[5] One error sometimes encountered in economic impacts studies is the treatment of a loss of tourism expenditure as a result of some inability to access tourism services as a net loss for the economy. Although this may be valid when considering international tourism, for domestic tourists the reduction in expenditure may simply be balanced by expenditure at alternative sites or regions and/or increased consumption of alternative commodities. The spatial extent of the society/community considered is important (i.e. whether the impacts analysed are for a particular local community, a region, or the whole nation).

capabilities for model calibration.

Uncertainty analysis Our ability to understand and interpret the outputs of MERIT would also be advanced by dedicated work on uncertainty analysis. The research team has identified this as a key topic for further work should research funds become available.

Road outages Transportation impacts are among the most challenging to analyse, because of the great diversity of transportation networks across space, and the high levels of substitutability within and between transportation types. Although this is not strictly part of the economic model itself, it is worth noting that the algorithms thus far developed for translating changes in road network levels of service into input parameters for the economic model do not yet cover all of the potential impacts of road outages on the economic system. When analysing road outages, particularly in an urban context, it is also important to consider whether the outage is likely to generate significant changes in the numbers of people choosing to travel to work, which can have significant economic consequences by reducing the labour factors of production. Further work is necessary to define with confidence the thresholds at which choices are made by individuals not to attend work when subject to changes in travel costs. Furthermore we recognise that businesses have some inherent and adaptive resilience to short term disruptions. For example, some employees can still undertake work at home, and production can be recaptured by working overtime and rescheduling. Further research is required to define the extent to which changes in travel to work result in losses in economic production, and how this also varies between economic industries.

Analysis of long-term and uncertain futures To conclude this section we provide some comments on the analysis of long-term scenarios under deep uncertainty. Given that the most important investment and policy decisions tend to involve long-term consequences, we see this as a topic that requires specific thought in regards to potential future applications of the Dynamic Economic model and MERIT. To recap, we analyse scenarios in MERIT by selecting appropriate input parameters to adjust, and also the degree of adjustment necessary for these parameters. While this may sound simple, in practice this is a complex process, often involving separate analysis or modules (e.g. the Business Behaviours Module), each with its associated sets of assumptions. Reflecting the scope of the ERI programme, the scenarios considered, and modelling undertaken, thus far have concentrated on relatively short-term impacts of infrastructure disruptions. Rebuild and recovery, for example, were deliberately omitted from consideration.

While longer-term impacts of infrastructure, hazard and other disruptions are clearly important, by their nature they also tend to be highly uncertain. It is likely that probability distributions of key variables and parameters will never be reliably quantified. For example, while we might attempt to estimate the proportion of manufacturing firms that will cease operations when subject to a water outage by building up evidence from historical data, estimates of future global economic growth would be neither reliable nor verifiable. The terms ‘deep uncertainty’ or ‘Knightian uncertainty’ have been coined for the types of future uncertainties about the future which we cannot expect to reliably quantify.

The extent to which people and businesses choose to permanently relocate out of disrupted city/region is an example of a long-term impact that is highly uncertain. These choices will be influenced by the political decisions that are made (e.g. whether to relocate key government services). Relocation dynamics are also likely to be non-linear, with a region able to sustain some disruption or stress but eventually a tipping point is achieved where significant out-migration occurs.

The uncertainty around long-term dynamics does not mean that models such as MERIT are not useful. Indeed these can be very useful in helping to focus a structure an informed dialogue on the consequences of disruptive events and the potential outcomes of policy decisions. What must be recognised is that for these processes to be successful, it will be necessary to invest significant time and effort in building consensus around the appropriate scenarios and assumptions to test in models and/or analysing outcomes under many different options and assumptions.[6]

2.5 Technical details

2.5.1 Conventions and notation used

Stocks are in bold font, with the first letter capitalised. Auxiliary (intermediate calculation) steps are named in all lower case italics. Variables with names in all capital letters and italics are exogenous inputs. Subscripts are used to indicate the dimensionality of a variable. For example, the stock $\mathbf{Pregdomcomm}_{sr,dr,c}$ denotes a price of regional domestic commodities specified by supply region (subscript 'sr'), demand region (subscript 'dr') and commodity type (subscript 'c'). Full information on subscripts is provided in Table 2.1.

Table 2.1 Subscripts used in the Dynamic Economic Model

Subscript indices	Description
$h = [CAP, LAB]$	Factors: Capital and Labour
$cap = [BuilC, NatC]$	Capital types: Built capital and Natural capital
$input = [FactsI, InterI]$	Input types: Factor inputs and Intermediate inputs
$g = [CentralG, LocalG]$	Governments: Central and Local
$sr = [SReg1, SReg2]$	Supply Regions: Region of interest and RoNZ
$dr = [DReg1, DReg2]$	Demand Regions: Region of interest and RoNZ
$i = [Ind1, Ind2, \ldots]$	Industries: see Table A.2 in Appendix A.1
$c = [Com1, Com2, \ldots]$	Commodities: see Table A.3 in Appendix A.1
$nct = [NatCap1, NatCap2, \ldots]$	Natural Capital types: see Table A.1 in Appendix A.1
$m = [Road, Rail]$	Transport margins: Road and Rail

As already indicated, two types of subscripts relate to regions, that is supply region (subscript 'sr') and demand region (subscript 'dr'). Occasionally within the model it is necessary to switch between these subscript types. The notation $sr \rightarrow dr$ in a subscript is used when the variables calculated for supply regions 1 and 2 respectively map to demand regions 1 and 2 (or vice versa when the subscripts are switched in the notation). The notation $DReg1 \leftrightarrow DReg2$ in a subscript is used when a quantity is transferred between regions i.e. the output from $DReg1$ goes to $DReg2$ and the output from $DReg2$ goes to $DReg1$.

2.5.2 Overview of computational method

The model is made up of a rate equation for each stock in the model that expresses how the value of that stock will change with time (known as a system of ordinary differential equations)

[6] We can identify two general approaches to decision making under uncertainty. One seeks to first reduce uncertainty by agreeing on assumptions about current and future conditions, and then analysing decision options. The alternative analyses many different options under different assumptions, deferring agreement to the very end of the process (Kalra *et al.*, 2014).

in the form:

$$\frac{d}{dt}\textbf{Stock} = rateofchange \tag{2.1}$$

The rates of change $rateofchange$ in this model can be (often nonlinear) functions of other stocks in the model at the current time or a past time (delays), as well as constant parameters and time varying exogenous inputs. Due to the nonlinearity in the model, these equations cannot typically be solved explicitly to find $\textbf{Stock}(t)$. However, these types of nonlinear dynamical systems arise almost ubiquitously in models of the real world and many methods have been developed to numerically approximate the solutions (values of $\textbf{Stock}(t)$). Numerical methods for solving differential equations must be convergent, i.e. the numerical solution must converge to the exact solution (the error must go to zero) as the step size Δt goes to zero.

We have chosen to use Euler's method to numerically approximate the solution to the Dynamic Economic Model, as it is computationally easy to calculate, can deal with delays, and is available in most systems dynamics software. Euler's method transforms the rate equations for the stocks into finite difference equations numerically approximates the solution as:

$$\textbf{Stock}(t + \Delta t) = \textbf{Stock}(t) + (rateofchange) \times \Delta t \tag{2.2}$$

Euler's method uses a fixed time step and only one calculation (function value) is required per time step. Additionally, it is what is known as an *explicit* method which means that calculating the value of the stocks at a time step only requires knowledge of the values at the previous time step, as in Eq. (2.2). This simplicity does however mean that the numerical error does not decrease with step size as quickly as some other methods, so a smaller step size is required for numerical accuracy.

When we are evaluating different scenarios, there may be sudden changes in exogenous inputs to the model, often in the form of discontinuities. It is know that the Euler method can be numerically unstable in the case of sudden changes, leading to numerical solutions that oscillate when the actual solution does not. We must take care to examine the results near any discontinuities, and reduce the step size further if there is any evidence of this unstable behaviour.

As discussed above the step size choice is always a trade off between numerical accuracy and stability, and computational time. After some investigation, we choose the time step $\Delta t = 0.0025$ years, which is approximately equal to a day. In the scenarios considered here this time step was found to produce stable numerical results for the Euler solution method we use to solve the model, whilst being able to be solved in a reasonable time. Additionally, in this model the time step is used explicitly to determine how fast various stocks adjust (e.g. the inflation rate, prices, industry production, and the GDP index). An adjustment time of a day is considered small enough to be realistic in these instances. In all cases, the adjustment times in the model ($\tau_{casurplus}$, τ_{income}, $\tau_{industry}$, $\tau_{interest}$, and τ_{prices}) must be greater than or equal to the time step, Δt, for the model to be numerically stable.

2.5.3 Constant elasticity of substitution and transformation functions

CES Like many CGE models, the model relies heavily on functions specified in a form that has become known as the CES functional form. The CES function is a particular type of aggregating function, which combines two or more types of goods, or two or more types of productive inputs into an aggregate quantity. As the name suggests, the function is characterised by the use of an elasticity of substitution, ϵ, which describes the percentage change in some quantity (e.g. the demand for a particular input) caused by a percentage change in the relative price of that quantity. The larger the elasticity, the greater the response to changes in price.

As an example, the CES production function for the demand for the ith composite good Q_i can be specified as:

$$Q_i = \gamma_i \left[\delta_i^m (M_i)^{\eta_i} + \delta_i^d (D_i)^{\eta_i} \right]^{\frac{1}{\eta_i}} \tag{2.3}$$

where M_i and D_i are the demand for the two input types (e.g. imported and domestic) available to produce good Q_i, and γ_i is the CES scale parameter. The CES input share coefficients for input types M_i and D_i are δ_m and δ_d, where $0 \leq \delta_i \leq 1$ and $\delta_i^m + \delta_i^d = 1$. Furthermore η_i is a parameter defined by the elasticity of substitution between inputs M_i and D_i:

$$\eta_i = \frac{\epsilon_i - 1}{\epsilon_i}, \qquad \eta_i \leq 1 \tag{2.4}$$

Now we can see that since $\eta_i \leq 1$, we have $1/(1 - \eta_i) \geq 0$, so as expected, when the price of an input p_i^d (or p_i^m) increases the demand for that input D_i (or M_i) decreases.

There are some special cases of the elasticity of substitution that are worth noting. Firstly, as $\epsilon \to \infty$, η goes to one and we have perfect (or linear) substitution. Conversely, in the extreme case where $\epsilon = 0$ (η goes to negative infinity) we have fixed shares of inputs, and Equation (2.3) becomes a Leontief function. Finally, in the limit where η goes to 0 Equation (2.3) becomes the well known Cobb-Douglas production function.

Let us assume that production choices are about maximising the value of outputs less input costs, and the demand for (composite) outputs produced are described by the CES production function above. The first-order conditions for this problem imply the following demand functions for the two input types:

$$\begin{aligned} M_i &= \left[(\gamma_i)^{\eta_i} \, \delta_i^m \, \frac{p_i^q}{p_i^m} \right]^{\frac{1}{1-\eta_i}} Q_i \\ D_i &= \left[(\gamma_i)^{\eta_i} \, \delta_i^d \, \frac{p_i^q}{p_i^d} \right]^{\frac{1}{1-\eta_i}} Q_i \end{aligned} \tag{2.5}$$

where p_i^m and p_i^d respectively denote the relative prices of input types M_i and D_i, and p_i^q is the price of the composite Q_i created by combining inputs M_i and D_i (before taxes or tariffs). The CES function-approach can be used either to (i) calculate the quantity of some 'composite' item produced from known quantities of inputs, or (ii) the relative demand for inputs of different types given the relative price of these inputs compared to the 'composite price' and given the total quantity of the composite item required.

CET The CET function is specified in the same form as the CES function (Equation (2.3)). However this time instead of M and D representing two alternative types of inputs for the production of Q_i, D_i and E_i represent two alternative types of products into which the output Z_i can be transformed (e.g. export or domestic goods). The gross output must satisfy the equation:

$$Z_i = \theta_i \left[\xi_i^e (E_i)^{\phi_i} + \xi_i^d (D_i)^{\phi_i} \right]^{\frac{1}{\phi_i}} \tag{2.6}$$

Where D_i and E_i are the supply of the two possible outputs, and θ_i is the scale parameter for the CET function. The scale parameters ξ_i^d and ξ_i^e have the properties $0 \leq \xi_i^d \leq 1$, $0 \leq \xi_i^e \leq 1$, and $\xi_i^d + \xi_i^e = 1$, as for the CES scale parameters.

As in the CES formulation, the elasticity of transformation, ψ, determines how sensitive the ratio of supply of outputs is to relative price changes, with larger ψ meaning that supply ratio responds more to price changes. However the parameter derived from the elasticity for use in the CET function (Equation (2.6)) is:

$$\phi_i = \frac{\psi_i + 1}{\psi_i}, \qquad \phi_i \geq 1 \tag{2.7}$$

By assuming profit maximisation we get the supply of D_i and E_i to be:

$$\begin{aligned} E_i &= \left[(\theta_i)^{\phi_i} \, \xi_i^e \, \frac{p_i^z}{p_i^e} \right]^{\frac{1}{1-\phi_i}} Z_i \\ D_i &= \left[(\theta_i)^{\phi_i} \, \xi_i^d \, \frac{p_i^z}{p_i^d} \right]^{\frac{1}{1-\phi_i}} Z_i \end{aligned} \tag{2.8}$$

where p_i^z is the price of the gross output Z_i (including and taxes or tariffs), and p_i^d and p_i^e are the prices of transformed outputs D_i and E_i respectively. However, since $\phi_i \geq 1$, the quantity $1/(1 - \phi_i) < 0$ and the relative quantity of D_i (or E_i) supplied will increases if the relative price of that product p_i^d (or p_i^e) increases compared to the gross output price p_i^z.

3 Modules

3.1 Household module

The Household module can be conceptually separated into two parts, one dealing with Household income and the other dealing with Household expenditure. The full set of equations are available in Appendix B.1. A tree diagram that provides a summary of the different contributions to income and expenditure is shown in Figure 3.1.

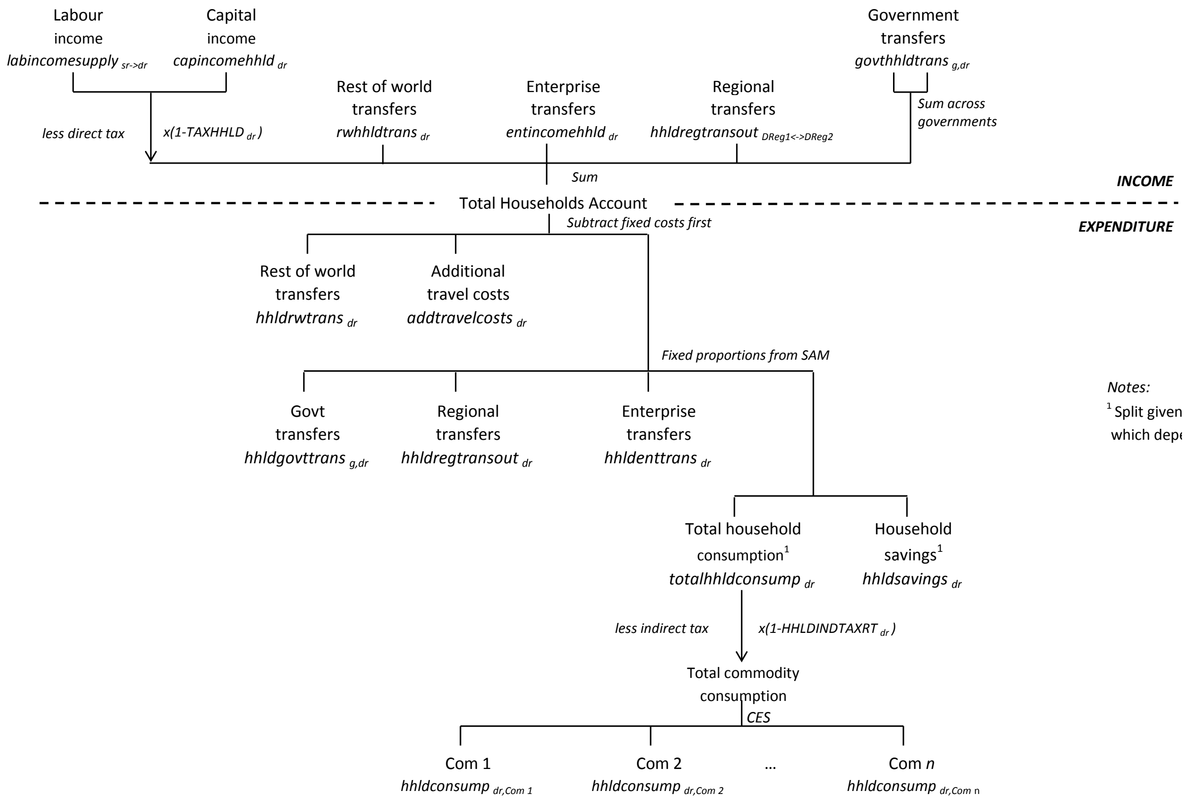

Figure 3.1 Tree diagram showing household income and expenditure. *CES*: Constant Elasticity of Substitution calculation. *SAM*: Social Accounting Matrix.

Starting with income, the principal sources of household income are factor payments to labour, $labincomesupply_{sr \rightarrow dr}$, capital income paid directly to households, $capincomehhld_{dr}$, and enterprise income that is transferred to households, $entincomehhld_{dr}$. Household incomes are further sourced from transfers from the rest of the world, $rwhhldtrans$, the government, $govthhldtrans_{g,dr}$, and interregional household transfers, $hhldregtransin_{dr}$. Altogether these sources of income, less direct taxes $hhlddirecttax_{dr}$, constitute the available household income $actualhhldincome$

(Eq. (B.3)):

$$actualhhldincome_{dr} = capincomehhld_{dr} + entincomehhld_{dr} + labincomesupply_{sr \to dr} + rwhhldtrans_{dr} + \sum_{g} (govthhldtrans_{g,dr}) + hhldregtransout_{DReg1 \leftrightarrow DReg2} - hhlddirecttax_{dr}$$

Direct tax is charged on the capital income paid directly to households, $capincomehhld_{dr}$, and the factor payments to labour, $labincomesupply_{sr \to dr}$, at the rate $TAXHHLD_{dr}$ (Eq. (B.22)):

$$hhlddirecttax_{dr} = (capincomehhld_{dr} + labincomesupply_{sr \to dr}) \times TAXHHLD_{dr}$$

Capital and enterprise income are made up of local transactions, and transactions from outside the region (Eq. (B.4) and Eq. (B.5)):

$$capincomehhld_{dr} = caplocalhhldtrans_{dr} + capreghhldtrans_{DReg1 \leftrightarrow DReg2}$$

$$entincomehhld_{dr} = enthhldtrans_{dr} + entreghhldtrans_{DReg1 \leftrightarrow DReg2}$$

To calculate the total value of labour income received by households within each region, $labincomeregion$, the model first determines the total value of income paid to labour within each region, $labincomedregion$, by multiplying the quantity of labour demanded in each region, $factorsu_{h=LAB,dr,i}$ by the current labour price, $preglabour_{dr}$. Note that an adjustment is made to account for a small proportion of labour supplied from outside of NZ thereby resulting in a transfer of labour income to the rest of the world. It is assumed that the proportion of labour income transferred to the rest of the world, $RWFACTRT$, remains constant with the base year (Eq. (B.7)):

$$labincomedregion_{dr} = \sum_{i} (factorsu_{h=LAB,dr,i} \, (1 - RWFACTRT_{h=LAB,dr}) \, preglabour_{dr})$$

Labour income that is 'paid out' within a (demand) region can be allocated either to local households or households from outside of the region. In the model the relative shares are determined simply according to each (supply) region's contribution to total labour supply for a given (demand) region (Eq. (B.6)):

$$labincomesupply_{sr} = \sum_{dr} \left(\frac{labincomedregion_{dr} \, reglaboursupply_{sr,dr}}{\sum_{sr}(reglaboursupply_{sr,dr})} \right)$$

Incomes received by households from the rest of the world, $rwhhldtrans_{dr}$, are assumed to grow from the base year amount, $RWHHLDTRANSBS_{dr}$, at the same rate of growth as world Gross Domestic Product (GDP). The index of world GDP, $WORLDGDPINDX(t)$, is exogenous and thus can be adjusted for different simulations. In order to also allow for some adjustment in transfers in response to changes in the exchange rate, it is assumed that a share of rest of world transfers to households, i.e. $FCSHRWHHLDTRANS$, is calculated in foreign currency (Eq. (B.9)):

$$rwhhldtrans_{dr} = RWHHLDTRANSBS_{dr} \, WORLDGDPINDX(t) \, (1 - FCSHRWHHLDTRANS_{dr}) \\ RWHHLDTRANSBS_{dr} \, WORLDGDPINDX(t) \left(\frac{1}{\textbf{Exchangert}} \right) FCSHRWHHLDTRANS_{dr}$$

Rather than using the auxiliary *actualhhldincome* to calculate expenditure in each time step based on instantaneous income in that same time step, this model uses the recognised household income **Rhhldincome**, which smooths the income over a longer time, τ_{income} (Eq. (B.1)):

$$\frac{d}{dt}\left(\mathbf{Rhhldincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualhhldincome_{dr} - \mathbf{Rhhldincome}_{dr}\right)$$

This has the advantage of avoiding stability issues due to simultaneous equations, and is more representative of household behaviour.

Turning now to household expenditure, six different 'sinks' for household income are recognised: household savings, *hhldsavings*, consumption of commodities, *totalhhldconsump*, transfers to the rest of the world, *hhldrwtrans*, transfers to government, *hhldgovttrans*, transfers to enterprises, *hhldenttrans*, and transfers to NZ households in other regions, *hhldregtransout*. This model also includes additional travel costs due to short term change such as a road outage, $addtravelcosts_{dr}$, which are described in Section 4.2.

In an analogous manner to transfers from the rest of the world to households, transfers from households to the rest of the world are assumed to vary from the base year in accordance with the change in world GDP and the exchange rate (Eq. (B.8)):

$$\begin{aligned}hhldrwtrans_{dr} = {} & HHLDRWTRANSBS_{dr}\,WORLDGDPINDX(t)\,(1 - FCSHHHLDRWTRANS_{dr}) \\ & + HHLDRWTRANSBS_{dr}\,WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHHHLDRWTRANS_{dr}\end{aligned}$$

Where $HHLDRWTRANSBS_{dr}$ is the transfers from households to the rest of the world in the base year, and $FCSHHHLDRWTRANS_{dr}$ is the proportion of transfers that are calculated in foreign currency.

The household transfers to the rest of the world, $hhldrwtrans_{dr}$, are subtracted from the recognised household income, **Rhhldincome**, first. In certain scenarios, e.g. road outages, there may be additional travel costs to the household, $addtravelcosts_{dr}$. These will be subtracted next. Once these expenditures have been subtracted, the exogenous constants $HHLDENTTRANSRT_{dr}$, $HHLDGOVTTRANSRT_{g,dr}$, and $HHLDREGTRANSRT_{dr}$ are used to define the shares of income reallocated from households to enterprises ($hhldenttrans_{dr}$), government ($hhldgovttrans_{g,dr}$), and other NZ households ($hhldregtransout_{dr}$), respectively. This gives (Eq. (B.10), Eq. (B.11), Eq. (B.12)):

$$\begin{aligned}hhldenttrans_{dr} = {} & (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\ & \times HHLDENTTRANSRT_{dr}\end{aligned}$$

$$\begin{aligned}hhldgovttrans_{g,dr} = {} & (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\ & \times HHLDGOVTTRANSRT_{g,dr}\end{aligned}$$

$$\begin{aligned}hhldregtransout_{dr} = {} & (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\ & \times HHLDREGTRANSRT_{dr}\end{aligned}$$

Of the remaining household income, the proportion that is allocated to consumption, rather than savings, is determined by the household consumption rate, *hhldconsumprt*, such that (Eq. (B.13), Eq. (B.14)):

$$hhldsavings_{dr} = \Big[\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - hhldregtransout_{dr} - hhldenttrans_{dr} - \sum_{g}(hhldgovttrans_{dr,g}) - addtravelcosts_{dr}\Big](1 - hhldconsumprt_{dr})$$

$$totalhhldconsump_{dr} = \Big[\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - hhldregtransout_{dr} - hhldenttrans_{dr} - \sum_{g}(hhldgovttrans_{dr,g}) - addtravelcosts_{dr}\Big]\, hhldconsumprt_{dr}$$

It is generally recognised that household consumption is negatively correlated with changes in the real interest rate. When interest rates increase, people will be spending more on repaying mortgages and thus there will be less money available to spend on consumption of goods. In this model the parameter $CIRELASTICITY_{dr}$ controls the degree to which household consumption changes in response to changes in the real interest rate, which gives the relationship (Eq. (B.15)):

$$hhldconsumprt_{dr} = \left[\left(\frac{realinterestrt}{BASEREALINTERESTRT} - 1\right) CIRELASTICITY_{dr} + 1\right] \times BASECONSUMPRT_{dr}$$

Where $BASEREALINTERESTRT$ is the interest rate in the base year, and $BASECONSUMPRT_{dr}$ is the household commodity consumption rate in the base year.

Once the total value of income spent on consumption is determined, the Households module also determines the proportion that is reallocated to government through the imposition of indirect taxes (e.g. GST). This is determined simply by multiplying the total value of consumption by a constant household indirect tax rate, $HHLDINDTAXRT_{dr}$ (Eq. (B.21)).

$$hhldindirecttax_{dr} = totalhhldconsump_{dr}\, HHLDINDTAXRT_{dr}$$

The model then applies a constant Constant Elasticity of Substitution (CES) function procedure to determine household consumption of individual commodities. In short this process involves first determining the total demand for 'composite' commodities, $hhldcompcommd_{dr}$, by dividing the value of total consumption (minus the indirect taxes) by the applicable composite commodity price, $\mathbf{Phhldcc}_{dr}$, (Eq. (B.20)):

$$hhldcompcommd_{dr} = \frac{totalhhldconsump_{dr} - hhldindirecttax_{dr}}{\mathbf{Phhldcc}_{dr}}$$

Next, the first-order conditions for the CES problem enables a function to be generated that specifies the quantity of each commodity consumed, $hhldconsump_{dr,c}$, given the price of the particular commodity, $\mathbf{Pcompcommd}_{dr,c}$, relative to the composite price, $\mathbf{Phhldcc}_{dr}$, and the applicable CES scale parameters, γ_{dr}^{hhldc}, share parameters, $\delta_{dr,c}^{hhldc}$, and elasticity of substitution, η_{dr}^{hhldc} (Eq. (B.18)):

$$hhldconsump_{dr,c} = \left[(\gamma_{dr}^{hhldc})^{\eta_{dr}^{hhldc}}\, \delta_{dr,c}^{hhldc}\, \frac{\mathbf{Phhldcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta_{dr}^{hhldc}}} hhldcompcommd_{dr}$$

Finally using the same CES scale and share parameters, and elasticity of substitution, the quantity of composite commodities consumed can then be calculated, (Eq. (B.17)):

$$qhhldcc_{dr} = \gamma_{dr}^{hhldc} \left[\sum_{c} \left(\delta_{dr,c}^{hhldc} \left(hhldconsump_{dr,c} \right)^{\eta_{dr}^{hhldc}} \right) \right]^{\frac{1}{\eta_{dr}^{hhldc}}}$$

Once we know the quantity of each composite commodity consumed, we can calculate the current household composite commodity consumption price using (Eq. (B.16)):

$$actualphhldcc_{dr} = \frac{\sum_c \left(\mathbf{Pcompcommd}_{dr,c} \, hhldconsump_{dr,c} \right)}{qhhldcc_{dr}}$$

The price stock $\mathbf{Phhldcc}_{dr}$ adjusts to the calculated current price $actualphhldcc_{dr}$ at the rate τ_{prices} (Eq. (B.2)):

$$\frac{d}{dt} \left(\mathbf{Phhldcc}_{dr} \right) = \frac{1}{\tau_{prices}} \left(actualphhldcc_{dr} - \mathbf{Phhldcc}_{dr} \right)$$

In order to have the prices respond almost instantaneously the adjustment time is set to be equal to the time step $\tau_{prices} = \Delta t$. This means that the price adjusts within one time step and in the case where we use Euler's method to numerically solve the rate equations this is exactly equivalent to setting $\mathbf{Phhldcc}_{dr}(t) = actualphhldcc_{dr}(t - \Delta t)$.

3.2 Government module

The Government module is very similar in structure to the Households module (Section 3.1). The module can also be conceptually separated into equations dealing with income and equations dealing with expenditure. The full set of equations are available in Appendix B.2. A tree diagram that provides a summary of the different contributions to income and expenditure is shown in Figure 3.2.

On the income side, governments receive income mainly from direct and indirect taxes, $directtaxincome_{g,dr}$ and $indirecttaxincome_{g,dr}$. In addition to income from taxes, the governments receive some transfers of income from capital, $capgovttrans_{g,dr}$, enterprise, $entgovttrans_{g,dr}$, and households, $hhldgovttrans_{g,dr}$. The model also accounts for some minor financial transfers between government agents, $betwgovttransout_{CentralG \leftrightarrow LocalG,dr}$. These transfers are simultaneously a source of income for the receiving government agent, and a source of expenditure for the providing government agent.

The actual government income is given by (Eq. (B.25)):

$$\begin{aligned} govtincome_{g,dr} = & directtaxincome_{g,dr} + indirecttaxincome_{g,dr} + capgovttrans_{g,dr} + entgovttrans_{g,dr} \\ & + hhldgovttrans_{g,dr} + betwgovttransout_{CentralG \leftrightarrow LocalG,dr} - govtdirecttax_{g,dr} \end{aligned}$$

Direct tax is charged on the capital and enterprise income transferred to governments at the rate $GOVTDIRECTTAXRT_{g,dr}$ (Eq. (B.28)):

$$govtdirecttax_{g,dr} = \left[capgovttrans_{g,dr} + entgovttrans_{g,dr} \right] \times GOVTDIRECTTAXRT_{g,dr}$$

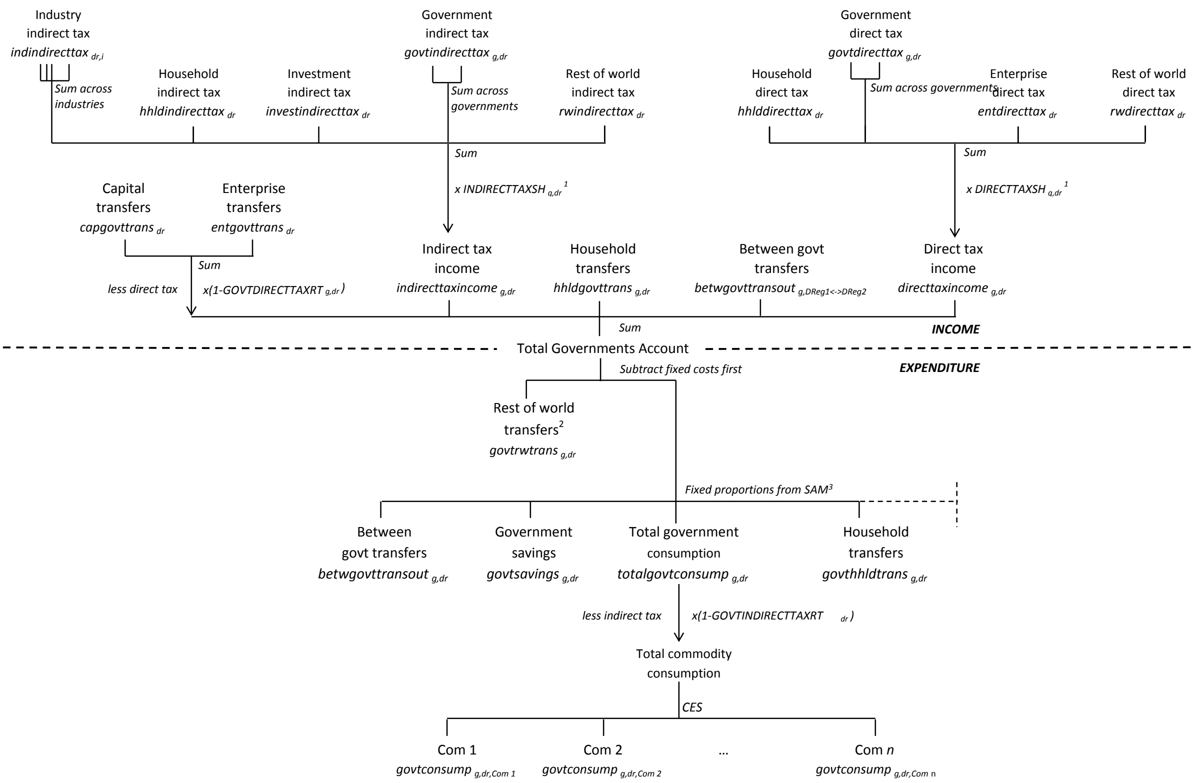

Figure 3.2 Tree diagram showing governments' income and expenditure. *CES*: Constant Elasticity of Substitution calculation. *SAM*: Social Accounting Matrix.
[1]How tax is split between local and central governments is exogenously determined.
[2]The amount that gets transferred to the rest of the world depends on the **Casurplus**.

In the calculation of all direct and indirect tax transfers to governments, exogenous tax rates, $DIRECTTAXSH_{g,dr}$ and $INDIRECTTAXSH_{g,dr}$, are applied to determine how the tax income is split between local and central governments. Direct tax comes from taxes on the income of enterprises, $entdirecttax_{dr}$, households, $hhlddirecttax_{dr}$, the rest of the world, $rwdirecttax_{dr}$, and the governments itself, $govtdirecttax_{g,dr}$ (Eq. (B.26)):

$$directtaxincome_{g,dr} = \left[entdirecttax_{dr} + hhlddirecttax_{dr} + rwdirecttax_{dr} + \sum_{g}(govtdirecttax_{g,dr})\right] \times DIRECTTAXSH_{g,dr}$$

While indirect taxes (including GST) come from the consumption of households, $hldindirecttax_{dr}$, investments, $investindirecttax_{dr}$, industries $indindirecttax_{dr,i}$, the rest of the world, $rwindirecttax_{dr}$, and the government, $govtindirecttax_{dr,g}$ (Eq. (B.27)):

$$indirecttaxincome_{g,dr} = \left[investindirecttax_{dr} + rwindirecttax_{dr} + hhldindirecttax_{dr} \sum_{i}(indindirecttax_{dr,i}) + \sum_{g}(govtindirecttax_{dr,g})\right] \times INDIRECTTAXSH_{g,dr}$$

Government expenditure includes transfers to households, $govthhldtrans_{g,dr}$, other government agents, $betwgovttransout_{g,dr}$, and the rest of the world, $govtrwtrans_{g,dr}$. The stock $\mathbf{Rgovtincome}_{g,dr}$, which is the recognised governments' (local and central) income, smoothed over the time τ_{income}

(Eq. (B.23)):

$$\frac{d}{dt}\left(\mathbf{Rgovtincome}_{g,dr}\right) = \frac{1}{\tau_{income}}\left(govtincome_{g,dr} - \mathbf{Rgovtincome}_{g,dr}\right)$$

is used to determine the allocations of income to different expenditures.

The transfers from the government to the rest of the world, $govtrwtrans_{g,dr}$, vary from the base year amount $GOVTRWTRANSBS_{g,dr}$, depending on the current account surplus **Casurplus** according to the elasticity parameter $EGOVTTRANS$ (Eq. (B.29)):

$$\begin{aligned} govtrwtrans_{g,dr} =& \operatorname{sgn}\left(\mathbf{Casurplus}\right) \left|\mathbf{Casurplus}\right|^{EGOVTTRANS} \left(\frac{GOVTRWTRANSBS_{g,dr}}{\sum_g \sum_{dr} \left(GOVTRWTRANSBS_{g,dr}\right)}\right) \\ &+ GOVTRWTRANSBS_{g,dr} \end{aligned}$$

This expenditure is removed first, then the share of remaining income allocated to other areas is calculated (Equations (B.30)-(B.33)):

$$govthhldtrans_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTHHLDTRANSRT_{g,dr}$$

$$betwgovttransout_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times BTWGOVTTRANSRT_{g,dr}$$

$$govtsavings_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTSAVRT_{g,dr}$$

$$totalgovtconsump_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTCONSUMPRT_{g,dr}$$

Once the total government consumption is calculated, the indirect tax paid on consumption is given by (Eq. (B.39)):

$$govtindirecttax_{g,dr} = totalgovtconsump_{g,dr}\, GOVTINDIRECTTAXRT_{g,dr}$$

As in the households module (Section 3.1) the CES function is used as a basis for allocating total government consumption expenditure among individual commodity types.

First the model determines the total demand for 'composite' commodities, $govtcompcommd_{g,dr}$, by dividing the total consumption (minus indirect taxes) by the applicable commodity price, $\mathbf{Pgovtcc}_{g,dr}$ (Eq. (B.38)):

$$govtcompcommd_{g,dr} = \frac{totalgovtconsump_{g,dr} - govtindirecttax_{g,dr}}{\mathbf{Pgovtcc}_{g,dr}}$$

Next, the consumption of individual commodities, $govtconsump_{g,dr,c}$ can be calculated from the demand for composite commodities, using a CES function that adjusts the particular commodity allocations based on the price of that commodity, $\mathbf{Pcompcommd}_{dr,c}$, relative to the composite price, $\mathbf{Pgovtcc}_{g,dr}$, using the CES scale parameters $\gamma^{govtc}_{g,dr}$, the CES share parameters $\delta^{govtc}_{g,dr,c}$, and the elasticity of substitution $\eta^{govtc}_{g,dr}$ (Eq. (B.36)):

$$govtconsump_{g,dr,c} = \left[(\gamma^{govtc}_{g,dr})^{\eta^{govtc}_{g,dr}}\ \delta^{govtc}_{g,dr,c} \frac{\mathbf{Pgovtcc}_{g,dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta^{govtc}_{g,dr}}} govtcompcommd_{g,dr}$$

The same scale and share parameters, and elasticity of substitution can then be used to calculate the quantity of composite commodities consumed (Eq. (B.35)):

$$qgovtcc_{g,dr} = \gamma^{govtc}_{g,dr} \left[\sum_c \left(\delta^{govtc}_{g,dr,c} \left(govtconsump_{g,dr,c} \right)^{\eta^{govtc}_{g,dr}} \right) \right]^{\frac{1}{\eta^{govtc}_{g,dr}}}$$

The actual price of government composite commodities is then calculated (Eq. (B.34)):

$$actualpgovtcc_{g,dr} = \frac{\sum_c \left(govtconsump_{g,dr,c} \, \mathbf{Pcompcommd}_{dr,c} \right)}{qgovtcc_{g,dr}}$$

And the price stock $\mathbf{Pgovtcc}_{g,dr}$ adjusts at rate τ_{prices} to meet this price (Eq. (B.24)):

$$\frac{d}{dt} \left(\mathbf{Pgovtcc}_{g,dr} \right) = \frac{1}{\tau_{prices}} \left(actualpgovtcc_{g,dr} - \mathbf{Pgovtcc}_{g,dr} \right)$$

3.3 Enterprise module

The Enterprise module follows a similar form to the Government module (Section 3.2) and the Household module (Section 3.1). It can be broken up into income and expenditure sections, however, as Enterprises do not consume commodities directly there is no consumption or commodity section. The full set of equations are available in Appendix B.3. A tree diagram that provides a summary of the different contributions to income and expenditure is shown in Figure 3.3.

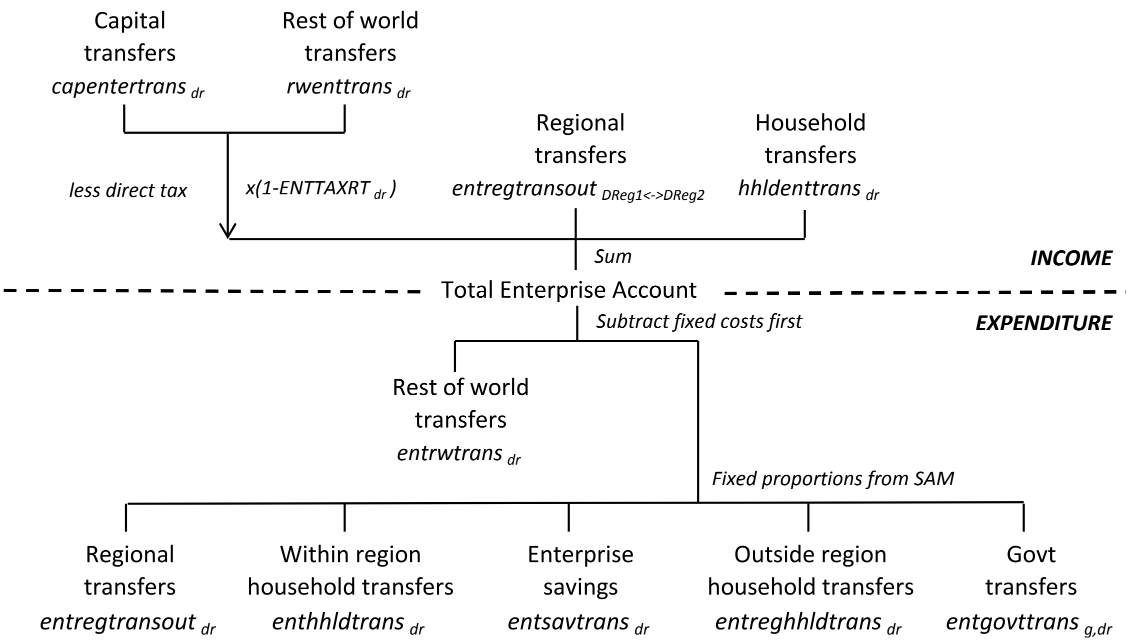

Figure 3.3 Tree diagram showing enterprise income and expenditure. *SAM*: Social Accounting Matrix.

The income of enterprises is made up of transfers of capital income, $capentertrans_{dr}$, transfers from households, $hhldenttrans_{dr}$, and transfers from the rest of the world, $rwenttrans_{dr}$. Additionally, this model includes enterprise income transfers between regions, which are simultaneously an expenditure for the providing region $entregtransout_{dr}$, and a source of income for the receiving region $entregtransout_{DReg1 \leftrightarrow DReg2}$\$. This gives the equation for the total enterprise income (Eq. (B.41)):

$$
\begin{aligned}
actualenterincome_{dr} =& capentertrans_{dr} + hhldenttrans_{dr} + rwenttrans_{dr} \\
& + entregtransout_{DReg1 \leftrightarrow DReg2} - entdirecttax_{dr}
\end{aligned}
$$

Where there is direct tax charged on capital and rest of world transfers only, at a rate $ENTTAXRT_{dr}$ (Eq. (B.42)):

$$
entdirecttax_{dr} = (capentertrans_{dr} + rwenttrans_{dr}) \times ENTTAXRT_{dr}
$$

Transfers to enterprises from the rest of the world, $rwenttrans_{dr}$, follow the same form as household transfers from the rest of the world (see Eq. (B.9)). Specifically, they are assumed to grow from the rate in the base year, $RWENTTRANSBS_{dr}$, following the GDP index $WORLDGDPINDX(t)$, with a fixed proportion, $FCSHRWENTTRANS_{dr}$, being set in foreign currency (Eq. (B.43)):

$$
\begin{aligned}
rwenttrans_{dr} =& RWENTTRANSBS_{dr} \, WORLDGDPINDX(t)(1 - FCSHRWENTTRANS_{dr}) \\
& RWENTTRANSBS_{dr} \, WORLDGDPINDX(t) \left(\frac{1}{\mathbf{Exchangert}} \right) FCSHRWENTTRANS_{dr}
\end{aligned}
$$

The recognised enterprise income stock, $\mathbf{Renterincome}_{dr}$, follows the actual enterprise income, $actualenterincome_{dr}$, adjusting over the time τ_{income} (Eq, (B.40)):

$$
\frac{d}{dt} (\mathbf{Renterincome}_{dr}) = \frac{1}{\tau_{income}} (actualenterincome_{dr} - \mathbf{Renterincome}_{dr})
$$

Enterprise income is distributed to six different 'sinks' of expenditure: transfers to the rest of the world, $entrwtrans_{dr}$, enterprise transfers between regions, $entregtransout_{dr}$, transfers to governments, $entgovttrans_{g,dr}$, transfers to households within the region, $enthhldtrans_{dr}$, and outside the region, $entreghhldtrans_{dr}$, with the remaining transferred to savings, $entsavtrans_{dr}$.

As in the Household and Government modules, we subtract the transfers to the rest of the world first, with the amount calculated in a similar way (Eq. (B.44)):

$$
\begin{aligned}
entrwtrans_{dr} =& ERWTRANSBS_{dr} \, WORLDGDPINDX(t)(1 - FCSHENTRWTRANS) \\
& ERWTRANSBS_{dr} \, WORLDGDPINDX(t) \left(\frac{1}{\mathbf{Exchangert}} \right) FCSHENTRWTRANS
\end{aligned}
$$

The remaining income is then allocated according to the proportions: $EREGTRANSRT_{dr}$, $EGOVTTRANSRT_{g,dr}$, $EHHLDTRANSRT_{dr}$, $ERHHLDTRANSRT_{dr}$, and $ESAVTRANSRT_{dr}$ (Equations (B.45)- (B.49)):

$$
entregtransout_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr} \right] \times EREGTRANSRT_{dr}
$$

$$
entgovttrans_{g,dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr} \right] \times EGOVTTRANSRT_{g,dr}
$$

$$
enthhldtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr} \right] \times EHHLDTRANSRT_{dr}
$$

$$
entreghhldtrans_{dr} = [\mathbf{Renterincome}_{dr} - entrwtrans_{dr}] \times ERHHLDTRANSRT_{dr}
$$

$$
entsavtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr} \right] \times ESAVTRANSRT_{dr}
$$

Where the proportions sum to exactly one, ensuring that all enterprise income is distributed to expenditure each time step.

3.4 Industries module

The industries module performs three primary functions: (1) Calculating the value of industry production within each region, (2) calculating the quantity of composite factors demanded by industries, and (3) determining the respective values of industry income and expenditure. The full set of equations are available in Appendix B.4.

Starting with the first, the model assumes that each industry will ascertain a desired level of production, $\mathbf{Desiredprod}_{dr,i}$, based on the value of industry sales. In turn the value of industry sales is calculated by first determining the value of commodity sales, including both domestic and export sales (Eq. (B.54)):

$$vcomdemand_{sr,c} = \sum_{dr} (pregdomcomminclmargin_{sr,dr,c}\, regdomcommd_{sr,dr,c}) \\ + expcommoditys_{sr,c}\, actualpexports_{sr,c}\, \mathbf{Exchangert}$$

Once the values of commodity sales are determined, these are allocated to industries based on each industry's relative share of total production, also termed supply coefficients, $supcoeff_{sr,i,c}$, thereby determining the total value of demand for industries' output, $vinddemand_{sr,i}$ (Eq. (B.53)):

$$vinddemand_{sr,i} = \sum_{c} (supcoeff_{sr,i,c}\, vcomdemand_{sr,c})$$

Rather than assuming that industries adjust production immediately to match the current value of sales, the model incorporates a time delay that acts to smooth industry production against short term fluctuations in commodity sales. This approach is well recognised in the System Dynamics literature and is termed 'psychological smoothing' (Forrester, 1961, p497). In short, psychological smoothing is a form of information delay that is recognised to occur whenever a decision that forms part of the feedback structure of a system is influenced by gradual adjustments of perceptions or beliefs. Thus, desired industry production is represented as a state of the system (i.e. stock) that either grows or declines based on the difference between it and the actual value of industry sales, as well as the assumed adjustment time for beliefs, $\tau_{industry}$ (Eq. (B.50)):

$$\frac{d}{dt}\left(\mathbf{Desiredprod}_{dr,i}\right) = \frac{1}{\tau_{industry}}\left(vinddemand_{sr\rightarrow dr,i} - \mathbf{Desiredprod}_{dr,i}\right)$$

The model further allows for the value of actual industry production, $actualprod_{dr,i}$, to vary from the desired level of production. The intention is to capture external influences on industries that prevent achievement of 'as normal' levels of production. Examples tested so far are disruptions in critical infrastructure services, such as water and electricity, which cause disruption to normal production activities. This is achieved essentially by enforcing a maximum level of production when the operability of an industry, $OPERABILITY_{sr,i}(t)$, which is a scalar that varies between zero (complete disruption) and one (no disruption). See further discussion in Section 4.2.2

Having determined the value of industry production, the quantity of industry production is determined simply by dividing by the unit cost of production, $unitcost_{dr,i}$. Given also the quantity of factors required per unit of production, $factinputshare_{dr,i}$, it is possible to determine the total demand for composite factors by industries, $compfactord_{dr,i}$ with the equation (Eq. (B.59)):

$$compfactord_{dr,i} = \frac{\mathbf{Desiredprod}_{dr,i}}{unitcost_{dr,i}}\, factinputshare_{dr,i} \times \left(\frac{1}{\mathbf{Multifactprod}_{dr,i}}\right)$$

With a scaling of $(1/\mathbf{Multifactprod}_{dr,i})$ to account for the current level of productivity of the factor inputs (multi-factor productivity). It is important that the model uses desired production rather than actual production in the calculation of composite factor demands. This means that industries 'hold on' to factor demands, at least in the short term, despite disruptions in operations. As rationale we would not expect, for example, firms to lay off staff immediately following a water supply disruption, even if those staff could not be effectively employed.

Nevertheless it is necessary to also calculate the effective composite factor demands, $effectcompfactord_{dr,i}$, based on actual industry production, as this forms an input to the Factors (Section 3.6), and then the Commodities module (Section 3.5) and the Industry module (Section 3.4). The actual levels of commodity production are calculated (Eq. (B.60)):

$$effectcompfactord_{dr,i} = \frac{actualprod_{dr,i}}{unitcost_{dr,i}} \, factinputshare_{dr,i} \times \left(\frac{1}{\mathbf{Multifactprod}_{dr,i}} \right)$$

The unit cost of production is made up of the unit cost of intermediate inputs and the unit cost of factors (Eq. (B.61)):

$$unitcost_{dr,i} = interinputunitcost_{dr,i} + factinputunitcost_{dr,i}$$

The contributions to unit cost are calculated based on the relative shares of intermediate inputs and factors necessary for production, $interinputshare_{dr,i}$ and $factinputshare_{dr,i}$, the relative prices of these inputs, $\mathbf{Pintinputs}_{dr,i}$ and $\mathbf{Pcfact}_{dr,i}$, and accounting for current state of multi-factor productivity and the imposition of indirect taxes on intermediate inputs (Equations (B.62), (B.63)):

$$\begin{aligned} interinputunitcost_{dr,i} = & \left(interinputshare_{dr,i} \, \mathbf{Pintinputs}_{dr,i} \, (1 + INDINDIRECTTAXRT_{dr,i}) \right) \\ & \times \left(\frac{1}{\mathbf{Multifactprod}_{dr,i}} \right) \end{aligned}$$

$$factinputunitcost_{dr,i} = \left(factinputshare_{dr,i} \, \mathbf{Pcfact}_{dr,i} \right) \times \left(\frac{1}{\mathbf{Multifactprod}_{dr,i}} \right)$$

The final components of the Industries module are concerned with calculating industry income, $industryinc_{dr,i}$, and expenditure, $indexpendu_{dr,i}$. Income is calculated as the minimum of the value of commodities supplied, and the value of commodities demanded (Eq. (B.57)):

$$industryinc_{dr,i} = \min(potentialsales_{sr \to dr,i}, vinddemand_{sr \to dr,i})$$

The value of commodities supplied, or in other words potential sales, $potentialsales_{sr,i}$, is determined simply by multiplying the quantity of commodities supplied,$indcommoditys_{sr,i,c}$, by the respective commodity prices, $\mathbf{Pcompcomms}_{sr,c}$ (Eq. (B.58)):

$$potentialsales_{sr,i} = \sum_{c} \left(indcommoditys_{sr,i,c} \, \mathbf{Pcompcomms}_{sr,c} \right)$$

The values for industry expenditure, on the other hand, are derived from the use of inputs to production and the costs of those inputs (Eq. (B.56)):

$$indexpendu_{dr,i} = \frac{compfactoru_{dr,i}}{factinputshare_{dr,i}} \, \mathbf{Multifactprod}_{dr,i} \times unitcost_{dr,i}$$

Additionally the model keeps track of any differences between industry income and expenditure, $\mathbf{Industrybalance}_{dr,i}$, as this surplus adds to industry 'profits' and thus needs to be considered in the calculation of Industry Value Added (Eq. (B.52)):

$$\frac{d}{dt}\left(\mathbf{Industrybalance}_{dr,i}\right) = \frac{1}{\tau}\left((industryinc_{dr,i} - indexpendu_{dr,i}) - \mathbf{Industrybalance}_{dr,i}\right)$$

3.5 Commodities module

The commodities module is the largest of all the modules in terms of the number of equations specified. However, many of the equations follow a repeated structure involving only different sets of quantities and prices. The full set of equations for the commodities module are available in Appendix B.5.

A visual representation of the key structure used to determine the regional domestic commodity supply and use (with associated domestic prices) is provided by the tree diagram in Figure 3.4.

As can be seen, the commodities module sets in place a nested structure of CES and CET functions. The CES functions deal with the demand side of commodities, while the CET functions deal with the supply side. Towards the middle of the diagram the supply and demand sides meet, because the CES and CET structures are each used to specify the same set of quantity information, that is the the supply of each commodity c from each supply region sr to each demand region dr. These quantities are termed regional domestic commodity supply, $regcdomcomms_{sr,dr,c}$, from the supply side, and regional domestic commodity demand, $regdomcommd_{sr,dr,c}$, from the demand side.

The prices for these commodity flows, $\mathbf{Pregdomcomm}_{sr,dr,c}$ are the base prices of the commodities module, influencing the prices for all other quantities calculated elsewhere within the module. Like other base prices within the model, these are calculated by comparing the supply and demand of quantities, where the ratio of supply to demand is:

$$excessproduction_{sr,dr,c} = \frac{regcdomcomms_{sr,dr,c}}{regdomcommd_{sr,dr,c}}$$

The prices $\mathbf{Pregdomcomm}_{sr,dr,c}$ change following the rate equation:

$$\frac{d}{dt}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\right) = \left(\left(\frac{1}{excessproduction_{sr,dr,c}}\right)^{\alpha^{pregdommcomm}} - 1\right)\mathbf{Pregdomcomm}_{sr,dr,c}$$

From this equation we can see that the price is adjusted upwards if demand is greater than supply ($excessproduction_{sr,dr,c} < 1$), and downwards if supply is greater than demand ($excessproduction_{sr,dr,c} > 1$). The parameter $\alpha^{pregdommcomm}$ controls the magnitude of price adjustment rates in response to imbalances between supply and demand.

Once the base commodity prices have been calculated, the model provides an option to add an additional margin onto the prices, as may be necessary to reflect the circumstances of a scenario under investigation. For example, if $DMARGINSHOCKCOEF_{sr,dr,c}$ is the quantity of additional road transport margins charged per unit of commodity under a road outage scenario, and $pimproadmargins_{sr,dr}$ is the price per unit of road transportation services, the adjusted price, $pregdomcomminclmargin_{sr,dr,c}$, can be calculated as:

$$\begin{aligned} pregdomcomminclmargin_{sr,dr,c} =& DMARGINSHOCKCOEF_{sr,dr,c}\, pimproadmargins_{sr,dr} \\ &+ \mathbf{Pregdomcomm}_{sr,dr,c} \end{aligned}$$

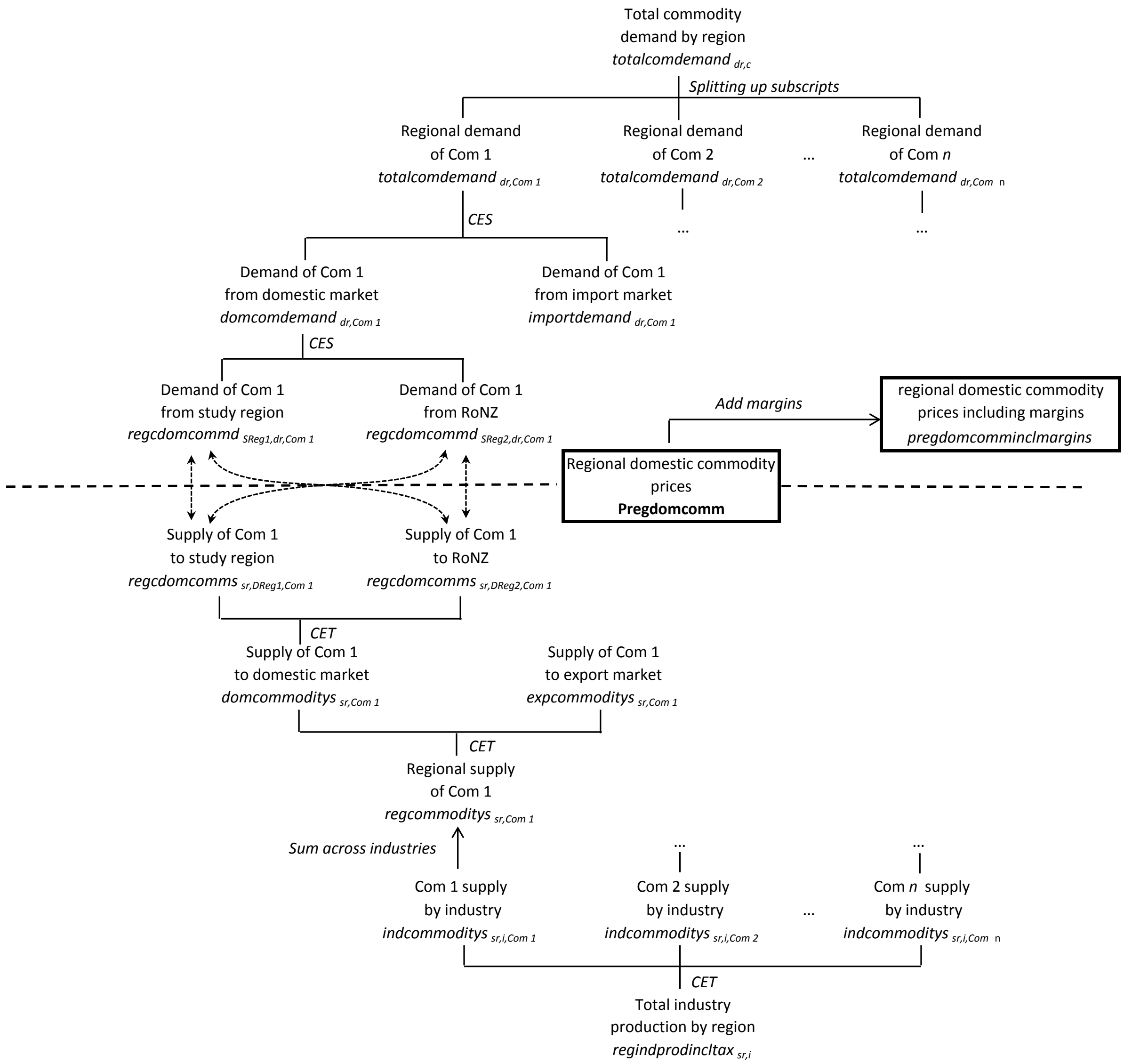

Figure 3.4 Tree diagram showing how the supply and demand of commodities are calculated. Here we just show the main structure for Commodity 1 (Com 1) due to space limitations, but the same Constant Elasticity of Transformation (CET) and CES transformations are applied to all commodities.

See Section 4.2 for a description of how these margins are calculated.

Now, turning back to the supply of, and demand for, domestic commodities, i.e. $regcdomcomms_{sr,dr,c}$ and $regdomcommd_{sr,dr,c}$, we can trace along the branches of the CET and CES trees in Figure 3.4 to understand the way in which these quantities are calculated within the module.

3.5.1 Supply side

At the base of the supply tree in Figure 3.4, is the level of industry production within each region, $regindprodincltax_{sr,i}$. Production levels achieved by industries are controlled in the model by industries' effective use of composite factors, $effectivecompfactoru_{dr\rightarrow sr,i}$ as determined under the Factors module (Section 3.6):

$$regindprodincltax_{sr,i} = \frac{effectcompfactoru_{dr\rightarrow sr,i}}{factinputshare_{dr\rightarrow sr,i}} PRODSCALAR_{dr\rightarrow sr,i} \, \mathbf{Multifactprod}_{dr\rightarrow sr,i}$$

Here $factinputshare_{dr\rightarrow sr,i}$ specifies the ratio of composite factor inputs per unit of production, and $PRODSCALAR_{dr\rightarrow sr,i}$ and $\mathbf{Multifactprod}_{dr\rightarrow sr,i}$ are added to enable the estimated industry production levels to be adjusted to respectively account for indirect taxes and changes in multi-factor productivity. Note that as all of the variables on the right hand side of the equation are specified according to the demand region categories, these need to be first matched to the appropriate supply region for calculation.

A CET function (Eq. (B.79)) describes how the quantities of total industry production within each region, $regindprodincltax_{sr,i}$, are transformed into quantities of specific commodities produced by each industry within each region, $indcommoditys_{sr,i,c}$:

$$indcommoditys_{sr,i,c} = \left[(\theta^{comsup}_{sr,i})^{\phi^{comsup}_{sr,i}} \xi^{comsup}_{sr,i,c} \frac{\mathbf{Pcindustrys}_{sr,i}}{\mathbf{Pcompcomms}_{sr,c}} \right]^{\frac{1}{1-\phi^{comsup}_{sr,i}}} regindprodincltax_{sr,i}$$

The outcome of the CET function is that, depending on the strength of the assumed elasticity of transformation, for a given industry the greater the price of a particular commodity relative to the composite price for all commodities supplied by that industry, the greater the proportion of industry output that will be devoted to supply of that commodity.

Having calculated $indcommoditys_{sr,i,c}$ we can also calculate the supply coefficients, $supcoeff_{sr,i,c}$, to be used in the industries module. These coefficients specify, of the total production of a particular commodity c in supply region sr, the proportion that is produced by industry i:

$$supcoeff_{sr,i,c} = \frac{indcommoditys_{sr,i,c}}{\sum_i (indcommoditys_{sr,i,c})}$$

Summing commodity production across all industries provides the total quantities of each commodity produced within each supply region, $regcommoditys_{sr,c}$ (Eq. (B.78)):

$$regcommoditys_{sr,c} = \sum_i (indcommoditys_{sr,i,c})$$

For each region, we are then able to split total commodity supply using a CET function into supply to the domestic market, $domcommoditys_{sr,c}$ (Eq. (B.87)), and supply to the export market, $expcommoditys_{sr,c}$ (Eq. (B.76)):

$$expcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsexp}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{pexpcommnz_{sr,c}} \right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c}$$

$$domcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsdom}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{\mathbf{Pcompdomcomms}_{sr,c}} \right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c}$$

Note that the share parameters in these CET functions sum to one: $\xi^{commsdom}_{sr,c} + \xi^{commsexp}_{sr,c} = 1$.

Finally we use a CET function to split the domestic commodity supply between the two domestic regions (Eq. (B.85)):

$$regcdomcomms_{sr,dr,c} = \left[(\theta^{commregs}_{sr,c})^{\phi^{regcom}_{sr,c}} \xi^{commregs}_{sr,dr,c} \frac{\mathbf{Pcompdomcomms}_{sr,c}}{\mathbf{Pregdomcomm}_{sr,dr,c}} \right]^{\frac{1}{1-\phi^{regcom}_{sr,c}}} \times domcommoditys_{sr,c}$$

3.5.2 Demand side

We now turn to the demand side components of the commodities module, as depicted on the top half of Figure 3.4. Assuming for the time being that we already know the total demand in each region for each commodity, $totalcomdemand_{dr,c}$, these demands are first split into demands from the domestic market, $domcomdemand_{dr,c}$, and demands from the import market, $importdemand_{dr,c}$ via CES functions:[7]

$$domcomdemand_{dr,c} = \left[(\gamma^{commd}_{dr,c})^{\eta^{com}_{dr,c}} \delta^{commddom}_{dr,c} \frac{\mathbf{Pcompcommd}_{dr,c}}{\mathbf{Pcompdomcommd}_{dr,c}} \right]^{\frac{1}{1-\eta^{com}_{dr,c}}} totalcomdemand_{dr,c}$$

$$importdemand_{dr,c} = \left[(\gamma^{commd}_{dr,c})^{\eta^{com}_{dr,c}} \delta^{commdimp}_{dr,c} \frac{\mathbf{Pcompcommd}_{dr,c}}{pimpcommnz_{dr,c}} \right]^{\frac{1}{1-\eta^{com}_{dr,c}}} totalcomdemand_{dr,c}$$

Note that the share parameters in these CES functions sum to one: $\delta^{commddom}_{dr,c} + \delta^{commdimp}_{dr,c} = 1$. Next, demands from the domestic market are split into demands from each individual region, $regdomcommd_{sr,dr,c}$, via another CES function:

$$regdomcommd_{sr,dr,c} = \left[(\gamma^{commregd}_{dr,c})^{\eta^{regcom}_{dr,c}} \delta^{commregd}_{sr,dr,c} \frac{\mathbf{Pcompdomcommd}_{dr,c}}{pregdomcomminclmargin_{sr,dr,c}} \right]^{\frac{1}{1-\eta^{regcom}_{dr,c}}} \times domcomdemand_{dr,c}$$

This completes all of the components of Figure 3.4 apart from the calculation of total commodity demands. Total commodity demands are a combination of the demands from households, investment, governments, and as intermediate inputs for industries. Some additional household consumption and margins are added for the case of infrastructure outages. This is described in Figure 3.5.

For each region, total demand for commodities is the sum of the demands from households, investment, governments and industries, with the first three determined under the respective modules for each of these agents. Additionally, the model allows for net increases in demand for transportation related commodities that may be incurred due to the circumstances of a scenario:

$$\begin{aligned} totalcomdemand_{dr,c} = &\sum_{g} (govtconsump_{g,dr,c}) + hhldconsump_{dr,c} + investconsumpq_{dr,c} \\ &+ \sum_{i} (indconsump_{dr,i,c}) + marginconsumpq_{dr,c} + ADDHHLDTRAVEL_{dr,c}(t) \end{aligned}$$

The variable $marginconsumpq_{dr,c}$ captures additional consumption of road or rail transportation services for moving freight (all of domestic, import and export freight), while $ADDHHLDTRAVEL_{dr,c}$ is the net increase in consumption of transportation related commodities (e.g. petroleum, vehicle maintenance services) incurred by households.

[7] Trade data often records the coexistence of imports and exports of the type of goods. To explain this anomaly, termed 'cross-hauling', it is proposed that such goods, despite fitting within the same classification, must in some way be different causing the goods to be *imperfectly* substitutable. Within economic modelling it tends to be substitution between imports and domestic goods, and between exports and domestic goods that is important. The degree of similarity between the these goods can be measured by a parameter such as the elasticity of substitution . The assumption about imperfect substitution between imports and domestic goods is called the Armington (1969) assumption, and it is often incorporated in Computable General Equilibrium (CGE) models via a CES function.

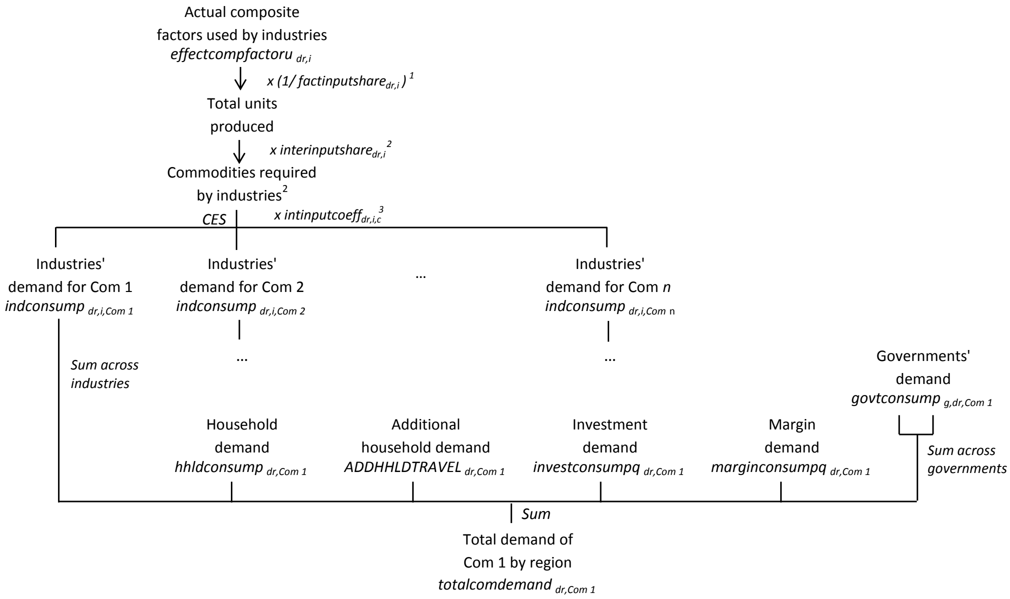

Figure 3.5 Tree diagram showing demand for commodities. For the final summation of demand we just show Commodity 1 (Com 1) due to space limitations, but the same transformations are applied to all commodities.

Commodities are used by industries as intermediate inputs of production. To calculate the demand from industries for commodities, $indconsump_{dr,i,c}$, required the following steps. First we divide the total use of composite factors by industries, $effectcompfactoru_{dr,i}$, by the quantity of factors required per unit of production, $factinputshare_{dr,i}$. This gives us the total units produced by industries. We then multiply this total number of units by the intermediate inputs required per unit of production, $interinputshare_{dr,i}$, which gives us the overall demand for intermediate inputs from industries. Finally the auxiliary variable $intinputcoeff_{dr,i,c}$, determines how this overall demand is split between the (54) different commodities. Putting this all together gives the equation (Eq. (B.90)):

$$indconsump_{dr,i,c} = \frac{effectcompfactoru_{dr,i}}{factinputshare_{dr,i}} interinputshare_{dr,i} \times intinputcoeff_{dr,i,c}$$

The total consumption of each commodity across all industries is required, which we obtain simply by summing across all industries (Eq. (B.91)):

$$totalindconsump_{dr,c} = \sum_{i} (indconsump_{dr,i,c})$$

The coefficients in the above equation, $factinputshare_{dr,i}$, $interinputshare_{dr,i}$ and $intinputcoeff_{dr,i,c}$, are also derived from CES functions. At the top tier, $factinputshare_{dr,i}$ and $intinputcoeff_{dr,i,c}$ are derived by splitting the total units of inputs required per unit of production into two different categories, factors and intermediate inputs:

$$factinputshare_{dr,i} = \left[(\gamma^{fi}_{dr,i})^{\eta^{fi}_{dr,i}} \delta^{fi}_{input=FactsI,dr,i} \frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pcfact}_{dr,i}} \right]^{\frac{1}{1-\eta^{fi}_{dr,i}}}$$

$$interinputshare_{dr,i} = \left[(\gamma^{fi}_{dr,i})^{\eta^{fi}_{dr,i}} \delta^{fi}_{input=InterI,dr,i} \frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pintinputs}_{dr,i}} \right]^{\frac{1}{1-\eta^{fi}_{dr,i}}}$$

We then calculate the share of intermediate inputs allocated to each commodity by the CES function (Eq. (B.111)):

$$intinputcoeff_{dr,i,c} = \left[(\gamma_{dr,i}^{cominput})^{\eta_{dr,i}^{cominput}} \delta_{dr,i,c}^{cominput} \frac{\mathbf{Pintinputs}_{dr,i}}{\mathbf{Pcompcommd}_{dr,c}} \right]^{\frac{1}{1-\eta_{dr,i}^{cominput}}}$$

3.5.3 Calculating prices

Export and Import prices In the model the world commodity price $PCOMMWORLD_c(t)$ is exogenously determined in US dollars, and can be changed by the modeller to represent different future scenarios (forecasts). The commodity import price in NZ dollars, $pimpcommnz_{dr,c}$ is calculated simply as:

$$pimpcommnz_{dr,c} = \frac{PCOMMWORLD_c(t)}{\mathbf{Exchangert}} + pimportmargins_{dr,c}$$

where the margin component of the price $pimportmargins_{dr,c}$ is intended to capture short term additions of net margins charged on imports, for example under a road outage scenario causing more costly transportation from ports.

In terms of export prices, one option would be to assume that NZ producers are completely 'price takers' for all commodities. Prices received in NZ for exports would thus be equal simply to the world commodity price divided by the exchange rate. However, to allow for some variation from this strict assumption, for example to account for lags in behaviour due to contracting or 'flooding' of markets, we have included a specific export commodity demand function in the model. Separate 'base' prices for export commodities, $\mathbf{Pexpcomm}_{sr,c}$, are then calculated (Eq. (B.71)):

$$\frac{d}{dt}\left(\mathbf{Pexpcomm}_{sr,c}\right) = \left(\left(\frac{1}{exportratio_{sr,c}} \right)^{\alpha^{pexpcomm}} - 1 \right) \mathbf{Pexpcomm}_{sr,c}$$

using the ratio of export commodity supply and demand, $exportratio_{sr,c}$ (Eq. (B.75)):

$$exportratio_{sr,c} = \frac{expcommoditys_{sr,c}}{expcommodityd_{sr,c}}$$

Under this approach export demands grow at the rate of world GDP growth from the amount in the base year ($BASEEXPORTS_{dr \to sr,c} \times WORLDGDPINDX(t)$), modified to account for changes in the relative price of NZ commodities as experienced by foreign purchases, $pexportcommd_{sr,c}$, compared to the world price. The exogenous parameter $EXPORTPc$ controls how responsive foreign demands are to changes in these relative prices as follows (Eq. (B.82)):

$$\begin{aligned} expcommodityd_{sr,c} =& BASEEXPORTS_{dr \to sr,c} WORLDGDPINDX(t) \\ & \times \left(\frac{PCOMMWORLD_c(t)}{pexportcommd_{sr,c}} \right)^{EXPORTP_c} \end{aligned}$$

Even after accounting for differences in currencies, under certain scenarios the price experienced by foreign purchases for NZ commodities $pexportcommd_{sr,c}$ can vary from the price received by NZ producers for those same commodities, $pexpcommnz_{sr,c}$, due to the imposition of net additional transportation margins (Eq. (B.83)):

$$pexportcommd_{sr,c} = \mathbf{Pexpcomm}_{sr,c} + pexportmargins_{sr,c} \mathbf{Exchangert}$$

$$pexpcommnz_{sr,c} = \mathbf{Pexpcomm}_{sr,c} \left(\frac{1}{\mathbf{Exchangert}} \right)$$

Composite prices All CES and CET-based functions that have been noted above for the commodities module depend on the input of a composite price. For example, the CES function which determines the apportionment of domestic commodity demand, $domcomdemand_{dr,c}$ into demand from specific regions, $regdomcommd_{sr,dr,c}$, requires as an input the composite demand price for all regions, $\mathbf{Pcommpdomcommd}_{sr,dr,c}$ (Eq. (B.86)). In a similar manner to the composite commodity consumption prices for households, governments and investment discussed in the respective modules above, all of the necessary composite prices in the commodities module are modelled as stocks that update to reflect changes in underlying quantities and prices.

In the case of the composite domestic commodity demand price, for example, the quantity of composite domestic commodity demand is calculated by combining the domestic commodity demand from source regions using a CES function (Eq. (B.99)):

$$qdomcommd_{dr,c} = \gamma_{dr,c}^{commregd} \left[\sum_{sr} \left(\delta_{sr,dr,c}^{commregd} (regdomcommd_{sr,dr,c})^{\eta_{dr,c}^{regcom}} \right) \right]^{\frac{1}{\eta_{dr,c}^{regcom}}}$$

Then the total price for this quantity of composite demand can be calculated by multiplying the price for the base commodity demand (including margins), $pregdomcomminclmargin_{sr,dr,c}$, by the base commodity demand $regdomcommd_{sr,dr,c}$, and then dividing this total value by the quantity (Eq. (B.98)):

$$actualpcdcd_{dr,c} = \frac{\sum_{sr} (pregdomcomminclmargin_{sr,dr,c}\, regdomcommd_{sr,dr,c})}{qdomcommd_{dr,c}}$$

Once again, in order to have the prices respond almost instantaneously, the adjustment time is set to be equal to the time step, i.e. $\tau_{prices} = \Delta t$, (Eq (B.69)):

$$\frac{d}{dt} \left(\mathbf{Pcompdomcommd}_{dr,c} \right) = \frac{1}{\tau_{prices}} \left(actualpcdcd_{dr,c} - \mathbf{Pcompdomcommd}_{dr,c} \right)$$

The equations to calculate the remaining composite prices ($\mathbf{Pcindustrys}_{sr,i}$, $\mathbf{Pcompcommd}_{dr,c}$, $\mathbf{Pcompcomms}_{sr,c}$, $\mathbf{Pcompdomcomms}_{sr,c}$, $\mathbf{Pfiinputs}_{dr,i}$, $\mathbf{Pintinputs}_{dr,i}$) follow the same steps and are provided in Appendix B.5.

3.6 Factors module

Two types of factors are recognised in the Dynamic Economic Model, namely labour and capital. The factors module essentially deals with the supply of and demand for these factors, as well as the supply of and demand for composite factors. Figure 3.6 provides a visual representation of the factors module, while the full set of equations are available in Appendix B.6.

As shown at the top of the diagram, a key input to the factors module is the quantity of composite factors demanded by industries, $compfactord_{dr,i}$, as derived from the industries module (Section 3.4). In the calculation of composite factor demands, an increase in factor productivity is deemed to reduce the quantity of factors required per unit of industry production. Actual changes in

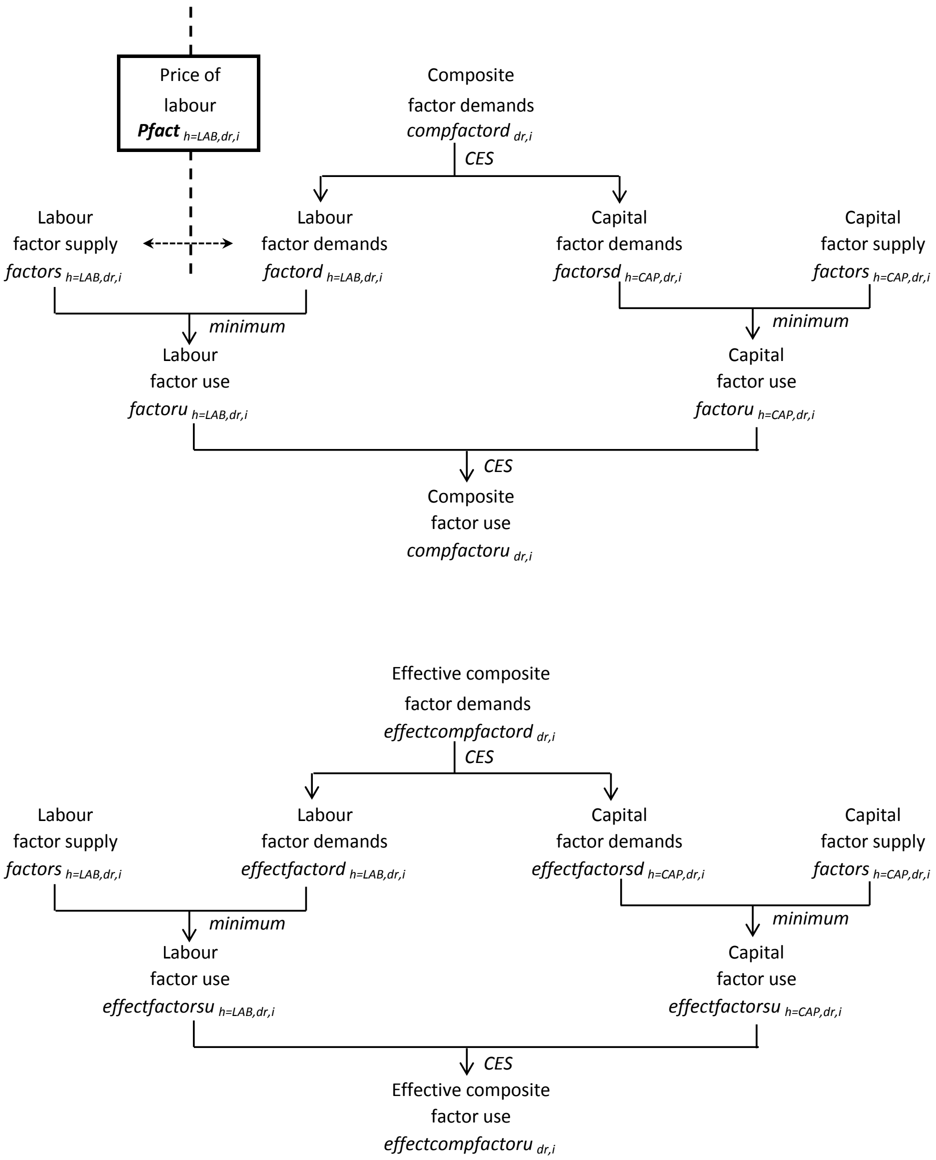

Figure 3.6 Tree diagram showing how the factors used and effective factors used are calculated.

multifactor productivity are, however, computed in the factors module according to the simple equation:

$$\frac{d}{dt}\left(\mathbf{Multifactprod}_{dr,i}\right) = \left(MFPGRRT_{dr,i}\left(MFPADJUST_{dr,i}(t)+1\right)\right)\mathbf{Multifactprod}_{dr,i}$$

where $MFPGRRT_{dr,i}$ is the annual average rate of growth in multifactor productivity, and $MFPADJUST_{dr,i}$ is included to allow for short term and/or scenario specific adjustments to productivity.

In order to split demands for composite factors,$compfactord_{dr,i}$, into demands for individual factors, i.e. labour and capital, the model once again relies on a CES function (Eq.(B.129)):

$$factorsd_{h,dr,i} = \left[(\gamma^{fact}_{dr,i})^{\eta^{fact}_{dr,i}}\ \delta^{fact}_{h,dr,i}\frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} compfactord_{dr,i}\left(1 - RWFACTRT_{h,dr}\right)$$

As part of this task, an adjustment is made to account for the proportion of factors supplied from abroad. The base SAM records a flow of income to the rest of the world associated with payments for labour. The relatively small proportion of the labour factor derived from overseas, $RWFACTRT_{h=LAB,dr}$, derived from these base year accounts is assumed to remain constant for each model run. Note also that the proportion of capital factors that are supplied from overseas is equal to zero ($RWFACTRT_{h=CAP,dr} = 0$), also in accordance with base SAM.

The price appearing in the numerator of the above equation, $\mathbf{Pcfact}_{dr,i}$, is the price for composite factors and is calculated in the same manner as other composite prices within the model. That is, we take the sum of the value (i.e. price x quantity) of the individual items that make up the composite, and divide by the composite quantity achieved as determined by the CES function:

$$\frac{d}{dt}\left(\mathbf{Pcfact}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcfact_{dr,i} - \mathbf{Pcfact}_{dr,i}\right)$$

$$actualpcfact_{dr,i} = \frac{\sum_h \left(factorsd_{h,dr,i}\,\mathbf{Pfact}_{h,dr,i}\right)}{qcompfactd_{dr,i}}$$

$$qcompfactd_{dr,i} = \gamma^{fact}_{dr,i}\left[\sum_h \left(\delta^{fact}_{h,dr,i}\left(factorsd_{h,dr,i}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}}$$

Interestingly the factor price contained in the denominator is a composite price in the case of capital, $\mathbf{Pfact}_{h=CAP,dr,i}$, but a base price in the case of labour $\mathbf{Pfact}_{h=LAB,dr,i}$. In the model capital is a composite item because it is further disaggregated into built capital and natural capital (see Section 3.8).

The composite capital demand price is computed as follows:

$$\frac{d}{dt}\left(\mathbf{Pfact}_{h=CAP,dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcapital_{dr,i} - \mathbf{Pfact}_{h=CAP,dr,i}\right)$$

$$actualpcapital_{dr,i} = \frac{capitaltyped_{cap=BuilC,dr,i}\,\mathbf{Pbuiltcap}_{dr,i}}{qcapitald_{dr,i}} + \frac{capitaltyped_{cap=NatC,dr,i}\,\mathbf{Pcompnaturalcapd}_{dr,i}}{qcapitald_{dr,i}}$$

$$qcapitald_{dr,i} = \gamma^{cc}_{dr,i}\left[\sum_{cap}\left(\delta^{cc}_{cap,dr,i}(capitaltyped_{cap,dr,i})^{\eta^{cc}_{dr,i}}\right)\right]^{\frac{1}{\eta^{cc}_{dr,i}}}$$

where the built capital and composite natural capital demand prices, $\mathbf{Pbuiltcap}_{dr,i}$ and $\mathbf{Pcompnaturalcapd}_{dr,i}$ are provided from the capital module (Section 3.8). The supply of (composite) capital, $factorss_{h=CAP,dr,i}$, is also taken directly from the capital module:

$$factorss_{h=CAP,dr,i} = ccapitals_{dr,i}$$

Although in theory labour could also be treated as a composite item, representing the combination of a variety of different labour skills, this has not been implemented in the present version of the model, with only one uniform labour type or skill recognised. For each industry the model thus computes the total ratio of labour supply to labour demand, $labratio_{dr,i}$:

$$labratio_{dr,i} = \frac{\sum_i (factorss_{h=LAB,dr,i})}{\sum_i (factorsd_{h=LAB,dr,i})}$$

Then, as with other base prices, the alpha parameter, α^{plab}, determines the degree to which the labour price changes in response to imbalances between supply and demand:

$$\frac{d}{dt}\left(\mathbf{Pfact}_{h=LAB,dr,i}\right) = \left(\left(\frac{1}{labratio_{dr,i}}\right)^{\alpha^{plab}} - 1\right)\mathbf{Pfact}_{h=LAB,dr,i}$$

where the supply component for labour, $factorss_{h=LAB,dr,i}$ is taken directly from the labour module:

$$factorss_{h=LAB,dr,i} = indlabours_{dr,i}$$

The model further computes the maximum use of factors by considering the minimum of supply and demand:

$$factorsu_{h,dr,i} = \frac{\min(factorsd_{h,dr,i}, factorss_{h,dr,i})}{1 - RWFACTRT_{h,dr}}$$

The use of the underlying labour and capital factors then allows us to calculate the composite factors used from the CES-composite function:

$$compfactoru_{dr,i} = \gamma^{fact}_{dr,i}\left[\sum_h \left(\delta^{fact}_{h,dr,i}\left(\frac{factorsu_{h,dr,i}}{1 - RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}}$$

The use quantities are important outputs of the factors module, being needed to calculate industry expenditure on factor inputs and, conversely, the income flows to capital and labour.

The remaining formulae within the factors module is concerned with calculating an auxiliary termed effective composite factors used, $effectcompfactoru_{dr,i}$, which serves as an input to both the industry and commodities modules.

First we use a CES function to split the effective composite demand into effective demand for labour and capital:

$$effectfactorsd_{h,dr,i} = \left[(\gamma^{fact}_{dr,i})^{\eta^{fact}_{dr,i}}\ \delta^{fact}_{h,dr,i}\frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} effectcompfactord_{dr,i}\left(1 - RWFACTRT_{h,dr}\right)$$

Then the effective factors used are found from the minimum of the supply and the effective demand:

$$effectfactorsu_{h,dr,i} = \frac{\min(effectfactorsd_{h,dr,i}, factorss_{h,dr,i})}{1 - RWFACTRT_{h,dr}}$$

And finally the effective factors used are combined to give the effective composite factor use:

$$effectcompfactoru_{dr,i} = \gamma^{fact}_{dr,i}\left[\sum_h \left(\delta^{fact}_{h,dr,i}\left(\frac{effectfactorsu_{h,dr,i}}{1 - RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}}$$

The purpose of calculating this auxiliary is to ensure that when an industry is subject to a short term disruption in operability, the estimates of industry production and consumption of intermediate inputs, respectively calculated within the industry and commodities modules, can be appropriately scaled downwards, while allowing the use of capital and labour $compfactoru_{dr,i}$ to remain at the undisrupted level. The steps for calculating the effective demand and use of factors and composite factors follow the same steps as for the undisrupted case.

3.7 Labour module

The Labour module tracks the amount of labour available, and apportions it to different regions and industries. The full set of equations are available in Appendix B.7.

The available labour, also known as the labour endowment, $\mathbf{Labour}_{sr}$, is assumed to change at an annual rate $NEWLABOUR_{sr}(t)$ which gives the rate equation (Eq, (B.135)):

$$\frac{d}{dt}\left(\mathbf{Labour}_{sr}\right) = NEWLABOUR_{sr}(t)$$

where the rate of change of labour is specified exogenously and can be varied by the modeller to account for different growth scenarios. The equation for $\mathbf{Labour}_{sr}$ is uncoupled from the rest of the model and can be solved analytically to find the labour endowment at any time t, given a known labour endowment at an earlier time $t_{initial}$:

$$\mathbf{Labour}_{sr}(t) = \int_{t_{initial}}^{t} NEWLABOUR_{sr}(t)\ dt + iLabour \tag{3.1}$$

where the labour endowment at the initial (start) time is $iLabour = \mathbf{Labour}_{sr}(t_{initial})$. It should be noted that we could calculate this labour endowment through time could be used as an exogenous input to the model, instead of needing to be calculated at each time step. This approach would reduce calculation time, but would add an extra step of data pre-processing if the available data forecasts for available labour are given as rates.

Once we know the total labour available, we use a CET function to allocate the supply of labour to different regions. The CET function uses the price of labour in the region of origin (supply region, sr), $\mathbf{Pavglabour}_{sr}$, relative to the price of labour in the region of use (demand region, dr), $preglabour_{dr}$, which is given by (Eq. (B.140)):

$$preglabour_{dr} = \mathbf{Pfact}_{h=LAB,dr,i=Ind1}$$

Note: $Ind1$ is chosen for convenience here, but any industry could be used as the price of labour in all industries within a region is the same in this model, i.e. $\mathbf{Pfact}_{h=LAB,dr,i=A} = \mathbf{Pfact}_{h=LAB,dr,i=B}$ for any industries A and B.

Using the scale and share parameters, θ_{sr}^{lab} and $\xi_{sr,dr}^{lab}$, and elasticity of transformation, ϕ^{lab}, the CET function gives the first estimate of the labour supply by region of labour origin and region of labour use (Eq. (B.141)):

$$reglabourest_{sr,dr} = \left[(\theta_{sr}^{lab})^{\phi_{sr}^{lab}}\ \xi_{sr,dr}^{lab}\ \frac{\mathbf{Pavglabour}_{sr}}{preglabour_{dr}} \right]^{\frac{1}{1-\phi_{sr}^{lab}}} \mathbf{Labour}_{sr}$$

We then multiply this first estimate, measured by actual people in the labour supply, by the labour stock-to-flow conversion parameter $LSFCONVERT_{dr}$ (Eq. (B.143)):

$$reglaboursupply_{sr,dr} = reglabourest_{sr,dr}\ LSFCONVERT_{dr}$$

This provides a first estimate of the actual labour supply by region of labour origin and region of labour use measured according to the annual value (in 2007 terms) of the labour endowment. A further adjustment is made to produce the final estimate of labour supply by region of labour origin and region of labour use, $indlabours_{dr,i}$ (Eq. B.142):

$$indlabours_{dr,i} = \sum_{sr} (reglaboursupply_{sr,dr}) \frac{factorsd_{h=LAB,dr,i}}{\sum_i (factorsd_{h=LAB,dr,i})}$$

The composite price of labour within a region of origin (supply), $\mathbf{Pavglabour}_{sr}$ can be calculated in the same manner as other composite prices contained within the model:

$$qlabour_{sr} = \theta^{lab}_{sr} \left[\sum_{dr} \left(\xi^{lab}_{sr,dr} \left(reglabourest_{sr,dr} \right)^{\phi^{lab}_{sr}} \right) \right]^{\frac{1}{\phi^{lab}_{sr}}}$$

$$actualpavglabour_{sr} = \frac{\sum_{dr} \left(reglabourest_{sr,dr} \, preglabour_{dr} \right)}{qlabour_{sr}}$$

$$\frac{d}{dt} \left(\mathbf{Pavglabour}_{sr} \right) = \frac{1}{\tau_{prices}} \left(actualpavglabour_{sr} - \mathbf{Pavglabour}_{sr} \right)$$

3.8 Capital module

As already explained, two forms of capital are recognised in the model, built capital and natural capital. Within the capital module natural capital is further separated into three types: agricultural land, coal, and oil/natural gas [8]. The capital module keeps track of capital stocks, computes factor prices and quantities for capital, and redistributes the income from capital to economic agents. The full set of equations are available in Appendix Section B.8.

Capital Stocks Stocks of built capital held by each industry, $\mathbf{Builtcapital}_{dr,i}$, grow from the addition of new capital items and decline as a result of depreciation:

$$\frac{d}{dt} \left(\mathbf{Builtcapital}_{dr,i} \right) = newcapital_{dr,i} - depreciation_{dr,i}$$

Depreciation on capital is calculated simply be assuming a constant depreciation rate, $RDEP_{dr,i}$, although one-off or scenario-specific adjustments to the depreciation rate are also available via the exogenous parameter $DEPSHFT_{dr,i}$:

$$depreciation_{dr,i} = \mathbf{Builtcapital}_{dr,i} \left[RDEP_{dr,i} \left(1 + DEPSHFT_{dr,i} \right) \right]$$

Total investment in new capital is determined under the investment and savings module through the calculation of aggregate investment value, $aggregateinvestv_{dr}$. In order to remove the effect of commodity price changes in determining the relative quantity of new capital items added to capital stocks each period, the investment value is divided by the current composite capital price, $\mathbf{Pinvestcc}_{dr}$, as determined under the savings and investment module, and further adjusted for exogenous set quantities of investment defined by $SETINVESTCQ_{dr,c}$.

Having determined the total quantity of new capital items, the next task is to determine how this new capital is distributed amongst industries. An extreme specification of the model would

[8] In order to integrate the economic model with a land use change model in the spatial version of MERIT, the number of natural capital types recognised in the economic model increases to seven, with agricultural land split into five categories: horticulture, sheep and beef, dairy, exotic forest, and other farming

allocate investment to industries simply according to each industry's share of total capital income. These shares are defined by the auxiliary $capincomesh_{dr,i}$ defined as follows:

$$capincomesh_{dr,i} = \frac{indcapincome_{dr,i}}{\sum_i (indcapincome_{dr,i})}$$

$$indcapincome_{dr,i} = builtuse_{dr,i}\,\mathbf{Pbuiltcap}_{dr,i}$$

$$builtuse_{dr,i} = min(\mathbf{Builtcapital}_{dr,i}\,KSFCONVERT_{dr,i}\,,\,capitaltyped_{cap=BuilC,dr,i})$$

where $\mathbf{Pbuiltcap}_{dr,i}$ is the industry specific built capital factor price, and $\mathbf{Builtcapital}_{dr,i} \times KSFCONVERT_{dr,i}$ specifies the annual quantity of capital factors provided.

Recognising however that some investment is mobile, only a proportion of new capital, specified by one minus the exogenous parameter $MOBILESH_{dr,i}$ is allocated to industries according to each industry's relevant share of capital income. Altogether the new capital allocated to each industry by this approach, termed $immobileinvest_{dr,i}$, is calculated as follows:

$$immobileinvest_{dr,i} = \left[\frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}} + \sum_c (SETINVESTCQ_{dr,c})\right] \times (1 - MOBILESH_{dr,i})\,capincomesh_{dr,i}$$

The remaining quantity of new capital provided by investment is allocated to industries based on relative capital returns. Industries with above-average capital returns receive a larger share of this mobile investment capital than their share in capital income, while the converse occurs for industries from which capital returns are below-average.The following set of equations determines each industry's share of mobile investment capital, $mobileinvestsh_{dr,i}$:

$$mobileinvestsh_{dr,i} = \frac{mobileinvest1_{dr,i}}{\sum_i (mobileinvest1_{dr,i})}\,ALLOCATESH_{dr} + INVESTCONSTSH_{dr,i}$$

$$mobileinvest1_{dr,i} = (INVESTPARAM_{dr,i}\,netreturn_{dr,i})^{EINVEST_{dr,i}} \times capincomesh_{dr,i}$$

$$netreturn_{dr,i} = grossreturn_{dr,i} - RDEP_{dr,i}$$

$$grossreturn_{dr,i} = \frac{\mathbf{Pbuiltcap}_{dr,i}\,KSFCONVERT_{dr,i}}{\mathbf{Pinvestcc}_{dr}}$$

For the above equations $ALLOCATESH_{dr}$ is the share of mobile investment that is allocated to industries based on the relative returns to capital in those industries, $INVESTCONSTSH_{dr,i}$ is the industry share of regional investment held constant, $EINVEST_{dr,i}$ is a parameter that controls the degree to which investment allocated to industries response to changes in the rate of return on capital, and $INVESTPARAM_{dr,i}$ is a parameter for scaling net return on capital.

Once the shares of mobile investment capital are determined, it is then possible to calculate mobile investment capital allocated to each industry, and also total investment capital allocated to each industry:

$$mobileinvest_{dr,i} = \left[\frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}} + \sum_c (SETINVESTCQ_{dr,c})\right] \times MOBILESH_{dr,i}\,mobileinvestsh_{dr,i}$$

$$newcapital_{dr,i} = mobileinvest_{dr,i} + immobileinvest_{dr,i}$$

To complete this section we note that stocks of natural capital are currently held constant in the model, i.e.

$$\frac{d}{dt}(\mathbf{Naturalcapital}_{dr,i}) = 0$$

Capital Factor Prices and Quantities As with other base prices in the model, the base prices for built capital and natural capital are determined by considering the balance of supply and demand:

$$\frac{d}{dt}\left(\mathbf{Pbuiltcap}_{dr,i}\right) = \left(\left(\frac{1}{builtratio_{dr,i}}\right)^{\alpha^{pbuiltcap}} - 1\right)\mathbf{Pbuiltcap}_{dr,i}$$

$$\frac{d}{dt}\left(\mathbf{Pnaturalcap}_{dr,i,nct}\right) = \left(\left(\frac{1}{naturalcapratio_{dr,i,nct}}\right)^{\alpha^{pnatcap}} - 1\right)\mathbf{Pnaturalcap}_{dr,i,nct}$$

where the ratios of supply and demand are:

$$builtratio_{dr,i} = \frac{builts_{dr,i}}{capitaltyped_{cap=BuilC,dr,i}}$$

$$naturalcapratio_{dr,i,nct} = \frac{indnaturalcaps_{dr,i,nct}}{naturalcaptyped_{dr,i,nct}}$$

A tree diagram showing how the supply ($builts_{dr,i}$ and $indnaturalcaps_{dr,i,nct}$) and demand ($capitaltyped_{cap=BuilC,dr,i}$ and $naturalcaptyped_{dr,i,nct}$) for capital factors is calculated is shown in Figure 3.7.

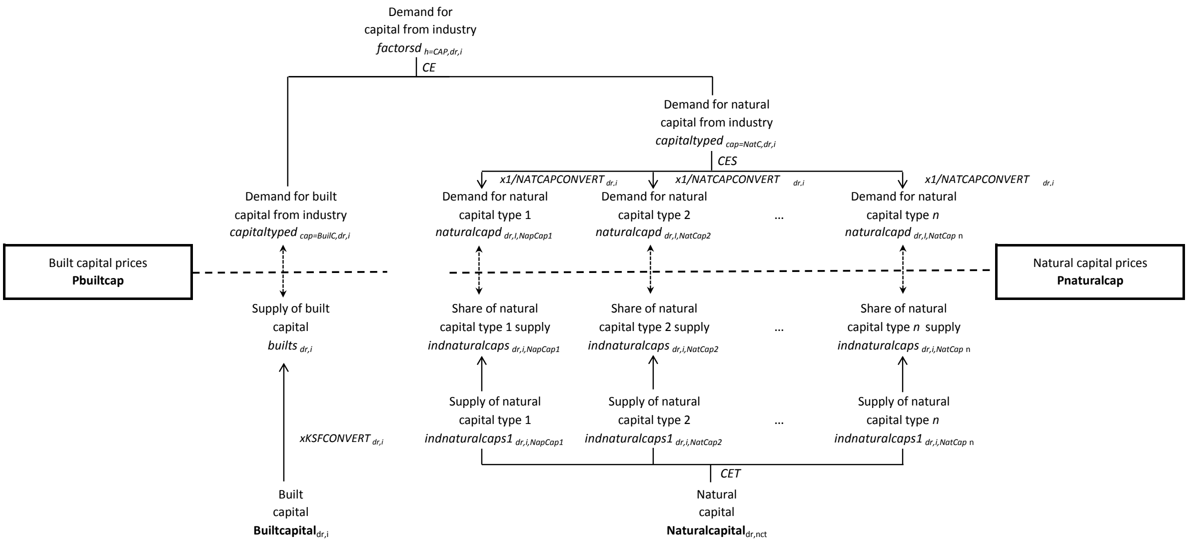

Figure 3.7 Tree diagram showing how the prices of built and natural capital are calculated from the supply and demand.

The demand equations pick up directly from the demand components of the factors module. Once again CES functions are used, first to split industry-specific composite capital demands,

$factorsd_{h=cap,dr,i}$, into demands for built capital, $capitaltyped_{cap=BuilC,dr,i}$, and composite natural capital, $capitaltyped_{cap=NatC,dr,i}$, by the following equations:

$$capitaltyped_{cap=BuilC,dr,i} = \left[(\gamma^{cc}_{dr,i})^{\eta^{cc}_{dr,i}} \delta^{cc}_{cap=BuilC,dr,i} \frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pbuiltcap}_{dr,i}} \right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \times factorsd_{h=CAP,dr,i}$$

$$capitaltyped_{cap=NatC,dr,i} = \left[(\gamma^{cc}_{dr,i})^{\eta^{cc}_{dr,i}} \delta^{cc}_{cap=NatC,dr,i} \frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pcompnaturalcapd}_{cap=NatC,dr,i}} \right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \times factorsd_{h=CAP,dr,i}$$

The key difference between these two equations is that demands for built capital are calculated by comparing the composite capital factor demand price, $\mathbf{Pfact}_{h=CAP,dr,i}$, with the price of built capital, $\mathbf{Pbuiltcap}_{dr,i}$, while demands for composite natural capital are calculated by comparing the composite capital factor demand price with the composite natural capital demand price, $\mathbf{Pcompnaturalcapd}_{dr,i}$. The price of built capital is a base price and thus is computed by comparing the supply of built capital to the demand for built capital (see Equations (B.146), (B.162), and (B.163)), while the composite natural capital demand price is computed like other CES composite demand prices in the model (see Equations (B.147), (B.154), and (B.155)).

To complete the demand side, composite natural capital demands are then disaggregated into demands for individual natural capital types using once again the CES approach:

$$naturalcapd_{dr,i,nct} = \left[(\gamma^{natcap}_{dr,i})^{\eta^{natcap}_{dr,i}} \delta^{natcap}_{dr,i,nct} \frac{\mathbf{Pcompnaturalcapd}_{dr,i}}{\mathbf{Pnaturalcap}_{dr,i,nct}} \right]^{\frac{1}{1-\eta^{natcap}_{dr,i}}} \frac{capitaltyped_{cap=NatC,dr,i}}{NATCAPCONVERT_{dr,i}}$$

Note that because the model permits natural capital to be measured according to a variety of units (e.g. hectares for land), the equation incorporates an exogenous conversion factor, $NATCAPCONVERT_{dr,i}$, that enables conversion to the selected unit of measurement for each natural capital type.

Turning now to the supply side, we have already noted that the capital module computes stocks of natural capital by type and industry-specific stocks of built capital. Using a similar approach to the labour module, the supply of natural capital is transformed into individual industry allocations of natural capital, $indnaturalcaps_{dr,i.nct}$, based on a CET function:

$$indnaturalcaps1_{dr,i,nct} = \left[(\theta^{natcap}_{dr,nct})^{\phi^{natcap}_{dr,nct}} \xi^{natcap}_{dr,i,nct} \frac{\mathbf{Pcompnaturalcaps}_{dr,nct}}{\mathbf{Pnaturalcap}_{dr,i,nct}} \right]^{\frac{1}{1-\phi^{natcap}_{dr,nct}}} \mathbf{Naturalcapital}_{dr,nct}$$

$$indnaturalcaps_{dr,i,nct} = \frac{indnaturalcaps1_{dr,i,nct}}{\sum_i (indnaturalcaps1_{dr,i,nct})} \mathbf{Naturalcapital}_{dr,nct}$$

We then have available both the supply- and demand-side components of natural capital, allowing for computation of the natural capital base price, $\mathbf{Pnaturalcap}_{dr,i,nct}$ (see Eqs (B.149) and (B.160)).

To complete the supply components so that appropriate inputs can be generated for the factors module, the industry-specific endowments of natural capital of different types are also combined using a CES function to determine the total supply of composite natural capital to industries (see Eqs (B.165) and (B.166)). Finally, the supply of composite capital is determined by combining built capital supply with composite natural capital supply, also using the CES function (Eq (B.164)) as shown in Figure 3.8. As part of this process, the price of composite natural capital supply, $\mathbf{Pcompnaturalcaps}_{dr,nct}$, is determined in the usual manner (Eqs (B.148), (B.151)).

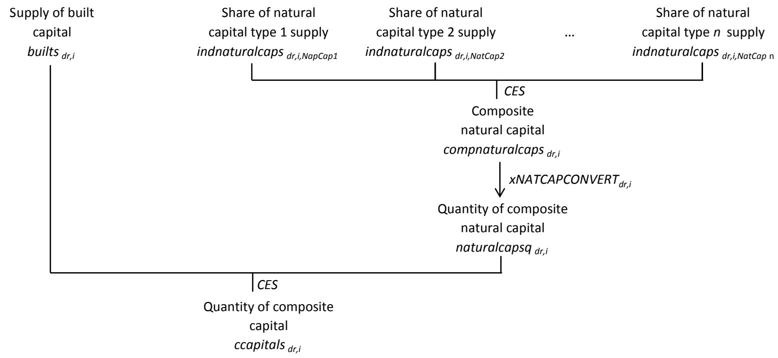

Figure 3.8 Tree diagram showing how composite capital supply is calculated from the supply of built and natural capital.

Capital Income and Expenditure Account A tree diagram that provides a summary of the different contributions to capital income and how capital is distributed is shown in Figure 3.9.

The principal source of income to the capital account is calculated simply by multiplying the quantity of capital factors used, as derived under the factors module, by the capital price, and adjusting for any income that flows directly overseas. Also added to the capital income account is any net surplus in industry income, i.e. $\sum_i(\mathbf{Industrybalance}_{dr,i})$, as calculated under the industry module:

$$capitalincome_{dr} = \frac{\sum_i (factorsu_{h=CAP,dr,i}\,\mathbf{Pfact}_{h=CAP,dr,i})}{1 - RWFACTRT_{h=CAP,dr}} + capregtransout_{DReg1\rightarrow SReg2,DReg2\rightarrow SReg1} + \sum_i(\mathbf{Industrybalance}_{dr,i})$$

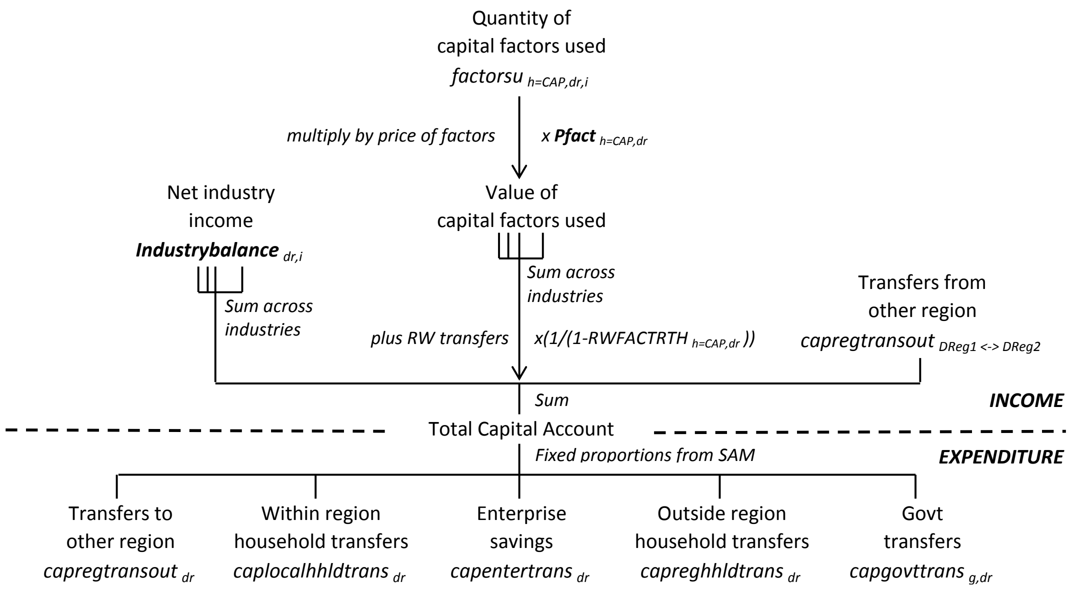

Figure 3.9 Tree diagram showing sources of capital and capital distribution. *SAM*: Social Accounting Matrix.

The recognised capital income account, $\mathbf{Rcapincome}_{dr}$, adjusts to reflect the actual value of capital income:

$$\frac{d}{dt}\left(\mathbf{Rcapincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(capitalincome_{dr} - \mathbf{Rcapincome}_{dr}\right)$$

To ensure that this occurs quickly the time for adjustment, τ_{income} is set equal to the time step. The capital income is then distributed according to fixed proportions derived from the base year SAM:

$$capregtransout_{dr} = \mathbf{Rcapincome}_{dr} \times CREGTRANSRT_{dr}$$

$$capentertrans_{dr} = \mathbf{Rcapincome}_{dr} \times CENTTRANSRT_{dr}$$

$$capgovttrans_{g,dr} = \mathbf{Rcapincome}_{dr} \times CGOVTTRANSRT_{g,dr}$$

$$caplocalhhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CHHLDTRANSRT_{dr}$$

$$capreghhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CRHTRANSRT_{dr}$$

where the proportions $CREGTRANSRT_{dr}$, $CENTTRANSRT_{dr}$, $CGOVTTRANSRT_{g,dr}$, $CHHLDTRANS$ and $CRHTRANSRT_{dr}$ sum to 1. The auxiliary $capregtransout_{dr}$ is the value of capital income transferred to the capital account in the other NZ region, and $capentertrans_{dr}$, $capgovttrans_{g,dr}$, $caplocalhhldtrans_{dr}$ and $capreghhldtrans_{dr}$ are respectively capital income transferred to enterprises, governments, within-region households and out-of-region households.

3.9 Investment & Savings module

We begin the description of this module by setting out the distinction between investment and savings. While in every day conversation the term investment might be used to refer to financial investments, such as shares and bonds, within the national accounts investment has quite a distinct and different meaning. In short, investment is the purchase of goods that will be used in the future to produce more goods and services. It therefore includes purchases of new houses, capital equipment, inventories and structures. Savings by contrast occur when an agent's income is greater than its expenditure, leading to a surplus of funds that are typically deposited in a bank. This module determines the values for both investment and savings, for each region. The full set of equations are available in Appendix Section B.9.

Savings To begin, total savings for each region are comprised of savings which accrue specifically by regions, $regsavings_{dr}$, less interregional transfers of savings, $savregtransout_{dr}$, and savings derived from the rest of the world, $rwsavings_{dr}$ (Eq. (B.192)):

$$savingstotal_{dr} = rwsavings_{dr} + regsavings_{dr} - savregtransout_{dr}$$

A tree diagram that summarises the different contributions to overall savings is shown in Figure 3.10.

To determine the savings derived from the rest of the world, the model assumes simply that a fixed shares, $RWREGSAVSH_{dr}$, of national savings from the rest of the world, $rwsavingstotal$, are allocated to each region (Eq. (B.193)):

$$rwsavings_{dr} = rwsavingstotal\, RWREGSAVSH_{dr}$$

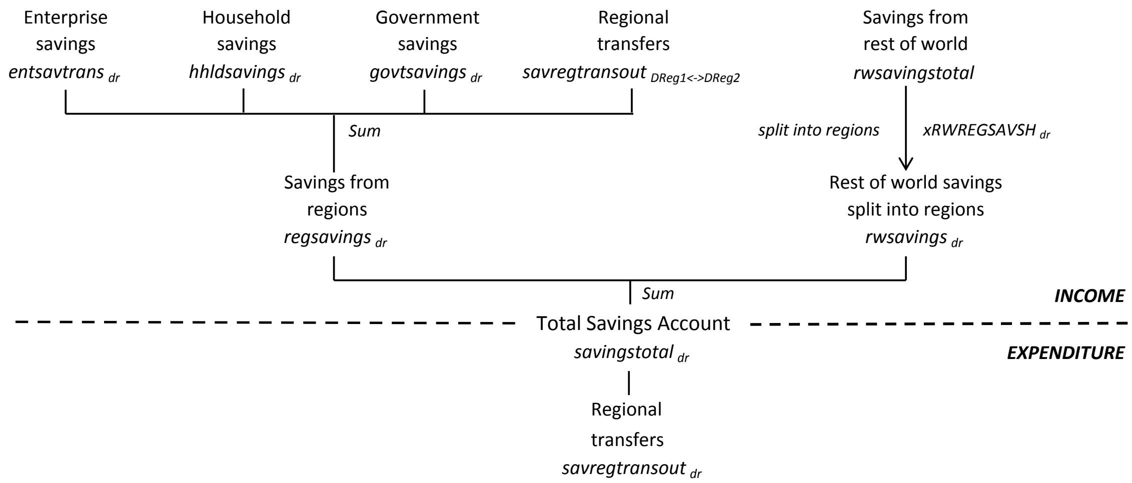

Figure 3.10 Tree diagram showing contributions to savings.

In turn, regionally-derived savings are the sum of enterprise, household, and government savings, i.e. $entsavtrans_{dr}$, $hhldsavings_{dr}$ and $govtsavings_{g,dr}$ respectively, plus the receipt of interregional transfers which are given by $savregtransout_{DReg1 \leftrightarrow DReg2}$ as transfers into $DReg2$ equal the transfers out of $DReg1$ ($savregtransout_{DReg1}$), and vice-versa. This gives the equation for regionally-derived savings (Eq. (B.195)):

$$regsavings_{dr} = entsavtrans_{dr} + hhldsavings_{dr} + \sum_{g}(govtsavings_{g,dr}) + savregtransout_{DReg1 \leftrightarrow DReg2}$$

Lastly, interregional transfers of savings are indexed to households within the origin region, where $SAVREGTRANSBS_{dr}$ defines the value of these transfers during the base year:

$$savregtransout_{dr} = SAVREGTRANSBS_{dr} \frac{\mathbf{Rhhldincome}_{dr}}{BASEHHLDACCOUNT_{dr}}$$

Total national rest of world savings, $rwsavingstotal$, is a relatively significant auxiliary within the model, deserving special consideration. We use a regression equation whereby total rest of world savings are calculated as a function of (1) the difference between the national interest rate and the world interest rate , $100\,\mathbf{Interestrt} - WORLDINTERESTRT(t)$, and (2) the world GDP index , $WORLDGDPINDX(t)$:

$$rwsavingstotal = \begin{cases} ACRWSAVINGS(t) & \text{for} \quad t \leq 6 \\ [100\,\mathbf{Interestrt} - WORLDINTERESTRT(t)] & \\ \times INTERESTWEIGHT & \text{for} \quad t > 6 \\ +WORLDGDPINDX(t)\,GDPWEIGHT + RWSAVCONST & \end{cases}$$

The rationale is that, the greater the differential between the interest rate achieved on saved funds in NZ compared to the interest rate achieved on funds elsewhere in the world, the more funds will be directed towards savings in NZ. At the same time, however, simple growth in the world economy and hence the ‘pool of funds’ available will also increase rest of world savings. The relative weights given to the interest rate and world GDP predictor variables are the exogenous parameters $INTERESTWEIGHT$ and $GDPWEIGHT$ respectively, while $RWSAVCONST$ is the regression constant. The model also provides the option to override the regression equation for estimating rest of world savings with a time series of actual values, $ACRWSAVINGS(t)$, for the historic time over which data is available (presently $t \leq 6$). Note that the since the model commences in 2006, the initial modelled years will cover the global financial crisis, a period during which savings behaviours were relatively unusual and hence not expected to be well represented by a generic regression equation based on historical data.

In NZ, market interest rates are influenced by the official cash rate, the interest rate set by the Reserve Bank of NZ. Importantly, changes in the official cash rate are the primary means by which the Reserve Bank maintains its core function of achieving and maintaining stability in the general level of prices. Increasing the interest rate can help to alleviate inflationary pressure via a number of routes. As already described under the households module, it is generally considered that household consumption is negatively correlated with interest rates, as households will need to devote a greater proportion of their budget towards repayment of mortgages when interest rates rise. A more subtle route is via the influence of interest rates on the exchange rate. Recall from the last paragraph that we model rest of world savings as dependant, in part, on the available interest rates in NZ. If there is an increased demand for NZ currency via rest of world savings this will help to appreciate the value of the NZ dollar relative to other currencies. In turn, by making NZ commodities relatively more expensive than foreign goods, this helps to reduce export demands and hence inflationary pressure. One further influence of the interest rate on commodity prices is described below in regards to investment demands.

Economist John Taylor proposed a policy rule whereby the short-term interest rate is adjusted to (1) movements of inflation from a desired value and (2) changes in the output gap (that is the difference between actual economic output ant its underlying trend) (Taylor, 1993). Although relatively simple, this rule and its variations has been widely regarded as a reasonable approximation of policy behaviour (cf. Bayoumi, 2004). Following a similar approach we therefore model the change in the interest rate as adjusting to meet a variable termed the target interest rate, $targetinterestrt$, with the period of adjustment controlled by the constant $\tau_{interest}$:

$$\frac{d}{dt}(\mathbf{Interestrt}) = \frac{1}{\tau_{interest}}(targetinterestrt - \mathbf{Interestrt})$$

and with the target interest rate fixed to the data $ACTUALINTERESTRT(t)$ for the first 6 years:

$$targetinterestrt = \begin{cases} ACTUALINTERESTRT(t) & \text{for} \quad t \leq 6 \\ desiredinterestrt & \text{for} \quad t > 6 \end{cases}$$

Apart from the years over which we insert real data, the target interest rate, in turn, is determined by differences between (1) the inflation rate and the target inflation rate (assumed to be 2% per annum) and (2) the GDP gap. The latter is the percentage shortfall of GDP from an estimate of its natural rate, $NATURALGDP(t)$:

$$\begin{aligned} desiredinterestrt = &\mathbf{Inflationrt} + INTERESTCONST + INTERESTINFLW\,(\mathbf{Inflationrt} - 0.02) \\ &- INTERESTGDPW\; gdpgap \end{aligned}$$

$$gdpgap = \frac{NATURALGDP(t)}{realgdp} - 1$$

The respective weights given to the inflation and GDP components of the calculation, $INTERESTINFLW$ and $INTERESTGDPW$, are set specific to the NZ context based on historic data.

Finally we can note that once we have calculated the interest rate and inflation rates (the latter is available from the output reporting module) we can also determine the real interest rate:

$$realinterestrt = \mathbf{Interestrt} - \mathbf{Inflationrt}$$

Investment A tree diagram in Figure 3.11 shows how available investment funds are calculated, and where investment is allocated. For each region, the primary input to the funds available for investment is labelled 'variable investment'. Investment funds are assumed to be positively correlated with changes in the total value of regional savings. This helps to capture general trends in the size of the economy but also reflects that with more savings, banks will have more funds available to loan for investment spending. At the same time, it is also assumed that the funds allocated to investment are negatively correlated with the real interest rate. This is because the opportunity costs of borrowing money to invest in capital goods increases when the real interest rate increases, and people will also be spending more on repaying existing loads and mortgages so will have less available to invest. These relationships are integrated into the model by the equation for $aggregateinvestv_{dr}$:

$$aggregateinvestv_{dr} = (realinterestrt\, ALPHA_{dr} + savingstotal_{dr}\, BETA_{dr} + INVESTCONST_{dr}) \times (1 - INVESTINDIRECTTAXRT_{dr})$$

where $ALPHA_{dr}$, $BETA_{dr}$ and $INVESTCONST_{dr}$ are respectively the weights given to the interest rate and savings, and the regression constant. The equation also includes an adjustment for the proportion of investment funds that are allocated towards payment of indirect taxes. This proportion, i.e. $INVESTINDIRECTTAXRT_{dr}$, is assumed to remain constant with the base year. The total value of indirect taxes on investment, investindirecttaxdr, is a necessary input for the government module and is calculated simply as follows:

$$investindirecttax_{dr} = aggregateinvestv_{dr} \frac{INVESTINDIRECTTAXRT_{dr}}{1 - INVESTINDIRECTTAXRT_{dr}}$$

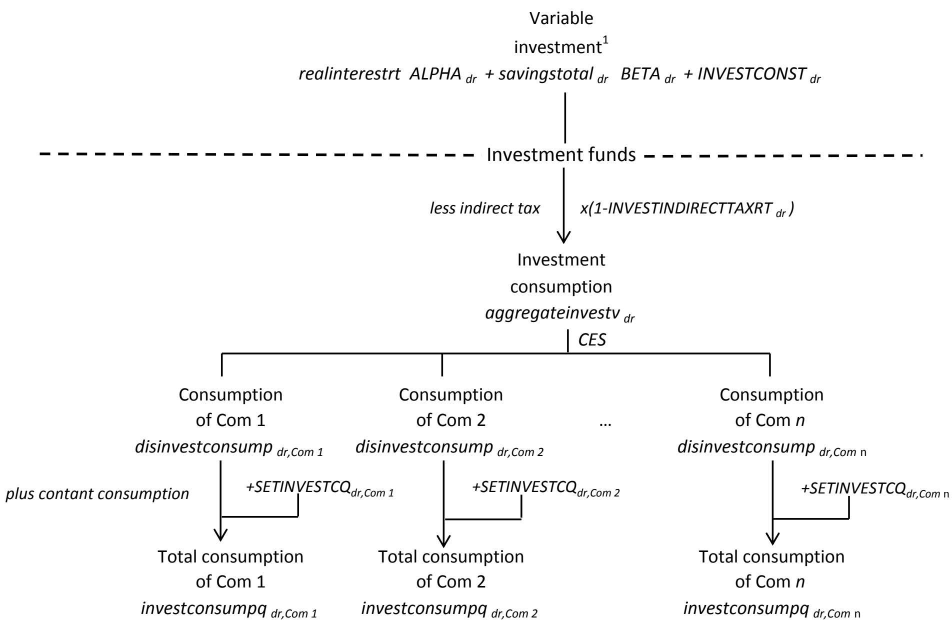

Figure 3.11 Tree diagram showing how available investment is calculated, and where it is allocated. *CES*: Constant Elasticity of Substitution calculation.
[1] Determined empirically from a linear regression fit to data.

Once the funds available for investment are determined, the module operates in a similar manner to the household and government modules. That is the composite quantity of investment commodities consumed is calculated simply by dividing the aggregate investment value by the composite investment price, $\mathbf{Pinvestcc}_{dr}$. This composite quantity is then disaggregated by a CES function so as to determine the individual commodity types consumed for investment, $disinvestconsump_{dr,c}$:

$$disinvestconsump_{dr,c} = \left[(\gamma_{dr}^{investc})^{\eta_{dr}^{investc}} \delta_{dr,c}^{investc} \frac{\mathbf{Pinvestcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta_{dr}^{investc}}} \frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}}$$

In this equation $\eta_{dr}^{investc}$ is the commodity substitution parameter for investment consumption, while $\gamma_{dr}^{investc}$ and $\delta_{dr,c}^{investc}$ are the CES scale and share parameters. Additionally $\mathbf{Pcompcommd}_{dr,c}$ is the commodity-specific price by agents demanding commodities, as obtained from the commodities module.

Like other composite prices, the composite investment price is calculated via a series of three equations:

$$\frac{d}{dt}(\mathbf{Pinvestcc}_{dr}) = \frac{1}{\tau_{prices}}(actualpinvestcc_{dr} - \mathbf{Pinvestcc}_{dr})$$

$$qinvestcc_{dr} = \gamma_{dr}^{investc}\left[\sum_c \left(\delta_{dr,c}^{investc}(disinvestconsump_{dr,c})^{\eta_{dr}^{investc}}\right)\right]^{\frac{1}{\eta_{dr}^{investc}}}$$

$$actualpinvestcc_{dr} = \frac{\sum_c (disinvestconsump_{dr,c}\,\mathbf{Pcompcommd}_{dr,c})}{qinvestcc_{dr}}$$

Once the composite investment demands have been disaggregated into demands for individual commodities a final adjustment is made to account for small quantities of goods that are assumed to be consumed each year and are set exogenously. This provides the final quantities of investment commodity consumption, $investconsumpq_{dr,c}$:

$$investconsumpq_{dr,c} = disinvestconsump_{dr,c} + SETINVESTCQ_{dr,c}$$

Presently this adjustment is made simply because the base SAM records small quantities of negative investment consumption for certain commodities. These are all negligible quantities and probably relate to changes in stocks during the base year.

3.10 Rest of world module

The rest of world module tracks flows of funds into and out of the NZ economic system. The full set of equations are available in Appendix Section B.10.

A primary task of the rest of world module is to determine changes in the exchange rate, **Exchangert**. The exchange rate can be conceptualised as the quantity of US dollars per NZ dollar, but rescaled so that the rate is equal to one for the base year. Importantly, the model treats the exchange rate in an analogous manner to a base prices in other modules. That is, the exchange rate fluctuates in response to differences in supply and demand quantities following the equation (Eq. (B.204)):

$$\frac{d}{dt}(\mathbf{Exchangert}) = \left(\left(\frac{1}{bopratio}\right)^{\alpha^{exchangert}} - 1\right)\mathbf{Exchangert}$$

with the rate of response dependant on the exogenous alpha parameter, $\alpha^{exchangert}$. In this case the supply and demand quantities compared are the supply and demand for NZ currency, also respectively termed rest of world expenditure and rest of world income. Also, the ratio of rest of world income to rest of world expenditure is termed the balance of payments ratio, *bopratio* (Eq. (B.205)):

$$bopratio = \frac{rwincome}{rwexpenditure}$$

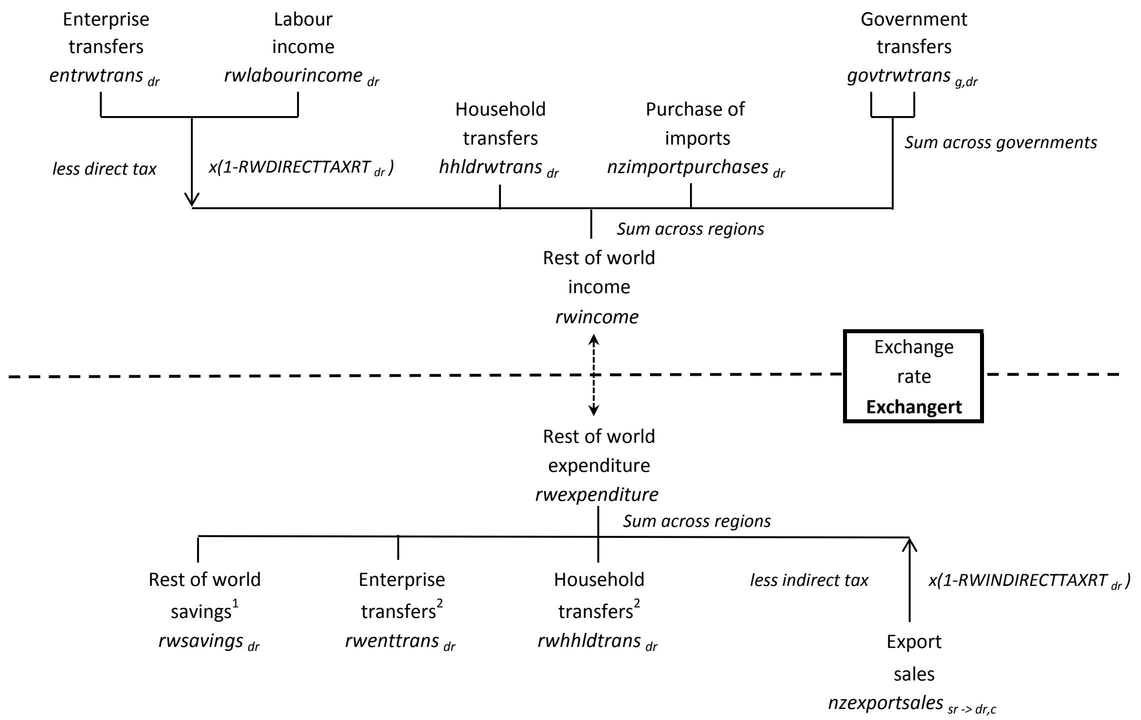

Figure 3.12 Tree diagram showing rest of world income and expenditure.
[1]Exogenously determined, see the Savings & Investment Module (Section 3.9) for details.
[2]The rate at which funds get transferred is exogenously determined and depends on the **Exchangert**.

A tree diagram that provides a summary of the different contributions to rest of world income and expenditure is shown in Figure 3.12. Rest of world income is calculated as the sum of income from labour and transfers from enterprises, less direct taxes, plus income from sale of commodities to NZ (i.e. NZ imports), household transfers and government transfers (Eq. (B.206)):

$$rwincome = \sum_{dr} \Big(rwlabourincome_{dr} + \sum_{c} (nzimportpurchases_{dr,c}) + entrwtrans_{dr} + hhldrwtrans_{dr} + \sum_{g} (govtrwtrans_{g,dr}) - rwdirecttax_{dr} \Big)$$

Transfers from enterprises, $entrwtrans_{dr}$, households, $hhldrwtrans_{dr}$, and governments $govtrwtrans_{g,dr}$, are taken directly from the respective modules for these agents, while rest of world labour income is calculating simply by multiplying rest of world labour supply by the relevant labour price (Eqs. (B.207), (B.208)):

$$rwlabourincome_{dr} = \sum_{i} (rwlaboursupply_{dr,i}\, preglabour_{dr})$$

$$rwlaboursupply_{dr,i} = factorsu_{h=LAB,dr,i}\frac{RWFACTRT_{h=LAB,dr}}{1-RWFACTRT_{h=LAB,dr}}$$

Similarly, rest of world income from imports is determined by multiplying the quantity of import commodities demanded, by the import price (Eq. (B.209)):

$$nzimportpurchases_{dr,c} = importdemand_{dr,c}\,pimpcommnz_{dr,c}$$

Finally rest of world direct taxes are determined by multiplying income from labour and enterprises by an assumed constant direct tax rate, $RWDIRECTTAXRT_{dr}$ (Eq. (B.210)):

$$rwdirecttax_{dr} = (entrwtrans_{dr} + rwlabourincome_{dr})\,RWDIRECTTAXRT_{dr}$$

On the expenditure side, funds are transferred into the economic system via transfers from enterprises, $entrwtrans_{dr}$, and households, $rwhhldtrans_{dr}$, from NZ export sales ,$nzexportsales_{sr,c}$, via payments of indirect taxes, $rwindirecttax_{dr}$, and from net rest of world savings, $rwsavings_{dr}$, (Eq. (B.211)):

$$rwexpenditure = \sum_{dr}\left(rwsavings_{dr} + rwenttrans_{dr} + rwhhldtrans_{dr} + rwindirecttax_{dr}\right) + \sum_{sr}\sum_{c}(nzexportsales_{sr,c})$$

To calculate indirect taxes, a constant indirect tax rate is applied to NZ's commodity export sales (Eqs. (B.212), (B.213)):

$$rwindirecttax_{dr} = \sum_{c}(rwindirecttaxdisag_{dr,c})$$

$$rwindirecttaxdisag_{dr,c} = nzexportsales_{sr\to dr,c}\,RWINDIRECTTAXRT_{dr}$$

In turn, export sales of each commodity are calculated as follows (Eqs. (B.214), (B.215), (B.216)):

$$nzexportsales_{sr,c} = \frac{expcommoditys_{sr,c}\,actualpexports_{sr,c}}{\mathbf{Exchangert}}$$

$$actualpexports_{sr,c} = \frac{actualexports_{sr,c}\,\mathbf{Pexpcomm}_{sr,c}}{expcommoditys_{sr,c}}$$

$$actualexports_{sr,c} = \min\left(expcommodityd_{sr,c}, expcommoditys_{sr,c}\right)$$

where the quantities of export demands and supply, $expcommodityd_{sr,c}$ and $expcommoditys_{sr,c}$, along with the export commodity price, $\mathbf{Pexpcomm}_{sr,c}$, are available from the commodities module (Section 3.5).

Completing the rest of world module is the calculation of the current account surplus, which is simply total rest of world expenditure less income:

$$actualsurplus = rwexpenditure - rwincome$$

The perceived current account surplus, **Casurplus** will adjust to reflect the actual current account surplus, depending on the time for adjustment, $\tau_{casurplus}$:

$$\frac{d}{dt}(\mathbf{Casurplus}) = \frac{1}{\tau_{casurplus}}(actualcasurplus - \mathbf{Casurplus})$$

3.11 Output variable module

The variable module contains the calculations for the inflation rate, Consumer Price Index (CPI), and GDP. Although each of these items are also used in calculations elsewhere in the model, the items have been selected for inclusion within this module as these do not fit neatly into the subject matter of the other modules. Furthermore each item, in its own right, is a useful indicator or reporting item for monitoring the state of an economic system.

If we begin with the inflation rate, this is modelled as a stock that quickly adjusts to match an auxiliary $targetinflationrt$. In turn, the latter auxiliary is calculated based on the rate of change in the CPI. As with the calculations for rest of world savings, the model also includes an option to override the inflation rate with real data for a historic time period, i.e. as given by the exogenous time series $ACINFLATIONRT(t)$, if appropriate:

$$\frac{d}{dt}\left(\mathbf{Inflationrt}\right) = \frac{1}{\tau}\left(targetinflationrt - \mathbf{Inflationrt}\right)$$

$$targetinflationrt = \begin{cases} ACINFLATIONRT(t) & \text{for} \quad t \leq 6 \\ desiredinflationrt & \text{for} \quad t > 6 \end{cases}$$

$$desiredinflationrt = 4\left(\frac{cpif(t) - cpif(t-0.25)}{cpif(t-0.25)}\right)$$

Both the CPI and GDP are calculated as Fisher indices. The Paasche and Laspeyres indices are alternative approaches for calculating price indices, but both have limitations. On the one hand a Paasche index fails to sufficiently account for substitution within the 'basket' of goods considered in the index, while on the other hand the a Laspeyres index tends to over-estimate inflation (IMF, 2004). The Fisher index takes the geometric average of the Paasche and Laspeyres indices, thereby seeking to offset the biases inherent in the two approaches. The CPI Fisher index, $cpif$, is thus calculated as:

$$cpif = \sqrt{cpip \times cpil}$$

where $cpip$ and $cpil$ are respectively the CPI Paasche and Laspeyres indices. In turn the CPI Paasche and Laspeyres indices are calculated as follows:

$$cpil = 1000\,\frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\,BASEHHLDCONSUMP_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\,BASEHHLDCONSUMP_{dr,c}\right)}$$

$$cpip = 1000\,\frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\,hhldconsump_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\,hhldconsump_{dr,c}\right)}$$

In the above equations $BASEHHLDCONSUMP_{dr,c}$ is, for each region, the quantity of household consumption of each commodity c at time zero (the base period), while $BASEPCOMPCOMMD_{dr,c}$ is the associated commodity price at time zero.

In the case of GDP the Fisher index is calculated, and then a stock, **Gdpindex**, quickly adjusts to meet the calculated Fisher index:

$$actualgdpindex = 1000\sqrt{gdpindexl \times gdpindexp}$$

$$\frac{d}{dt}(\mathbf{Gdpindex}) = \frac{1}{\tau}(actualgdpindex - \mathbf{Gdpindex})$$

The specification of the GDP index as a stock is necessary to avoid computational problems elsewhere within the model. Essentially the GDP index is considered a general indicator of the growth of size of the economic system.

The quantities and prices that are considered in the calculations of the underlying Paasche and Laspeyres indices are selected based on the expenditure method for calculating GDP (cf Viet, 2009). Thus both the Paasche and Laspeyres indices consider a 'basket of goods' that is made up of commodities consumed by households, governments and for investment. This approach has been undertaken simply to reduce the size of the equation.

The Laspeyres index is given by the equation:

$$gdpindexl = \Bigg[\sum_{dr}\sum_{c}\Bigg(\Bigg(BASEHHLDCONSUMP_{dr,c} + BASEINVESTCONSUMP_{dr,c} + \sum_{g}(BASEGOVTCONSUMP_{g,dr,c})\Bigg)\mathbf{Pcompcommd}_{dr,c}\Bigg) + \sum_{dr}\sum_{c}\Bigg((BASEEXPORTS_{dr,c} - BASEIMPORTS_{dr,c})\frac{PCOMMWORLD_{c}(t)}{\mathbf{Exchangert}}\Bigg)\Bigg]$$
$$\div \Bigg[\sum_{dr}\sum_{c}\Bigg(\Bigg(BASEHHLDCONSUMP_{dr,c} + BASEINVESTCONSUMP_{dr,c} + \sum_{g}(BASEGOVTCONSUMP_{g,dr,c})\Bigg)BASEPCOMPCOMMD_{dr,c}\Bigg) + \sum_{dr}\sum_{c}\Bigg((BASEEXPORTS_{dr,c} - BASEIMPORTS_{dr,c})\frac{BASEPCOMMWORLD_{c}}{BASEEXCHANGERT}\Bigg)\Bigg]$$

where once again the term '$BASE$' is added to the start of all names of exogenous input data generated from the benchmark or base year accounts (i.e. when time = zero). For example $BASEHHLDCONSUMP_{dr,c}$ is the same as $hhldconsump_{dr,c}$ except that where as the latter is determined for the current time period, the former is generated from the base year accounts.

The Paasche index is given by the equation:

$$gdpindexp = \Bigg[totalexpenditure - \sum_{dr}\Bigg(hhldindirecttax_{dr} + investindirecttax_{dr} + rwindirecttax_{dr} + \sum_{g}\left(govtindirecttax_{dr,g}\right)\Bigg)\Bigg]$$
$$\div$$
$$\Bigg[\sum_{dr}\sum_{c}\Bigg(\Bigg(hhldconsump_{dr,c} + investconsumpq_{dr,c} + stockchangessupply_{sr\to dr,c} + \sum_{g}(govtconsump_{g,dr,c})\Bigg) BASEPCOMPCOMMD_{dr,c}\Bigg) + \sum_{dr}\sum_{c}\Bigg(\left(expcommoditys_{sr\to dr,c} - importdemand_{dr,c}\right)\frac{BASEPCOMMWORLD_{c}}{BASEEXCHANGERT}\Bigg)\Bigg]$$

Included in this GDP calculation is the quantity of net commodity exports (i.e. exports less imports). Note that the formula for the Paasche index is written in two steps, the first two involving calculation of the auxiliaries $stockchangessupply_{sr,c}$ and $totalexpenditure$:

$$stockchangessupply_{sr,c} = \sum_{dr}(regcdomcomms_{sr,dr,c} - regdomcommd_{sr,dr,c})$$

$$totalexpenditure = \sum_{c}\Bigg(\sum_{dr}\Bigg(stockchangessupply_{sr\to dr,c} - nzimportpurchases_{dr,c} \Bigg(hhldconsump_{dr,c} + \sum_{g}\left(govtconsump_{g,dr,c}\right) + investconsumpq_{dr,c}\Bigg)\mathbf{Pcompcommd}_{dr,c}\Bigg) + \sum_{sr}\left(nzexportsales_{sr,c}\right)\Bigg) + \sum_{dr}\Bigg(hhldindirecttax_{dr} + investindirecttax_{dr} + rwindirecttax_{dr} + \sum_{g}\left(govtindirecttax_{g,dr}\right)\Bigg)$$

The real GDP index that is used in the calculation of the desired interest rate is calculated simply as follows:

$$realgdp = \frac{1000}{actualgdpindex} \times totalexpenditure$$

4 Applying the model

4.1 Input data

Many of the exogenous constants and base (initial condition) data are set from the regional Social Accounting Matrices (SAMs). Some exogenous constants are calculated in a calibration process.

4.2 Scenario settings

The model is a relatively generic economic model that could be used to evaluate a wide range of potential scenarios and policy questions. In this section we describe only those types of input changes that have been investigated in scenarios thus far analysed within the Economics of Resilient Infrastructure (ERI) research programme. To implement alternative scenarios in the model it is possible that further scenario 'levers' will also be identified, some of these could potentially involve alternations to the input data described in Section 4.1.

Presently the scenario settings, some of which require additional inputs and calculations, can be divided into three categories: general scenario settings, business operability, and transportation margins and travel costs. Each of these categories are discussed below.

4.2.1 General scenario settings

These are a group of input data or settings that broadly determine the economic growth path for NZ. The following sets of input data need to be selected by the model user:

- *Multi-factor productivity* - the model incorporates industry-specific default estimates of the annual rate of growth in multi-factor productivity. These can effectively be altered via the exogenous input $MFPADJUST_{dr,i}$.
- *World commodity prices* - the model also incorporates econometric projections of world commodity prices, $PCOMMWORLD_c$. These default projects can be revised to reflect alternative scenarios.
- *World GDP* - similarly, the default projections for the world GDP index, $WORLDGDPINDX$ are derived via econometric projections but can be altered to reflect new scenarios.
- *World interest rate* - once again the world interest rate, $WORLDINTERESTRT$ can be altered to reflect user-defined scenarios.

4.2.2 Business operability

Section 3.4 requires the calculation of the actual value of industry production within each region, $actualprod_{dr,i}$. If the exogenous 'business operability' parameter, $OPERABILITY_{sr,i}$, is set at a level below one for a given industry, the model enforces a maximum level of production that can be achieved in that industry (Eq. (B.55)):

$$actualprod_{dr,i} = \begin{cases} \mathbf{Desiredprod}_{dr,i} & \text{if} \quad OPERABILITY_{sr\to dr,i}(t) = 1 \\ \min(maxprod_{dr,i}, \mathbf{Desiredprod}_{dr,i}) & \text{if} \quad OPERABILITY_{sr\to dr,i}(t) < 1 \end{cases}$$

Importantly, the population of these business operability parameters constitutes the link between the Dynamic Economic Model and the Business Behaviours Model within 'MERIT' (Modelling the Economics of Resilient Infrastructure Tool). The operability parameters are a set of scalars that vary between zero (complete disruption to operations) and one (no change in 'as normal' operations) for each industry.

In this calculation we need a measure of the quantity that could have been produced if there was no disruption. To estimate this we take the maximum of the quantity of production desired at the time at which the disruption event commences, $t = SHOCKINITIATION$, and the quantity of production desired now (after the disruption):

$$qasplannedprod_{dr,i} = \max\left(qdesiredprod_{dr,i}(t = SHOCKINITIATION), qdesiredprod_{dr,i}\right)$$

We use this estimate instead of simply $qdesiredprod_{dr,i}$ to avoid underestimating production capability in cases that the quantity of production desired decreases due to the disruption. The quantity of production desired is calculated as the desired value of production divided by the price:

$$qdesiredprod_{dr,i} = \frac{\mathbf{Desiredprod}_{dr,i}}{Pcindustrys_{sr\to dr,i}}$$

And finally the the maximum value of production, $maxprod_{dr,i}$ is calculated as:

$$maxprod_{dr,i} = \left(qasplannedprod_{dr,i} \times OPERABILITY_{sr\to dr,i}(t)\right) Pcindustrys_{sr\to dr,i}$$

4.2.3 Transportation costs

In the case of a road or rail disruption, there will be costs to households for additional travel, and increases in the costs of transporting any commodities that would otherwise use that route. The Dynamic Economic Model does not, itself, calculate the magnitude of these additional transportation requirements, but once these have been determined the associated costs are incorporated in the model.

Starting with households, $ADDHHLDTRAVEL_{dr,c}(t)$ is the predetermined quantity of additional travel-related commodities consumed by households. This, combined with the price of composite commodities, $\mathbf{Pcompcommd}_{dr,c}$, sums to give the total additional travel cost to households (Eq. (B.230)):

$$addtravelcosts_{dr} = \sum_{c} \left(ADDHHLDTRAVEL_{dr,c}(t)\, \mathbf{Pcompcommd}_{dr,c}\right)$$

The additional transport costs on commodity freight are captured through the exogenous inputs $EMARGINSHOCKCOEF_{sr,c,m}(t)$, $IMARGINSHOCKCOEF_{dr,c,m}(t)$, and

$DMARGINSHOCKCOEF_{sr,dr,c}(t)$. These define net additions to the margins on export, import, and domestically-produced and consumed commodities, respectively. In all cases the margins are measured as the net additional demand for freight services per unit of commodity. Thus once we determine the quantities of export, import and domestic commodities, the net additional demand for margin-related transportation services is determined as follows:

$$exportmargindemand_{sr,m} = \sum_{c} (expcommoditys_{sr,c}\, EMARGINSHOCKCOEF_{sr,c,m}(t))$$

$$importmargindemand_{dr,m} = \sum_{c} \left(\mathbf{Estimports}_{dr,c}\, IMARGINSHOCKCOEF_{dr,c,m}(t)\right)$$

$$dommarginq_{dr} = \sum_{sr} \sum_{c} (regcdomcomms_{sr,dr,c}\, DMARGINSHOCKCOEF_{sr,dr,c}(t))$$

Further calculations are necessary to allocate these additional margin-related transportation services to the commodity definitions used in the model (see Equations B.236, B.238, and B.239), eventually enabling the total quantity of additional transportation services, $marginconsumpq_{dr,c}$, demanded to be found.

Finally, it is necessary to calculate the prices of net additional export and import margins, $pimportmargins_{dr,c}$ and $pexportmargins_{sr,c}$. This is determined simply based on the prices of the transportation services (road and rail) responsible for providing the additional margins, and the supply- share of those services in the provision of the margins (see Equations B.240, B.241, B.242, B.243, B.244, and B.245). Once determined, these prices are added to the relevant prices of import and export commodities within the commodities module (Equations B.82 and B.92). In a similar manner the commodities module also includes an equation that adds the value of additional domestic margins to the domestic commodity price (Equation B.100).

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A Index of names and definitions

A.1 Subscripts and concordances

Most subscripts are described in Table 2.1 on page 11. However, the natural capital types, and industry and commodity groupings can vary between different model applications and are described here.

Table A.1 Natural capital types

Name	Description
$NatCap1$	Agricultural land
$NatCap2$	Coal
$NatCap3$	Oil/gas

Table A.2 Industry categories

Name	Description
$Ind1$	Horticulture and fruit growing
$Ind2$	Sheep, beef cattle and grain farming
$Ind3$	Dairy cattle farming
$Ind4$	Poultry, deer and other livestock farming
$Ind5$	Forestry and logging
$Ind6$	Fishing and aquaculture
$Ind7$	Agriculture, forestry and fishing support services
$Ind8$	Mining, quarrying, exploration and other mining support services
$Ind9$	Oil and gas extraction
$Ind10$	Meat and meat product manufacturing
$Ind11$	Dairy product manufacturing
$Ind12$	Other food manufacturing
$Ind13$	Textile, leather, clothing and footwear manufacturing
$Ind14$	Wood and paper manufacturing
$Ind15$	Petroleum and coal product manufacturing
$Ind16$	Chemical, polymer and rubber product manufacturing

(continued on next page)

Table A.2 (continued)

Name	Description
$Ind17$	Non-metallic mineral product manufacturing
$Ind18$	Metal and metal product manufacturing
$Ind19$	Transport, equipment and machinery manufacturing
$Ind20$	Other manufacturing
$Ind21$	Electricity generation and supply
$Ind22$	Gas supply
$Ind23$	Water, sewerage, drainage and waste services
$Ind24$	Construction
$Ind25$	Wholesale trade
$Ind26$	Retail Trade
$Ind27$	Accommodation and food services
$Ind28$	Road transport
$Ind29$	Other transport, postal, courier, transport support and warehousing services.
$Ind30$	Air and space transport
$Ind31$	Information media and telecommunications
$Ind32$	Finance and insurance
$Ind33$	Rental, hiring and real estate services
$Ind34$	Ownership of owner-occupied dwellings
$Ind35$	Professional, scientific, technical, administrative and support services
$Ind36$	Central government administration, defence and public safety
$Ind37$	Local government administration
$Ind38$	Education and training
$Ind39$	Health care and social assistance
$Ind40$	Arts and recreation services
$Ind41$	Personal and other services

Table A.3 Commodity categories

Name	Description
$Com1$	Retail margin
$Com2$	Wholesale trade and margins
$Com3$	Horticulture and fruit
$Com4$	Sheep and cattle
$Com5$	Raw milk
$Com6$	Other livestock and animal products

(continued on next page)

Table A.3 (continued)

Name	Description
$Com7$	Wood and non-wood forest products
$Com8$	Fishing and fish products
$Com9$	Agriculture and forestry support services
$Com10$	Coal products
$Com11$	Oil and gas products
$Com12$	Other mining and quarrying
$Com13$	Non metallic mineral products
$Com14$	Meat products
$Com15$	Dairy products
$Com16$	Other food
$Com17$	Textiles
$Com18$	Clothing and footwear
$Com19$	Wood and paper products
$Com20$	Petroleum products
$Com21$	Chemical, rubber and plastic products
$Com22$	Metal and metal products
$Com23$	Machinery and equipment
$Com24$	Other manufactures
$Com25$	Electricity
$Com26$	Gas
$Com27$	Water
$Com28$	Sewerage services
$Com29$	Waste disposal, recycling and environmental protection services
$Com30$	Construction
$Com31$	Accommodation and restaurant services
$Com32$	Road transport freight services
$Com33$	Road passenger transport
$Com34$	Railway transport freight services
$Com35$	Railway passenger transport
$Com36$	Sea transport freight services
$Com37$	Passenger transportation by waterborne vessels
$Com38$	Air transport freight services
$Com39$	Air passenger transport
$Com40$	Postal and courier services
$Com41$	Cargo handling services

(continued on next page)

Table A.3 (continued)

Name	Description
$Com42$	Other transport services, storage and warehousing
$Com43$	Printing, publishing and broadcasting services
$Com44$	Internet and telecommunications services
$Com45$	Finance and insurance services
$Com46$	Real estate and rental services
$Com47$	Business services
$Com48$	Owner - occupied dwellings
$Com49$	Local government administration services
$Com50$	Central government administration services
$Com51$	Education
$Com52$	Health
$Com53$	Sport and recreation services
$Com54$	Personal and community services

A.2 Stocks

Table A.4 Description of stocks, and equation references

Name	Description	Eq.ref
$\mathbf{Builtcapital}_{dr,i}$	Built Capital	(B.144)
Casurplus	Current account surplus	(B.184)
$\mathbf{Desiredprod}_{dr,i}$	Value of desired industry production	(B.50)
$\mathbf{Estimports}_{dr,c}$	Commodity imports	(B.65)
Exchangert	Exchange rate	(B.204)
Gdpindex	Gross domestic product (Fisher) index	(B.217)
$\mathbf{Industryaccount}_{dr,i}$	Available industry funds	(B.51)
$\mathbf{Industrybalance}_{dr,i}$	Difference between industry income and industry expenditure	(B.52)
Inflationrt	Inflation rate	(B.218)
Interestrt	Interest rate	(B.185)
$\mathbf{Labour}_{sr}$	Labour endowment	(B.135)
$\mathbf{Multifactprod}_{dr,i}$	Multifactor productivity	(B.119)
$\mathbf{Naturalcapital}_{dr,i}$	Natural Capital	(B.145)
$\mathbf{Pavglabour}_{sr}$	Average labour price	(B.136)
$\mathbf{Pbuiltcap}_{dr,i}$	Industry built capital price	(B.146)

(continued on next page)

Table A.4 (continued)

Name	Description	Eq.ref
Pcfact$_{dr,i}$	Industry composite factor price	(B.120)
Pcindustrys$_{sr,i}$	Composite industry supply price	(B.66)
Pcompcommd$_{dr,c}$	Composite commodity demand price	(B.67)
Pcompcomms$_{sr,c}$	Composite commodity supply price	(B.68)
Pcompdomcommd$_{dr,c}$	Composite domestic commodity demand price	(B.69)
Pcompdomcomms$_{sr,c}$	Composite domestic commodity supply price	(B.70)
Pcompnaturalcapd$_{dr,i}$	Composite natural capital demand price	(B.147)
Pcompnaturalcaps$_{dr,nct}$	Composite natural capital supply price	(B.148)
Pexpcomm$_{sr,c}$	Price of export commodities	(B.71)
Pfact$_{h,dr,i}$	Price of factors	(B.121)
Pfiinputs$_{dr,i}$	Price of composite inputs	(B.72)
Pgovtcc$_{g,dr}$	Perceived government composite commodity consumption price	(B.24)
Phhldcc$_{dr}$	Household composite commodity consumption price	(B.2)
Pintinputs$_{dr,i}$	Price of composite intermediate inputs	(B.73)
Pinvestcc$_{dr}$	Investment composite commodity consumption price	(B.186)
Pnaturalcap$_{dr,i,nct}$	Price of natural capital types	(B.149)
Pregdomcomm$_{sr,dr,c}$	Price of commodities produced in and for the domestic market	(B.74)
Rcapincome$_{dr}$	Recognised capital income	(B.150)
Renterincome$_{dr}$	Recognised enterprise income	(B.40)
Rgovtincome$_{g,dr}$	Recognised government income	(B.23)
Rhhldincome$_{dr}$	Recognised household income	(B.1)

Table A.5 Initial condition settings for Stocks

Name	Notes
$iBuiltcapital_{dr,i}$	net capital stock by 31 industry types obtained from Statistics New Zealand (series SG07NAC04K90). This is disaggregated to 106 industries on a pro rata basis according to each industry's share of total capital factor payments. The national industry estimates are then further disaggregated to 15 New Zealand regions on a pro rata basis according to the regional share of each industry's total consumption of fixed capital. The latter data is obtained from multi-regional supply use tables (Smith *et al.*, 2015)
$iCasurplus$	set to 0 in base year
$iDesiredprod_{dr,i}$	set to BASEPRODUCTION in base year
$iEstimports_{dr,c}$	set to BASEIMPORTS in base year
$iExchangert$	set to 1 in base year

(continued on next page)

Table A.5 (continued)

Name	Notes
$iGdpindex$	set to 1000 in base year
$iIndustryaccount_{dr,i}$	derived from base social accounting matrix
$iIndustrybalance_{dr,i}$	set to 0 in base year
$iInflationrt$	set as 3.2 % (`http://www.rbnz.govt.nz/statistics/`)
$iInterestrt$	set as 7.3 % (`http://www.rbnz.govt.nz/statistics/`)
$iLabour_{sr}$	employment by industry and (demand) region obtained from Statistics New Zealand's Annual Business Frame (http://www.stats.govt.nz/). This data is disaggregated to regions from which employment sourced (supply region) according to the proportion of total labour factor payments from each region allocated to each supply region, as specified in the multi-regional supply and use tables (Smith *et al.*, 2015).
$iMultifactprod_{dr,i}$	set to 1 in base year
$iNaturalcapital_{dr,i}$	agricultural land use data is sourced from Smith and McDonald ().
$iPavglabour_{sr}$	prices in base year set to 1
$iPbuiltcap_{dr,i}$	prices in base year set to 1
$iPcfact_{dr,i}$	prices in base year set to 1
$iPcompcommd_{dr,c}$	prices in base year set to 1
$iPcompcomms_{sr,c}$	prices in base year set to 1
$iPcompdomcommd_{dr,c}$	prices in base year set to 1
$iPcompdomcomms_{sr,c}$	prices in base year set to 1
$iPcompnaturalcapd_{dr,i}$	prices in base year set to 1
$iPcompnaturalcaps_{sr,i}$	prices in base year set to 1
$iPexpcomm_{sr,c}$	prices in base year set to 1
$iPfact_{h,dr,i}$	prices in base year set to 1
$iPfiinputs_{dr,i}$	prices in base year set to 1
$iPgovtcc_{g,dr}$	prices in base year set to 1
$iPhhldcc_{dr}$	prices in base year set to 1
$iPintinputs_{dr,i}$	prices in base year set to 1
$iPinvestcc_{dr}$	prices in base year set to 1
$iPnaturalcap_{dr,i,nct}$	prices in base year set to 1
$iPregdomcomm_{sr,dr,c}$	prices in base year set to 1
$iRcapincome_{dr}$	derived from base social accounting matrix
$iRenterincome_{dr}$	derived from base social accounting matrix
$iRgovtincome_{g,dr}$	derived from base social accounting matrix
$iRhhldincome_{dr}$	set to BASEHHLDACCOUNT in base year

A.3 Auxiliaries

Table A.6 Description of auxiliaries, and equation references

Name	Description	Eq. ref
$actualenterincome_{dr}$	Current enterprise income	(B.41)
$actualexports_{sr,c}$	Current commodity exports	(B.216)
$actualgdpindex$	Current gross domestic product (Fisher) index	(B.224)
$actualhhldincome_{dr}$	Current household income	(B.3)
$actualindcompnaturalcaps_{dr,i}$	Current supply of composite natural capital to industries	(B.155)
$actualpavglabour_{sr}$	Current average labour price	(B.137)
$actualpcapital_{dr,i}$	Current capital price	(B.125)
$actualpccd_{dr,c}$	Current composite commodity demand price	(B.93)
$actualpccs_{sr,c}$	Current composite commodity supply price	(B.96)
$actualpcdcd_{dr,c}$	Current composite domestic commodity demand price	(B.98)
$actualpcdcs_{sr,c}$	Current composite domestic industry supply price	(B.101)
$actualpcfact_{dr,i}$	Current composite factor price	(B.122)
$actualpcnaturalcapd_{dr,i}$	Current composite natural capital demand price	(B.154)
$actualpcnaturalcaps_{dr,nct}$	Current composite natural capital supply price	(B.151)
$actualpexports_{sr,c}$	Current exports price	(B.215)
$actualpfiinputs_{dr,i}$	Current composite factor and intermediate inputs price	(B.105)
$actualpgovtcc_{g,dr}$	Current government composite commodity consumption price	(B.34)
$actualphhldcc_{dr}$	Current household composite commodity consumption price	(B.16)
$actualpintinputs_{dr,i}$	Current composite intermediate inputs price	(B.109)
$actualpinvestcc_{dr}$	Current investment composite commodity consumption price	(B.200)
$actualprod_{dr,i}$	Actual production	(B.55)
$actualcasurplus$	Actual current account surplus	(B.187)
$addtravelcosts_{dr}$	Additional household travel costs	(B.230)
$aggregateinvestv_{dr}$	Total value of investment	(B.203)
$betwgovttransout_{g,dr}$	Within-region transfers between government agents, by receiving agent	(B.30)
$bopratio$	Balance of payments ratio	(B.205)
$builtratio_{dr,i}$	Ratio of industry built capital supply to industry built capital demand	(B.162)
$builts_{dr,i}$	Supply quantity of built capital	(B.163)
$builtuse_{dr,i}$	Built capital used	(B.182)

(continued on next page)

Table A.6 (continued)

Name	Description	Eq. ref
$capentertrans_{dr}$	Transfers of capital income to enterprises	(B.169)
$capgovttrans_{g,dr}$	Transfers of capital income to government	(B.170)
$capitalincome_{dr}$	Total capital income	(B.167)
$capincomehhld_{dr}$	Transfers of capital income to households, by receiving region	(B.4)
$capincomesh_{dr,i}$	Industry share of total regional capital income	(B.180)
$capitaltyped_{cap,dr,i}$	Industry capital demand	(B.158)
$caplocalhhldtrans_{dr}$	Capital income transferred to within-region households	(B.171)
$capreghhldtrans_{dr}$	Capital income transferred to out-of-region households, by region of capital income receipt	(B.172)
$capregtransout_{dr}$	Capital income transferred between regions, by receiving region	(B.168)
$ccapitals_{dr,i}$	Composite capital supply	(B.164)
$compfactord_{dr,i}$	Industry demand for composite factors	(B.59)
$compfactoru_{dr,i}$	Industry use of composite factors	(B.131)
$compnaturalcaps_{dr,i}$	Composite natural capital supply	(B.166)
$cpif$	Consumer price (Fisher) index	(B.221)
$cpil$	Consumer price (Laspeyres) index	(B.222)
$cpip$	Consumer price (Paasche) index	(B.223)
$depreciation_{dr,i}$	Depreciation	(B.183)
$desiredinflationrt$	Desired inflation rate	(B.220)
$desiredinterestrt$	Desired interest rate	(B.189)
$directtaxincome_{g,dr}$	Government income from direct taxes	(B.26)
$disinvestconsump_{dr,c}$	Investment consumption by commodity	(B.199)
$domcomdemand_{dr,c}$	Domestic commodity demand	(B.95)
$domcommoditys_{sr,c}$	Domestic commodity supply	(B.87)
$dommarginq_{dr}$	Quantity of domestic margins	(B.234)
$effectcompfactord_{dr,i}$	Effective composite factor demand during a disruption that affects operability	(B.60)
$effectcompfactoru_{dr,i}$	Effective composite factor use during a disruption that affects operability	(B.134)
$effectfactorsd_{h,dr,i}$	Effective factor demand during a disruption that affects operability	(B.132)
$effectfactorsu_{h,dr,i}$	Effective factor use during a disruption that affects operability	(B.133)
$entdirecttax_{dr}$	Enterprise direct taxes	(B.42)

(continued on next page)

Table A.6 (continued)

Name	Description	Eq. ref
$entgovttrans_{g,dr}$	Enterprise transfers to government	(B.46)
$enthhldtrans_{dr}$	Enterprise transfers to households	(B.47)
$entincomehhld_{dr}$	Total enterprise transfers to households, by receiving region	(B.5)
$entreghhldtrans_{dr}$	Enterprise income transferred to out-of-region households, by region of enterprise income receipt	(B.48)
$entregtransout_{dr}$	Transfers between enterprises within different regions, by paying region	(B.45)
$entrwtrans_{dr}$	Transfers from enterprises to the rest of the world	(B.44)
$entsavtrans_{dr}$	Enterprise savings	(B.49)
$excessproduction_{sr,dr,c}$	Excess of commodity production over use	(B.84)
$expcommodityd_{sr,c}$	Export commodity demand	(B.82)
$expcommoditys_{sr,c}$	Export commodity supply	(B.76)
$exportmargindemand_{sr,m}$	Demand for transport margins on exports	(B.235)
$exportmarginsupply_{dr,m}$	Supply of transport margins on exports	(B.236)
$exportratio_{sr,c}$	Ratio of export supply to demand	(B.75)
$factinputshare_{dr,i}$	Share of total inputs comprised of factors	(B.107)
$factinputunitcost_{dr,i}$	Unit cost of factor inputs	(B.63)
$factorsd_{h,dr,i}$	Demand for factors	(B.129)
$factorss_{h,dr,i}$	Supply of factors	(B.128)
$factorsu_{h,dr,i}$	Use of factors	(B.130)
$gdpindexl$	Gross domestic product (Laspeyres) index	(B.225)
$gdpindexp$	Gross domestic product (Paasche) index	(B.226)
$gdpgap$	Difference between perceived and actual gross domestic product index	(B.190)
$govtcompcommd_{g,dr}$	Government composite commodity demand	(B.38)
$govtconsump_{g,dr,c}$	Government commodity consumption	(B.36)
$govtdirecttax_{g,dr}$	Direct taxes on government	(B.28)
$govthhldtrans_{g,dr}$	Transfers from government to households	(B.31)
$govtincome_{g,dr}$	Government income	(B.25)
$govtindirecttax_{g,dr}$	Indirect taxes on government	(B.39)
$govtrwtrans_{g,dr}$	Transfers from government to the rest of the world	(B.29)
$govtsavings_{g,dr}$	Government savings	(B.32)
$grossreturn_{dr,i}$	Gross returns on capital	(B.179)
$hhldcompcommd_{dr}$	Household composite commodity demand	(B.20)
$hhldconsump_{dr,c}$	Household commodity consumption	(B.18)

(continued on next page)

Table A.6 (continued)

Name	Description	Eq. ref
$hhldconsumprt_{dr}$	Household consumption rate	(B.15)
$hhlddirecttax_{dr}$	Direct taxes on households	(B.22)
$hhldenttrans_{dr}$	Transfers from households to enterprises	(B.10)
$hhldgovttrans_{g,dr}$	Transfers from households to government	(B.11)
$hhldindirecttax_{dr}$	Indirect taxes on households	(B.21)
$hhldregtransout_{dr}$	Transfers between households within different regions, by paying region	(B.12)
$hhldrwtrans_{dr}$	Transfers from households to the rest of the world	(B.8)
$hhldsavings_{dr}$	Household savings	(B.13)
$immobileinvest_{dr,i}$	Investment that is fixed to industry responsible for capital income	(B.175)
$importdemand_{dr,c}$	Demand for commodity imports	(B.88)
$importmargindemand_{dr,m}$	Demand for margins on imports	(B.237)
$importmarginsupply_{sr,m}$	Supply of margins on imports	(B.238)
$indcapincome_{dr,i}$	Current rate of income on capital, by industry	(B.181)
$indcommoditys_{sr,i,c}$	Supply of commodities by industries	(B.79)
$indconsump_{dr,i,c}$	Use of commodities by industries	(B.90)
$indexpendu_{dr,i}$	Industry expenditure	(B.56)
$indlabours_{dr,i}$	Supply of labour to industries	(B.142)
$indindirecttax_{dr,i}$	Indirect taxes on industries	(B.64)
$indirecttaxincome_{g,dr}$	Government income from direct taxes on industries	(B.27)
$indnaturalcaps_{dr,i,nct}$	Industry supply of natural capital, final estimate	(B.161)
$indnaturalcaps1_{dr,i,nct}$	Supply of natural capital to industries, first estimate	(B.152)
$industryinc_{dr,i}$	Industry income	(B.57)
$interinputshare_{dr,i}$	Share of industry inputs comprised of intermediate inputs	(B.108)
$interinputunitcost_{dr,i}$	Unit cost of intermediate inputs	(B.62)
$intinputcoeff_{dr,i,c}$	Commodity share of intermediate inputs	(B.111)
$investconsumpq_{dr,c}$	Quantity of commodities consumed for investment	(B.201)
$investindirecttax_{dr}$	Indirect taxes on investment	(B.202)
$labincomedregion_{dr}$	Total labour income by region of income generation	(B.7)
$labincomesupply_{sr}$	Total labour income paid to households, by region of labour supply	(B.6)
$labratio_{dr,i}$	Ratio of labour supply to labour demand	(B.127)
$marginconsumpq_{dr,c}$	Consumption of margin commodities	(B.239)
$maxprod_{dr,i}$	Maximum production	(B.231)

(continued on next page)

Table A.6 (continued)

Name	Description	Eq. ref
$mobileinvest_{dr,i}$	Mobile investment	(B.174)
$mobileinvest1_{dr,i}$	First estimate of mobile investment	(B.177)
$mobileinvestsh_{dr,i}$	Industry share of total mobile investment	(B.176)
$naturalcapitalsq_{dr,i}$	Supply quantity of natural capital in normalised units	(B.165)
$naturalcapd_{dr,i,nct}$	Demand for natural capital	(B.157)
$naturalcapratio_{dr,i,nct}$	Ratio of natural capital supply to demand	(B.160)
$netreturn_{dr,i}$	Net return on investment	(B.178)
$newcapital_{dr,i}$	New capital	(B.173)
$nzexportsales_{sr,c}$	Value of commodity exports	(B.214)
$nzimportpurchases_{dr,c}$	Value of commodity imports	(B.209)
$pexpcommnz_{sr,c}$	Price of export commodities in New Zealand currency	(B.77)
$pexportcommd_{sr,c}$	Price of export commodities including margins	(B.83)
$pexportmargins_{sr,c}$	Price of export margins	(B.245)
$pexprailmargins_{sr,dr}$	Price of rail margins on composite commodity demand	(B.243)
$pexproadmargins_{sr,dr}$	Price of road margins on composite commodity demand	(B.244)
$pimpcommnz_{dr,c}$	Price of import commodities in New Zealand currency	(B.92)
$pimportmargins_{dr,c}$	Price of import margins	(B.242)
$pimprailmargins_{sr,dr}$	Price of rail margins on commodities produced in and for the domestic market	(B.240)
$pimproadmargins_{sr,dr}$	Price of road margins on commodities produced in and for the domestic market	(B.241)
$potentialsales_{dr,i}$	Potential value of industry sales	(B.58)
$pregdomcomminclmargin_{sr,dr,c}$	Price of regional domestic commodities including margins	(B.100)
$preglabour_{dr}$	Regional labour price	(B.140)
$qcapitald_{dr,i}$	Quantity of capital demand	(B.126)
$qcompcommd_{dr,c}$	Quantity of composite commodity demand	(B.94)
$qcompcomms_{sr,c}$	Quantity of composite commodity supply	(B.97)
$qcompfactd_{dr,i}$	Quantity of composite factor demand	(B.123)
$qasplannedprod_{dr,i}$	Available quantity of production - based on production at onset of disruption	(B.232)
$qdesiredprod_{dr,i}$	Quantity of production desired	(B.233)
$qdomcommd_{dr,c}$	Quantity of domestic commodity demand	(B.99)
$qdomcomms_{sr,c}$	Quantity of domestic commodity supply	(B.102)
$qfiinputs_{dr,i}$	Quantity of composite factor and intermediate input supply	(B.106)
$qgovtcc_{g,dr}$	Quantity of composite government consumption	(B.35)

(continued on next page)

Table A.6 (continued)

Name	Description	Eq. ref
$qhhldcc_{dr}$	Quantity of composite household consumption	(B.17)
$qintinputs_{dr,i}$	Quantity of composite intermediate inputs	(B.110)
$qinvestcc_{dr}$	Quantity of composite investment consumption	(B.197)
$qlabour_{sr}$	Current supply quantity of labour	(B.138)
$realgdp$	Real gross domestic product	(B.227)
$realinterestrt$	Real interest rate	(B.191)
$regcdomcomms_{sr,dr,c}$	Domestic commodity supply, by region of commodity production (sr) and commodity consumption (dr)	(B.85)
$regcommoditys_{sr,c}$	Total commodity production within New Zealand regions	(B.78)
$regdomcommd_{sr,dr,c}$	Domestic commodity demand, by region of commodity production (sr) and commodity consumption (dr)	(B.86)
$regindprodincltax_{sr,i}$	Composite industry production	(B.81)
$reglabourest_{sr,dr}$	Labour supply, by region of labour origin (sr) and labour use (dr), first estimate	(B.141)
$reglaboursupply_{sr,dr}$	Labour supply, by region of labour origin (sr) and labour use (dr)	(B.143)
$regsavings_{dr}$	Total regional savings	(B.195)
$rwdirecttax_{dr}$	Rest of world direct tax	(B.210)
$rwenttrans_{dr}$	Transfers from the rest of the world to enterprises	(B.43)
$rwexpenditure$	Total expenditure for the rest of the world	(B.211)
$rwhhldtrans_{dr}$	Transfers from the rest of the world to households	(B.9)
$rwincome$	Total income for the rest of the world	(B.206)
$rwindirecttax_{dr}$	Total indirect taxes on exports	(B.212)
$rwindirecttaxdisag_{dr,c}$	Indirect taxes on export commodities	(B.213)
$rwlabourincome_{dr}$	Rest of world labour income	(B.207)
$rwlaboursupply_{dr,i}$	Rest of world labour supply	(B.208)
$rwsavings_{dr}$	Rest of world savings, by New Zealand region	(B.193)
$rwsavingstotal$	Total rest of world savings	(B.194)
$savingstotal_{dr}$	Total savings	(B.192)
$savregtransout_{dr}$	Transfers of savings between regions, by paying region	(B.196)
$stockchangessupply_{sr,c}$	Commodity production less commodity use	(B.229)
$supcoeff_{sr,i,c}$	Commodity share of composite industry production	(B.80)
$targetinflationrt$	Target inflation rate	(B.188)
$totalcomdemand_{dr,c}$	Total regional commodity demand	(B.89)
$totalexpenditure$	Total domestic expenditure	(B.228)

(continued on next page)

Table A.6 (continued)

Name	Description	Eq. ref
$totalgovtconsump_{g,dr}$	Total value of government consumption	(B.33)
$totalhhldconsump_{dr}$	Total value of household consumption	(B.14)
$totalindconsump_{dr,c}$	Total value of industry consumption	(B.91)
$unitcost_{dr,i}$	Unit cost of production	(B.61)
$vcomdemand_{sr,c}$	Value of commodity demand	(B.54)
$vinddemand_{sr,i}$	Value of industry demand	(B.53)
$\eta^{cc}_{dr,i}$	Parameter for input substitution between capital types	(B.159)
$\eta^{com}_{dr,c}$	Parameter for input substitution between domestic and imported commodities	(B.112)
$\eta^{cominput}_{dr,i}$	Parameter for input substitution between different types of commodities	(B.113)
$\eta^{fact}_{dr,i}$	Parameter for input substitution between factors	(B.124)
$\eta^{fi}_{dr,i}$	Parameter for input substitution between composite factors and composite intermediate inputs	(B.114)
$\eta^{govtc}_{g,dr}$	Parameter for commodity substitution in government consumption	(B.37)
η^{hhldc}_{dr}	Parameter for commodity substitution in household consumption	(B.19)
$\eta^{investc}_{dr}$	Parameter for commodity substitution investment consumption	(B.198)
$\eta^{natcap}_{dr,i}$	Parameter for input substitution between natural capital types	(B.156)
$\eta^{regcom}_{dr,c}$	Parameter for input substitution between commodities produced in different New Zealand regions	(B.115)
$\phi^{com}_{sr,c}$	Parameter for transformation of commodity supply between export and domestic markets	(B.117)
$\phi^{comsup}_{sr,c}$	Parameter for transformation of industry supply between different types of commodities	(B.116)
ϕ^{lab}_{sr}	Parameter for transformation of labour supply between different regional labour markets	(B.139)
$\phi^{natcap}_{dr,nct}$	Parameter for transformation of natural capital supply between different industries	(B.153)
$\phi^{regcom}_{sr,c}$	Parameter for transformation of commodity supply between different regional commodity markets	(B.118)

A.4 Exogenous inputs

Table A.7 Description of exogeneous constants

Name	Description
$ALLOCATESH_{dr}$	*Share of total mobile investment that is allocated to industries based on the relative returns to capital in those industries.* Set to one less the estimated proportion of investment in the base year that is assigned to industries with negative net returns
$ALPHA_{dr}$	*Weight given to the real interest weight in determining the total value of regional investment.* Estimated from linear regression of historic data
$BASECOMMFLOW_{sr,dr,c}$	*Commodities used by (dr) regions and sourced from production (sr) regions during the base year.* Derived from base year social accounting matrix
$BASECONSUMPRT_{dr}$	*Rate of household consumption during the base year.* Derived from base year social accounting matrix
$BASEEXPORTS_{dr,c}$	*Value of commodity exports during the base year.* Derived from base year social accounting matrix
$BASEGDP_{dr}$	*Gross domestic product for the base year.* Derived from base year social accounting matrix
$BASEGOVTCONSUMP_{g,dr,c}$	*Government commodity consumption for the base year.* Derived from base year social accounting matrix
$BASEHHLDACCOUNT_{dr}$	*Household income available for current consumption and savings during the base year.* Derived from base year social accounting matrix
$BASEHHLDCONSUMP_{dr,c}$	*Household commodity consumption for the base year.* Derived from base year social accounting matrix
$BASEINVESTCONSUMP_{dr,c}$	*Investment commodity consumption for the base year.* Derived from base year social accounting matrix
$BASEPCOMPCOMMD_{dr,c}$	*Base year composite commodity demand price.* Derived from base year social accounting matrix
$BASEPRODUCTION_{dr,i}$	*Base year industry production.* Derived from base year social accounting matrix
$BASEREALINTERESTRT$	*Base year real interest rate.* Set at 4.2% (`www.rbnz.govt.nz/statistics/`)
$BETA_{dr}$	*Weight given to the value of total regional savings in determining the total value of regional investment.* Estimated from linear regression of historic data
$BTWGOVTTRANSRT_{g,dr}$	*Rate of transfers between government agents.* Derived from base year social accounting matrix

(continued on next page)

Table A.7 (continued)

Name	Description
$CENTTRANSRT_{dr}$	*Share of capital income transferred to enterprises.* Derived from base year social accounting matrix
$CGOVTTRANSRT_{g,dr}$	*Share of capital income transferred to government.* Derived from base year social accounting matrix
$CHHLDTRANSRT_{dr}$	*Share of household income transferred to households.* Derived from base year social accounting matrix
$CIRELASTICITY_{dr}$	*Parameter that controls the degree to which household consumption changes in response to changes in the real interest rate.* Set at -0.075 (Creedy *et al.*, 2015)
$CREGTRANSRT_{dr}$	*Share of capital income transferred out of the region.* Derived from base year social accounting matrix
$CRHTRANSRT_{dr}$	*Share of capital income transferred to households out of the region.* Derived from base year social accounting matrix
$DEPSHFT$	*Adjustment to depreciation rate.* Default parameters derived from model calibration
$DIRECTTAXSH_{g,dr}$	*Share of total regional direct tax income allocated to government agents.* Derived from base year social accounting matrix
$EGOVTTRANS$	*Parameter that controls the degree to which transfers from government to the rest of the world change in response to changes in the current account surplus.* Set to 1
$EGOVTTRANSRT_{g,dr}$	*Government savings rate.* Derived from base year social accounting matrix
$EHHLDTRANSRT_{dr}$	*Share of enterprise income transferred to households.* Derived from base year social accounting matrix
$EINVEST_{dr,i}$	*Parameter that controls the degree to which investment allocated to industries responds to changes in the rate of return on capital.* Derived from model calibration
$ENTTAXRT_{dr}$	*Enterprise direct tax rate.* Derived from base year social accounting matrix
$EREGTRANSRT_{dr}$	*Share of enterprise income transferred out of the region.* Derived from base year social accounting matrix
$ERHHLDTRANSRT_{dr}$	*Share of enterprise income transferred to households located out of the region.* Derived from base year social accounting matrix

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Table A.7 (continued)

Name	Description
$ERWTRANSBS_{dr}$	*Transfers from enterprises to the rest of the world during the base year.* Derived from base year social accounting matrix
$ESAVTRANSRT_{dr}$	*Share of enterprise income transferred to savings.* Derived from base year social accounting matrix
$EXPORTP_{c}$	*Parameter that controls the change in export demand in response to the price of New Zealand export commodities relative to the world price.* Derived from Horridge and Zhai (2005)
$FCSHENTRWTRANS$	*Foreign currency share of transfers from enterprises to the rest of the world.* Set at 0.5
$FCSHHHLDRWTRANS_{dr}$	*Foreign currency share of transfers from households to the rest of the world.* Set at 0.5
$FCSHRWENTTRANS_{dr}$	*Foreign currency share of transfers from the rest of the world to enterprises.* Set at 0.5
$FCSHRWHHLDTRANS_{dr}$	*Foreign currency share of transfers from the rest of the world to households.* Set at 0.5
$GDPWEIGHT$	*Weight given to the world gross domestic product index in determining the value of savings from the rest of the world.* Estimated from linear regression of historic data
$GOVTCONSUMPRT_{g,dr}$	*Share of government income allocated to consumption.* Derived from base year social accounting matrix
$GOVTDIRECTTAXRT_{g,dr}$	*Government direct tax rate.* Derived from base year social accounting matrix
$GOVTHHLDTRANSRT_{g,dr}$	*Share of government income transferred to households.* Derived from base year social accounting matrix
$GOVTINDIRECTTAXRT_{g,dr}$	*Indirect tax rate on government commodity consumption.* Derived from base year social accounting matrix
$GOVTRWTRANSBS_{g,dr}$	*Transfers from government to the rest of the world during the base year.* Derived from base year social accounting matrix
$GOVTSAVRT_{g,dr}$	*Rate of savings on government income.* Derived from base year social accounting matrix
$HHLDENTTRANSRT_{dr}$	*Share of household income transferred to enterprises.*

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Table A.7 (continued)

Name	Description
	Derived from base year social accounting matrix
$HHLDGOVTTRANSRT_{g,dr}$	*share of household income transferred to government.* Derived from base year social accounting matrix
$HHLDINDTAXRT_{dr}$	*Indirect tax rate on household commodity consumption.* Derived from base year social accounting matrix
$HHLDREGTRANSRT_{dr}$	*Share of household income transferred to households located outside of the local region.* Derived from base year social accounting matrix
$HHLDRWTRANSBS_{dr}$	*Transfers from households to the rest of the world during the base year.* Derived from base year social accounting matrix
$INDINDIRECTTAXRT_{dr,i}$	*Indirect tax rate on industry commodity consumption.* Derived from base year social accounting matrix
$INDIRECTTAXSH_{g,dr}$	*Share of indirect tax income allocated to government agents.* Derived from base year social accounting matrix
$INTERESTCONST$	*Interest rate constant in calculation of desired interest rate.* Estimated from linear regression of historic data
$INTERESTGDPW$	*Parameter applied to GDP gap in the calculation of desired interest rate.* Estimated from linear regression of historic data
$INTERESTINFLW$	*Parameter applied to inflation rate gap in the calculation of desired interest rate.* Estimated from linear regression of historic data
$INTERESTWEIGHT$	*Weight given to the interest rate in determining the value of savings from the rest of the world.* Estimated from linear regression of historic data
$INVESTCONST_{dr}$	*Value of regional investment held constant.* Derived from base year data given assumed $ALPHA_{dr}$ and $BETA_{dr}$
$INVESTCONSTSH_{dr,i}$	*Industry share of regional investment held constant.* Model calibration
$INVESTINDIRECTTAXRT_{dr}$	*Indirect tax rate on investment commodity consumption.* Derived from base year social accounting matrix
$INVESTPARAM_{dr,i}$	*Parameter for scaling net return on capital.* Model calibration
$KSFCONVERT_{dr,i}$	*Built capital stock-to-flow conversion parameter.*

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Table A.7 (continued)

Name	Description
	Derived from base year estimates of stock values by industry (refer to input data for $iBuiltcapital_{dr,i}$) and factor payments from the base year social accounting matrix
$LSFCONVERT_{dr}$	*Labour stock-to-flow conversion parameter.* Derived from base year estimates of labour supply (refer to input data for $iLabour_{sr}$) and labour factor payments from the base social accounting matrix
$MFPGRRT_{dr,i}$	*Rate of growth in multi-factor productivity.* Derived from Statistics New Zealand's industry productivity statistics for 37 industry types (https://nzdotstat.stats.govt.nz/). Average growth rates in productivity over 1978-2012 for each industry selected
$MOBILESH_{dr,i}$	*Share of investment that is mobile between industries.* Assumed to be 0.7 but can be adjusted in calibration
$NATCAPCONVERT_{dr,i}$	*Natural capital stock-to-flow conversion parameter.* Derived from base year estimates of natural capital (refer to input data for $iNaturalcapital_{dr,nct}$) and natural capital payments calculated for the base year
$PRODSCALAR_{dr,i}$	*Scalar to adjust industry production to account for indirect taxes.* Derived from base year social accounting matrix
$RAILMAP_{c}$	*Concordance defining rail freight service commodity.* For default 54 commodity model, all commodities set to 0 except commodity 36 which is set to one
$RDEP_{dr,i}$	*Rate of capital depreciation.* Derived from base capital stock estimates (refer to input data for $iBuiltcapital_{dr,i}$) and region and industry
$ROADMAP_{c}$	*Concordance defining road freight service commodity.* For default 54 commodity model, all commodities set to 0 except commodity 34 which is set to one
$RWDIRECTTAXRT_{dr}$	*Direct tax rate for the rest of the world.* Derived from base year social accounting matrix
$RWENTTRANSBS_{dr}$	*Transfers from the rest of the world to enterprises during the base year.* Derived from base year social accounting matrix
$RWFACTRT_{h,dr}$	*Share of factors supplied by the rest of the world.* Derived from base year social accounting matrix
$RWHHLDTRANSBS_{dr}$	*Transfers from the rest of the world to households during e base year.* Derived from base year social accounting matrix
$RWINDIRECTTAXRT_{dr}$	*Indirect tax rate on rest of world commodity consumption.* Derived from base year social accounting matrix

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Table A.7 (continued)

Name	Description
$RWREGSAVSH_{dr}$	*Share of total rest of world savings allocated to regions.* Derived from base year social accounting matrix
$RWSAVCONST$	*Rest of world savings held constant.* Estimated from linear regression of historic data
$SAVREGTRANSBS_{dr}$	*Savings transferred out of the region during the base year.* Derived from base year social accounting matrix
$SETINVESTCQ_{dr,c}$	*Quantity of investment commodity consumption held constant.* Set equal to base year investment commodity consumption if this is less than zero, otherwise set to zero
$SHOCKINITIATION$	*Time at beginning of disruption (shock).* Set specifically for each scenario
$TAXHHLD_{dr}$	*Direct tax rate on households.* Derived from base year social accounting matrix
$\alpha^{exchangert}$	*Exchange rate change parameter.* Derived from model calibration
$\alpha^{pbuiltcap}$	*Built capital price change parameter.* Derived from model calibration
$\alpha^{pexpcomm}$	*Export commodity price change parameter.* Derived from model calibration
α^{plab}	*Labour price change parameter.* Derived from model calibration
$\alpha^{pnatcap}_{nct}$	*Natural capital price change parameter.* Derived from model calibration
$\alpha^{pregdomcomm}$	*Price of regional domestic commodities change parameter.* Derived from model calibration
δ^{cc}_{dr}	*Share parameter for capital types within the CES function for composite capital.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{cominput}_{dr,i,c}$	*Share parameter for commodities within the CES function for composite intermediate inputs.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{commddom}_{dr,c}$	*Share parameter for domestic commodities within the CES function for composite domestic and imported commodities.*

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Table A.7 (continued)

Name	Description
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{commdimp}_{dr,c}$	*Share parameter for imported commodities within the CES function for composite domestic and imported commodities.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{commregd}_{sr,dr,c}$	*Share parameter for regional commodities within the CES function for composite domestic commodities.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{fact}_{h,dr,i}$	*Share parameter for factors within the CES function for composite factors.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{fi}_{input,dr,i}$	*Share parameter for factors/intermediate inputs within the CES function for composite inputs.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{govtc}_{g,dr,c}$	*Share parameter for commodities within the CES function for composite government consumption.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{hhldc}_{dr,c}$	*Share parameter for commodities within the CES function for composite household consumption.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{investc}_{dr,c}$	*Share parameter for commodities within the CES function for composite investment consumption.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\delta^{natcap}_{dr,i,nct}$	*Share parameter for natural capital types within the CES function for composite natural capital.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\epsilon^{cc}_{dr,i}$	*Elasticity of substitution between capital types.* Set to 0.1 for energy industries and for all other industries set to 0.3 (informed by Rae and Strutt (2005) and Hertel *et al.* (2012))
$\epsilon^{com}_{dr,c}$	*Elasticity of substitution between imported and domestic commodities.* Derived from Robson (2012, p.95) and Hertel *et al.* (2012)
$\epsilon^{cominput}_{dr,i}$	*Elasticity of substitution between commodities of different types.*

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Table A.7 (continued)

Name	Description
	Set to 0.8 for all industries
$\epsilon_{dr,i}^{fact}$	*Elasticity of substitution between factors.*
	Derived from Hertel *et al.* (2012)
$\epsilon_{dr,i}^{factint}$	*Elasticity of substitution between factors and intermediate inputs.*
	Set to 0.5 for all industries (Robson, 2012)
$\epsilon_{g,dr}^{govtc}$	*Elasticity of substitution between commodities in government consumption.*
	Set to 0.5
ϵ_{dr}^{hhldc}	*Elasticity of substitution between commodities in household consumption.*
	Set to 0.5 (Robson, 2012)
$\epsilon_{dr}^{investc}$	*Elasticity of substitution between commodities in investment consumption.*
	Set to 0.5
$\epsilon_{dr,i}^{natcap}$	*Elasticity of natural capital substitution.*
	Set to 0.4 for forestry and logging and to 20 for other agriculture industries (Rae and Strutt, 2011)
$\epsilon_{dr,c}^{regcom}$	*Elasticity of substitution between commodities from different regions.*
	Derived from Robson (2012)
γ_{dr}^{cc}	*Scale parameter for the CES function for composite capital.*
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma_{dr,i}^{cominput}$	*Scale parameter for the CES function for composite intermediate inputs.*
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma_{dr,c}^{commd}$	*Scale parameter for the CES function for composite domestic and imported commodities.*
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma_{dr,c}^{commregd}$	*Scale parameter for the CES function for composite domestic commodities.*
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma_{dr,i}^{fact}$	*Scale parameter for the CES function for composite factors.*
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma_{dr,i}^{fi}$	*Scale parameter for the CES function for composite factors and intermediate inputs.*

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Table A.7 (continued)

Name	Description
	Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma^{govtc}_{g,dr}$	*Scale parameter for the CES function for composite government consumption.* Calculated from base year social accounting matrix given assumed elasticity of substitution
γ^{hhldc}_{dr}	*Scale parameter for the CES function for composite household consumption.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma^{investc}_{dr}$	*Scale parameter for the CES function for composite investment consumption.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\gamma^{natcap}_{dr,i}$	*Scale parameter for the CES function for composite natural capital.* Calculated from base year social accounting matrix given assumed elasticity of substitution
$\psi^{com}_{sr,c}$	*Elasticity of transformation between export and domestic commodity supply.* For each commodity, elasticities are assumed to be similar to that assigned for substitution between imports and domestic goods, i.e. $\epsilon^{com}_{dr,c}$
$\psi^{comsup}_{sr,i}$	*Elasticity of transformation in the supply of different types of commodities by industries.* Set to 0.8
ψ^{lab}_{sr}	*Elasticity of transformation in the supply of labour to different regions.* Set to 0.6
$\psi^{natcap}_{dr,nct}$	*Elasticity of transformation in the supply of natural capital to different industries.* Set to 0.4 (informed by Rae and Strutt (2011))
$\psi^{regcom}_{sr,c}$	*Elasticity of transformation in the supply of commodities to different regions.* For each commodity, set equal to the elasticity of substitution between different region's goods, $\epsilon^{regcom}_{dr,c}$
$\theta^{commregs}_{sr,c}$	*Scale parameter for the CET function for supply of commodities to different regions.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\theta^{commsdexp}_{sr,c}$	*Scale parameter for the CET function for supply of commodities to either the domestic or export market.*

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Table A.7 (continued)

Name	Description
	Calculated from base year social accounting matrix given assumed elasticity of transformation
$\theta^{comsup}_{sr,i}$	*Scale parameter for the CET function for the supply of different types of commodities by industries.* Calculated from base year social accounting matrix given assumed elasticity of transformation
θ^{lab}_{sr}	*Scale parameter for the CET function for the supply of labour to different regions.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\theta^{natcap}_{dr,nct}$	*Scale parameter for the CET function for the supply of natural capital to different industries.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\xi^{commregs}_{sr,dr,c}$	*Share parameter for regions in the CET function for the supply of domestic commodities to different New Zealand regions.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\xi^{commsdom}_{sr,c}$	*Share parameter for the domestic market in the CET function for the supply of commodities to the domestic or export market.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\xi^{commsexp}_{sr,c}$	*Share parameter for the export market in the CET function for the supply of commodities to the domestic or export market.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\xi^{comsup}_{sr,i,c}$	*Share parameter for commodities in the CET function for the supply of different types of commodities by industries.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\xi^{lab}_{sr,dr}$	*Share parameter for regions in the CET function for the supply of labour to different regions.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\xi^{natcap}_{dr,i,nct}$	*Share parameter for industries in the CET function for the supply of natural capital to different industries.* Calculated from base year social accounting matrix given assumed elasticity of transformation
$\tau_{casurplus}$	*Time to adjust perceived current account surplus to actual current account surplus.* Derived from model calibration

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Table A.7 (continued)

Name	Description
τ_{income}	*Time to adjust recognised income to actual income.* Derived from model calibration
$\tau_{industry}$	*Time to adjust desired industry production to value of industry demand.* Derived from model calibration
$\tau_{interest}$	*Time to adjust interest rate to desired interest rate.* Derived from model calibration
τ_{prices}	*Time to adjust price stocks to calculated prices.* Set to the time step Δt.
τ	*Time adjustment constant for stocks that we wish to adjust almost instantaneously.* Set to the time step Δt.

Table A.8 Description of time-varying exogeneous inputs

Name	Description
$ACINFLATIONRT(t)$	*Actual inflation rate* Origin
$ACRWSAVINGS(t)$	*Actual rest of world savings.* Origin
$ACTUALINTERESTRT(t)$	*Actual interest rate.* Origin
$ADDHHLDTRAVEL_{dr,c}(t)$	*Net additional household consumption of transport-related commodities.* Origin
$DMARGINSHOCKCOEF_{sr,dr,c}(t)$	*Net additional domestic transport margins per unit of commodity.* Origin
$EMARGINSHOCKCOEF_{sr,c,m}(t)$	*Net additional export transport margins per unit of commodity.* Origin
$IMARGINSHOCKCOEF_{dr,c,m}(t)$	*Net additional import transport margins per unit of commodity.* Origin
$MFPADJUST_{dr,i}(t)$	*Adjustment to multifactor productivity.* Origin
$NATURALGDP(t)$	*Natural gross domestic product.* Origin
$NEWLABOUR_{sr}(t)$	*New labour.*

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Table A.8 (continued)

Name	Description
	Origin
$OPERABILITY_{sr,i}(t)$	*Industry operability index.* Origin
$PCOMMWORLD_{c}(t)$	*World commodity price.* Origin
$REGSHEXPMAR_{sr,dr,m}(t)$	*Regional supply share of export margins.* Origin
$REGSHIMPMAR_{sr,dr,m}(t)$	*Regional supply share of import margins.* Origin
$WORLDGDPINDX(t)$	*World gross domestic product index.* Origin
$WORLDINTERESTRT(t)$	*World interest rate.* Origin

B Model equations

B.1 Household module equations

B.1.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Rhhldincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualhhldincome_{dr} - \mathbf{Rhhldincome}_{dr}\right) \tag{B.1}$$

$$\frac{d}{dt}\left(\mathbf{Phhldcc}_{dr}\right) = \frac{1}{\tau_{prices}}\left(actualphhldcc_{dr} - \mathbf{Phhldcc}_{dr}\right) \tag{B.2}$$

B.1.2 Auxiliaries

$$\begin{aligned} actualhhldincome_{dr} &= capincomehhld_{dr} + entincomehhld_{dr} + labincomesupply_{sr\to dr} + rwhhldtrans_{dr} \\ &+ \sum_{g}\left(govthhldtrans_{g,dr}\right) + hhldregtransout_{DReg1\leftrightarrow DReg2} - hhlddirecttax_{dr} \end{aligned} \tag{B.3}$$

$$capincomehhld_{dr} = caplocalhhldtrans_{dr} + capreghhldtrans_{DReg1\leftrightarrow DReg2} \tag{B.4}$$

$$entincomehhld_{dr} = enthhldtrans_{dr} + entreghhldtrans_{DReg1\leftrightarrow DReg2} \tag{B.5}$$

$$labincomesupply_{sr} = \sum_{dr}\left(\frac{labincomedregion_{dr}\, reglaboursupply_{sr,dr}}{\sum_{sr}\left(reglaboursupply_{sr,dr}\right)}\right) \tag{B.6}$$

$$labincomedregion_{dr} = \sum_{i}\left(factorsu_{h=LAB,dr,i}\left(1 - RWFACTRT_{h=LAB,dr}\right) preglabour_{dr}\right) \tag{B.7}$$

$$\begin{aligned} hhldrwtrans_{dr} &= HHLDRWTRANSBS_{dr}\, WORLDGDPINDX(t)\left(1 - FCSHHHLDRWTRANS_{dr}\right) \\ & HHLDRWTRANSBS_{dr}\, WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHHHLDRWTRANS_{dr} \end{aligned} \tag{B.8}$$

$$\begin{aligned} rwhhldtrans_{dr} &= RWHHLDTRANSBS_{dr}\, WORLDGDPINDX(t)\left(1 - FCSHRWHHLDTRANS_{dr}\right) \\ & RWHHLDTRANSBS_{dr}\, WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHRWHHLDTRANS_{dr} \end{aligned} \tag{B.9}$$

$$
\begin{aligned}
hhldenttrans_{dr} = &(\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\
&\times HHLDENTTRANSRT_{dr}
\end{aligned} \tag{B.10}
$$

$$
\begin{aligned}
hhldgovttrans_{g,dr} = &(\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\
&\times HHLDGOVTTRANSRT_{g,dr}
\end{aligned} \tag{B.11}
$$

$$
\begin{aligned}
hhldregtransout_{dr} = &(\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\
&\times HHLDREGTRANSRT_{dr}
\end{aligned} \tag{B.12}
$$

$$
\begin{aligned}
hhldsavings_{dr} = &\Bigg[\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - hhldregtransout_{dr} - hhldenttrans_{dr} \\
&- \sum_{g}(hhldgovttrans_{dr,g}) - addtravelcosts_{dr}\Bigg] (1 - hhldconsumprt_{dr})
\end{aligned} \tag{B.13}
$$

$$
\begin{aligned}
totalhhldconsump_{dr} = &\Bigg[\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - hhldregtransout_{dr} - hhldenttrans_{dr} \\
&- \sum_{g}(hhldgovttrans_{dr,g}) - addtravelcosts_{dr}\Bigg] hhldconsumprt_{dr}
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
hhldconsumprt_{dr} = &\left[\left(\frac{realinterestrt}{BASEREALINTERESTRT} - 1\right) CIRELASTICITY_{dr} + 1\right] \\
&\times BASECONSUMPRT_{dr}
\end{aligned} \tag{B.15}
$$

$$
actualphhldcc_{dr} = \frac{\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, hhldconsump_{dr,c}\right)}{qhhldcc_{dr}} \tag{B.16}
$$

$$
qhhldcc_{dr} = \gamma_{dr}^{hhldc}\left[\sum_{c}\left(\delta_{dr,c}^{hhldc}\left(hhldconsump_{dr,c}\right)^{\eta_{dr}^{hhldc}}\right)\right]^{\frac{1}{\eta_{dr}^{hhldc}}} \tag{B.17}
$$

$$
hhldconsump_{dr,c} = \left[(\gamma_{dr}^{hhldc})^{\eta_{dr}^{hhldc}}\, \delta_{dr,c}^{hhldc}\, \frac{\mathbf{Phhldcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta_{dr}^{hhldc}}} hhldcompcommd_{dr} \tag{B.18}
$$

$$
\eta_{dr}^{hhldc} = \frac{\epsilon_{dr}^{hhldc} - 1}{\epsilon_{dr}^{hhldc}} \tag{B.19}
$$

$$
hhldcompcommd_{dr} = \frac{totalhhldconsump_{dr} - hhldindirecttax_{dr}}{\mathbf{Phhldcc}_{dr}} \tag{B.20}
$$

$$
hhldindirecttax_{dr} = totalhhldconsump_{dr}\, HHLDINDTAXRT_{dr} \tag{B.21}
$$

$$
hhlddirecttax_{dr} = (capincomehhld_{dr} + labincomesupply_{sr\rightarrow dr}) \times TAXHHLD_{dr} \tag{B.22}
$$

B.2 Government module equations

B.2.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Rgovtincome}_{g,dr}\right) = \frac{1}{\tau_{income}}\left(govtincome_{g,dr} - \mathbf{Rgovtincome}_{g,dr}\right) \tag{B.23}$$

$$\frac{d}{dt}\left(\mathbf{Pgovtcc}_{g,dr}\right) = \frac{1}{\tau_{prices}}\left(actualpgovtcc_{g,dr} - \mathbf{Pgovtcc}_{g,dr}\right) \tag{B.24}$$

B.2.2 Auxiliaries

$$\begin{aligned} govtincome_{g,dr} = & directtaxincome_{g,dr} + indirecttaxincome_{g,dr} + capgovttrans_{g,dr} + entgovttrans_{g,dr} \\ & + hhldgovttrans_{g,dr} + betwgovttransout_{CentralG \leftrightarrow LocalG,dr} - govtdirecttax_{g,dr} \end{aligned} \tag{B.25}$$

$$\begin{aligned} directtaxincome_{g,dr} = & \left[entdirecttax_{dr} + hhlddirecttax_{dr} + rwdirecttax_{dr} + \sum_{g}\left(govtdirecttax_{g,dr}\right)\right] \\ & \times DIRECTTAXSH_{g,dr} \end{aligned} \tag{B.26}$$

$$\begin{aligned} indirecttaxincome_{g,dr} = & \Bigg[investindirecttax_{dr} + rwindirecttax_{dr} + hhldindirecttax_{dr} \\ & \sum_{i}(indindirecttax_{dr,i}) + \sum_{g}(govtindirecttax_{dr,g})\Bigg] \times INDIRECTTAXSH_{g,dr} \end{aligned} \tag{B.27}$$

$$govtdirecttax_{g,dr} = \left[capgovttrans_{g,dr} + entgovttrans_{g,dr}\right] \times GOVTDIRECTTAXRT_{g,dr} \tag{B.28}$$

$$\begin{aligned} govtrwtrans_{g,dr} = & \operatorname{sgn}\left(\mathbf{Casurplus}\right)\left|\mathbf{Casurplus}\right|^{EGOVTTRANS}\left(\frac{GOVTRWTRANSBS_{g,dr}}{\sum_{g}\sum_{dr}\left(GOVTRWTRANSBS_{g,dr}\right)}\right) \\ & + GOVTRWTRANSBS_{g,dr} \end{aligned} \tag{B.29}$$

$$betwgovttransout_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times BTWGOVTTRANSRT_{g,dr} \tag{B.30}$$

$$govthhldtrans_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTHHLDTRANSRT_{g,dr} \tag{B.31}$$

$$govtsavings_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTSAVRT_{g,dr} \tag{B.32}$$

$$totalgovtconsump_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTCONSUMPRT_{g,dr} \tag{B.33}$$

$$actualpgovtcc_{g,dr} = \frac{\sum_c \left(govtconsump_{g,dr,c}\, \mathbf{Pcompcommd}_{dr,c}\right)}{qgovtcc_{g,dr}} \tag{B.34}$$

$$qgovtcc_{g,dr} = \gamma^{govtc}_{g,dr} \left[\sum_c \left(\delta^{govtc}_{g,dr,c} \left(govtconsump_{g,dr,c}\right)^{\eta^{govtc}_{g,dr}}\right)\right]^{\frac{1}{\eta^{govtc}_{g,dr}}} \tag{B.35}$$

$$govtconsump_{g,dr,c} = \left[(\gamma^{govtc}_{g,dr})^{\eta^{govtc}_{g,dr}}\ \delta^{govtc}_{g,dr,c} \frac{\mathbf{Pgovtcc}_{g,dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta^{govtc}_{g,dr}}} govtcompcommd_{g,dr} \tag{B.36}$$

$$\eta^{govtc}_{g,dr} = \frac{\epsilon^{govtc}_{g,dr} - 1}{\epsilon^{govtc}_{g,dr}} \tag{B.37}$$

$$govtcompcommd_{g,dr} = \frac{totalgovtconsump_{g,dr} - govtindirecttax_{g,dr}}{\mathbf{Pgovtcc}_{g,dr}} \tag{B.38}$$

$$govtindirecttax_{g,dr} = totalgovtconsump_{g,dr}\, GOVTINDIRECTTAXRT_{g,dr} \tag{B.39}$$

B.3 Enterprise module equations

B.3.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Renterincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualenterincome_{dr} - \mathbf{Renterincome}_{dr}\right) \tag{B.40}$$

B.3.2 Auxiliaries

$$\begin{aligned} actualenterincome_{dr} =& capentertrans_{dr} + hhldenttrans_{dr} + rwenttrans_{dr} \\ &+ entregtransout_{DReg1 \leftrightarrow DReg2} - entdirecttax_{dr} \end{aligned} \tag{B.41}$$

$$entdirecttax_{dr} = (capentertrans_{dr} + rwenttrans_{dr}) \times ENTTAXRT_{dr} \tag{B.42}$$

$$\begin{aligned} rwenttrans_{dr} =& RWENTTRANSBS_{dr}\, WORLDGDPINDX(t)(1 - FCSHRWENTTRANS_{dr}) \\ & RWENTTRANSBS_{dr}\, WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHRWENTTRANS_{dr} \end{aligned} \tag{B.43}$$

$$\begin{aligned} entrwtrans_{dr} =& ERWTRANSBS_{dr}\, WORLDGDPINDX(t)(1 - FCSHENTRWTRANS) \\ & ERWTRANSBS_{dr}\, WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHENTRWTRANS \end{aligned} \tag{B.44}$$

$$entregtransout_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EREGTRANSRT_{dr} \tag{B.45}$$

$$entgovttrans_{g,dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EGOVTTRANSRT_{g,dr} \tag{B.46}$$

$$enthhldtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EHHLDTRANSRT_{dr} \tag{B.47}$$

$$entreghhldtrans_{dr} = [\mathbf{Renterincome}_{dr} - entrwtrans_{dr}] \times ERHHLDTRANSRT_{dr} \tag{B.48}$$

$$entsavtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times ESAVTRANSRT_{dr} \tag{B.49}$$

B.4 Industry module equations

B.4.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Desiredprod}_{dr,i}\right) = \frac{1}{\tau_{industry}}\left(vinddemand_{sr\rightarrow dr,i} - \mathbf{Desiredprod}_{dr,i}\right) \tag{B.50}$$

$$\frac{d}{dt}\left(\mathbf{Industryaccount}_{dr,i}\right) = industryinc_{dr,i} - indexpendu_{dr,i} \tag{B.51}$$

$$\frac{d}{dt}\left(\mathbf{Industrybalance}_{dr,i}\right) = \frac{1}{\tau}\left(\left(industryinc_{dr,i} - indexpendu_{dr,i}\right) - \mathbf{Industrybalance}_{dr,i}\right) \tag{B.52}$$

B.4.2 Auxiliaries

$$vinddemand_{sr,i} = \sum_{c}\left(supcoeff_{sr,i,c}\, vcomdemand_{sr,c}\right) \tag{B.53}$$

$$\begin{aligned} vcomdemand_{sr,c} = &\sum_{dr}\left(pregdomcomminclmargin_{sr,dr,c}\, regdomcommd_{sr,dr,c}\right) \\ &+ expcommoditys_{sr,c}\, actualpexports_{sr,c}\, \mathbf{Exchangert} \end{aligned} \tag{B.54}$$

$$actualprod_{dr,i} = \begin{cases} \mathbf{Desiredprod}_{dr,i} & \text{if} \quad OPERABILITY_{sr\rightarrow dr,i}(t) = 1 \\ \min(prod_{dr,i}, \mathbf{Desiredprod}_{dr,i}) & \text{if} \quad OPERABILITY_{sr\rightarrow dr,i}(t) < 1 \end{cases} \tag{B.55}$$

$$indexpendu_{dr,i} = compfactoru_{dr,i}\frac{\mathbf{Multifactprod}_{dr,i}}{factinputshare_{dr,i}}\, unitcost_{dr,i} \tag{B.56}$$

$$industryinc_{dr,i} = \min(potentialsales_{sr\rightarrow dr,i}, vinddemand_{sr\rightarrow dr,i}) \tag{B.57}$$

$$potentialsales_{sr,i} = \sum_{c}\left(indcommoditys_{sr,i,c}\, \mathbf{Pcompcomms}_{sr,c}\right) \tag{B.58}$$

$$compfactord_{dr,i} = \frac{\mathbf{Desiredprod}_{dr,i}}{unitcost_{dr,i}}\frac{factinputshare_{dr,i}}{\mathbf{Multifactprod}_{dr,i}} \tag{B.59}$$

$$effectcompfactord_{dr,i} = \frac{actualprod_{dr,i}}{unitcost_{dr,i}}\frac{factinputshare_{dr,i}}{\mathbf{Multifactprod}_{dr,i}} \tag{B.60}$$

$$unitcost_{dr,i} = interinputunitcost_{dr,i} + factinputunitcost_{dr,i} \tag{B.61}$$

$$\begin{aligned} interinputunitcost_{dr,i} = &\left(interinputshare_{dr,i}\, \mathbf{Pintinputs}_{dr,i}\left(1 + INDINDIRECTTAXRT_{dr,i}\right)\right) \\ &\times \left(\frac{1}{\mathbf{Multifactprod}_{dr,i}}\right) \end{aligned} \tag{B.62}$$

$$factinputunitcost_{dr,i} = (factinputshare_{dr,i}\,\mathbf{Pcfact}_{dr,i}) \times \left(\frac{1}{\mathbf{Multifactprod}_{dr,i}}\right) \tag{B.63}$$

$$indindirecttax_{dr,i} = \left[\sum_{c}\left(indconsump_{dr,i,c}\mathbf{Pcompdomcommd}_{dr,c}\right)\right] \times INDINDIRECTTAXRT_{dr,i} \tag{B.64}$$

B.5 Commodities module equations

B.5.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Estimports}_{dr,c}\right) = \left(importdemand_{dr,c} - \mathbf{Estimports}_{dr,c}\right) \tag{B.65}$$

$$\frac{d}{dt}\left(\mathbf{Pcindustrys}_{sr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcindustrys_{sr,i} - \mathbf{Pcindustrys}_{sr,i}\right) \tag{B.66}$$

$$\frac{d}{dt}\left(\mathbf{Pcompcommd}_{dr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpccd_{dr,c} - \mathbf{Pcompcommd}_{dr,c}\right) \tag{B.67}$$

$$\frac{d}{dt}\left(\mathbf{Pcompcomms}_{sr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpccs_{sr,c} - \mathbf{Pcompcomms}_{sr,c}\right) \tag{B.68}$$

$$\frac{d}{dt}\left(\mathbf{Pcompdomcommd}_{dr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpcdcd_{dr,c} - \mathbf{Pcompdomcommd}_{dr,c}\right) \tag{B.69}$$

$$\frac{d}{dt}\left(\mathbf{Pcompdomcomms}_{sr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpcdcs_{sr,c} - \mathbf{Pcompdomcomms}_{sr,c}\right) \tag{B.70}$$

$$\frac{d}{dt}\left(\mathbf{Pexpcomm}_{sr,c}\right) = \left(\left(\frac{1}{exportratio_{sr,c}}\right)^{\alpha^{pexpcomm}} - 1\right)\mathbf{Pexpcomm}_{sr,c} \tag{B.71}$$

$$\frac{d}{dt}\left(\mathbf{Pfiinputs}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpfiinputs_{dr,i} - \mathbf{Pfiinputs}_{dr,i}\right) \tag{B.72}$$

$$\frac{d}{dt}\left(\mathbf{Pintinputs}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpintinputs_{dr,i} - \mathbf{Pintinputs}_{dr,i}\right) \tag{B.73}$$

$$\frac{d}{dt}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\right) = \left(\left(\frac{1}{excessproduction_{sr,dr,c}}\right)^{\alpha^{pregdommcomm}} - 1\right)\mathbf{Pregdomcomm}_{sr,dr,c} \tag{B.74}$$

B.5.2 Auxiliaries

$$exportratio_{sr,c} = \frac{expcommoditys_{sr,c}}{expcommodityd_{sr,c}} \tag{B.75}$$

$$expcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsexp}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{pexpcommnz_{sr,c}} \right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c} \tag{B.76}$$

$$pexpcommnz_{sr,c} = \mathbf{Pexpcomm}_{sr,c} \left(\frac{1}{\mathbf{Exchangert}} \right) \tag{B.77}$$

$$regcommoditys_{sr,c} = \sum_i (indcommoditys_{sr,i,c}) \tag{B.78}$$

$$indcommoditys_{sr,i,c} = \left[(\theta^{comsup}_{sr,i})^{\phi^{comsup}_{sr,i}} \xi^{comsup}_{sr,i,c} \frac{\mathbf{Pcindustrys}_{sr,i}}{\mathbf{Pcompcomms}_{sr,c}} \right]^{\frac{1}{1-\phi^{comsup}_{sr,i}}} regindprodincltax_{sr,i} \tag{B.79}$$

$$supcoeff_{sr,i,c} = \frac{indcommoditys_{sr,i,c}}{\sum_i (indcommoditys_{sr,i,c})} \tag{B.80}$$

$$regindprodincltax_{sr,i} = \frac{effectcompfactoru_{dr \to sr,i}}{factinputshare_{dr \to sr,i}} PRODSCALAR_{dr \to sr,i} \mathbf{Multifactprod}_{dr \to sr,i} \tag{B.81}$$

$$\begin{aligned} expcommodityd_{sr,c} =& BASEEXPORTS_{dr \to sr,c} \left(\frac{PCOMMWORLD_c(t)}{pexportcommd_{sr,c}} \right)^{EXPORTP_c} \\ & \times WORLDGDPINDX(t) \end{aligned} \tag{B.82}$$

$$pexportcommd_{sr,c} = \mathbf{Pexpcomm}_{sr,c} + pexportmargins_{sr,c} \mathbf{Exchangert} \tag{B.83}$$

$$excessproduction_{sr,dr,c} = \frac{regcdomcomms_{sr,dr,c}}{regdomcommd_{sr,dr,c}} \tag{B.84}$$

$$\begin{aligned} regcdomcomms_{sr,dr,c} =& \left[(\theta^{commregs}_{sr,c})^{\phi^{regcom}_{sr,c}} \xi^{commregs}_{sr,dr,c} \frac{\mathbf{Pcompdomcomms}_{sr,c}}{\mathbf{Pregdomcomm}_{sr,dr,c}} \right]^{\frac{1}{1-\phi^{regcom}_{sr,c}}} \\ & \times domcommoditys_{sr,c} \end{aligned} \tag{B.85}$$

$$\begin{aligned} regdomcommd_{sr,dr,c} =& \left[(\gamma^{commregd}_{dr,c})^{\eta^{regcom}_{dr,c}} \delta^{commregd}_{sr,dr,c} \frac{\mathbf{Pcompdomcommd}_{dr,c}}{pregdomcomminclmargin_{sr,dr,c}} \right]^{\frac{1}{1-\eta^{regcom}_{dr,c}}} \\ & \times domcomdemand_{dr,c} \end{aligned} \tag{B.86}$$

$$domcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsdom}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{\mathbf{Pregdomcomm}_{sr,c}} \right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c} \tag{B.87}$$

$$importdemand_{dr,c} = \left[(\gamma^{commd}_{dr,c})^{\eta^{com}_{dr,c}} \delta^{commdimp}_{dr,c} \frac{\mathbf{Pcompcommd}_{dr,c}}{pimpcommnz_{dr,c}} \right]^{\frac{1}{1-\eta^{com}_{dr,c}}} totalcomdemand_{dr,c} \tag{B.88}$$

$$\begin{aligned} totalcomdemand_{dr,c} = &\sum_g (govtconsump_{g,dr,c}) + hhldconsump_{dr,c} + investconsumpq_{dr,c} \\ &+ \sum_i (indconsump_{dr,i,c}) + marginconsumpq_{dr,c} + ADDHHLDTRAVEL_{dr,c}(t) \end{aligned} \tag{B.89}$$

$$indconsump_{dr,i,c} = effectcompfactoru_{dr,i} \frac{interinputshare_{dr,i}}{factinputshare_{dr,i}} intinputcoeff_{dr,i,c} \tag{B.90}$$

$$totalindconsump_{dr,c} = \sum_i (indconsump_{dr,i,c}) \tag{B.91}$$

$$pimpcommnz_{dr,c} = \frac{PCOMMWORLD_c(t)}{\mathbf{Exchangert}} + pimportmargins_{dr,c} \tag{B.92}$$

$$actualpccd_{dr,c} = \frac{pimpcommnz_{dr,c}\, importdemand_{dr,c} + \mathbf{Pcompdomcommd}_{dr,c}\, domcomdemand_{dr,c}}{qcompcommd_{dr,c}} \tag{B.93}$$

$$\begin{aligned} qcompcommd_{dr,c} = \gamma^{commd}_{dr,c} &\left[\delta^{commdimp}_{dr,c} (importdemand_{dr,c})^{\eta^{com}_{dr,c}} \right. \\ &\left. + \delta^{commddom}_{dr,c} (domcomdemand_{dr,c})^{\eta^{com}_{dr,c}} \right]^{\frac{1}{\eta^{com}_{dr,c}}} \end{aligned} \tag{B.94}$$

$$domcomdemand_{dr,c} = \left[(\gamma^{commd}_{dr,c})^{\eta^{com}_{dr,c}} \delta^{commddom}_{dr,c} \frac{\mathbf{Pcompcommd}_{dr,c}}{\mathbf{Pcompdomcommd}_{dr,c}} \right]^{\frac{1}{1-\eta^{com}_{dr,c}}} totalcomdemand_{dr,c} \tag{B.95}$$

$$actualpccs_{sr,c} = \frac{pexpcommnz_{sr,c}\, expcommoditys_{sr,c} + \mathbf{Pcompdomcomms}_{sr,c}\, domcommoditys_{sr,c}}{qcompcomms_{sr,c}} \tag{B.96}$$

$$\begin{aligned} qcompcomms_{sr,c} = \theta^{commsdexp}_{sr,c} &\left[\xi^{commsexp}_{sr,c} (expcommoditys_{sr,c})^{\phi^{com}_{sr,c}} \right. \\ &\left. + \xi^{commsdom}_{sr,c} (domcommoditys_{sr,c})^{\phi^{com}_{sr,c}} \right]^{\frac{1}{\phi^{com}_{sr,c}}} \end{aligned} \tag{B.97}$$

$$actualpcdcd_{dr,c} = \frac{\sum_{sr} (pregdomcomminclmargin_{sr,dr,c}\, regdomcommd_{sr,dr,c})}{qdomcommd_{dr,c}} \tag{B.98}$$

$$qdomcommd_{dr,c} = \gamma^{commregd}_{dr,c} \left[\sum_{sr} \left(\delta^{commregd}_{sr,dr,c} (regdomcommd_{sr,dr,c})^{\eta^{regcom}_{dr,c}} \right) \right]^{\frac{1}{\eta^{regcom}_{dr,c}}} \tag{B.99}$$

$$pregdomcomminclmargin_{sr,dr,c} = DMARGINSHOCKCOEF_{sr,dr,c}\, pimproadmargins_{sr,dr} + \mathbf{Pregdomcomm}_{sr,dr,c} \tag{B.100}$$

$$actualpcdcs_{sr,c} = \frac{\sum_{dr}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\, regcdomcomms_{sr,dr,c}\right)}{qdomcomms_{sr,c}} \tag{B.101}$$

$$qdomcomms_{sr,c} = \theta^{commregs}_{sr,c}\left[\sum_{dr}\left(\xi^{commregs}_{sr,dr,c}(regcdomcomms_{sr,dr,c})^{\phi^{regcom}_{sr,c}}\right)\right]^{\frac{1}{\phi^{regcom}_{sr,c}}} \tag{B.102}$$

$$actualpcindustrys_{sr,i} = \frac{\sum_{dr}\left(indcommoditys_{sr,i,c}\,\mathbf{Pcompcomms}_{sr,i}\right)}{qcindustrys_{sr,i}} \tag{B.103}$$

$$qcindustrys_{sr,i} = \theta^{commsup}_{sr,i}\left[\sum_{c}\left(\xi^{commsup}_{sr,i,c}(indcommoditys_{sr,i,c})^{\phi^{comsup}_{sr,i}}\right)\right]^{\frac{1}{\phi^{comsup}_{sr,i}}} \tag{B.104}$$

$$actualpfiinputs_{dr,i} = \frac{factinputshare_{dr,i}\,\mathbf{Pcfact}_{dr,i} + interinputshare_{dr,i}\,\mathbf{Pintinputs}_{dr,i}}{qfiinputs_{dr,i}} \tag{B.105}$$

$$qfiinputs_{dr,i} = \gamma^{fi}_{dr,i}\left[\delta^{fi}_{input=FactI,dr,i}(factinputshare_{dr,i})^{\eta^{fi}_{dr,i}} + \delta^{fi}_{input=InterI,dr,i}(interinputshare_{dr,i})^{\eta^{fi}_{dr,i}}\right]^{\frac{1}{\eta^{fi}_{dr,i}}} \tag{B.106}$$

$$factinputshare_{dr,i} = \left[(\gamma^{fi}_{dr,i})^{\eta^{fi}_{dr,i}}\delta^{fi}_{input=FactsI,dr,i}\frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pcfact}_{dr,i}}\right]^{\frac{1}{1-\eta^{fi}_{dr,i}}} \tag{B.107}$$

$$interinputshare_{dr,i} = \left[(\gamma^{fi}_{dr,i})^{\eta^{fi}_{dr,i}}\delta^{fi}_{input=InterI,dr,i}\frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pintinputs}_{dr,i}}\right]^{\frac{1}{1-\eta^{fi}_{dr,i}}} \tag{B.108}$$

$$actualpintinputs_{dr,i} = \frac{\sum_{c}\left(intinputcoeff_{dr,i,c}\,\mathbf{Pcompcommd}_{dr,c}\right)}{qintinputs_{dr,i}} \tag{B.109}$$

$$qintinputs_{dr,i} = \gamma^{cominput}_{dr,i}\left[\sum_{c}(\delta^{cominput}_{dr,i,c}\left(intinputcoeff_{dr,i,c})^{\eta^{cominput}_{dr,i}}\right)\right]^{\frac{1}{\eta^{cominput}_{dr,i}}} \tag{B.110}$$

$$intinputcoeff_{dr,i,c} = \left[(\gamma^{cominput}_{dr,i})^{\eta^{cominput}_{dr,i}}\delta^{cominput}_{dr,i,c}\frac{\mathbf{Pintinputs}_{dr,i}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta^{cominput}_{dr,i}}} \tag{B.111}$$

$$\eta^{com}_{dr,c} = \frac{\epsilon^{comsub}_{dr,c} - 1}{\epsilon^{comsub}_{dr,c}} \tag{B.112}$$

$$\eta^{cominput}_{dr,i} = \frac{\epsilon^{cominput}_{dr,i} - 1}{\epsilon^{cominput}_{dr,i}} \tag{B.113}$$

$$\eta^{fi}_{dr,i} = \frac{\epsilon^{fi}_{dr,i} - 1}{\epsilon^{fi}_{dr,i}} \tag{B.114}$$

$$\eta^{regcom}_{dr,c} = \frac{\epsilon^{regcom}_{dr,c} - 1}{\epsilon^{regcom}_{dr,c}} \tag{B.115}$$

$$\phi^{comsup}_{sr,i} = \frac{\psi^{comsup}_{sr,i} + 1}{\psi^{comsup}_{sr,i}} \tag{B.116}$$

$$\phi^{com}_{sr,c} = \frac{\psi^{com}_{sr,c} + 1}{\psi^{com}_{sr,c}} \tag{B.117}$$

$$\phi^{regcom}_{sr,c} = \frac{\psi^{regcom}_{sr,c} + 1}{\psi^{regcom}_{sr,c}} \tag{B.118}$$

B.6 Factors module equations

B.6.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Multifactprod}_{dr,i}\right) = \left(MFPGRRT_{dr,i}\left(MFPADJUST_{dr,i}(t) + 1\right)\right)\mathbf{Multifactprod}_{dr,i} \tag{B.119}$$

$$\frac{d}{dt}\left(\mathbf{Pcfact}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcfact_{dr,i} - \mathbf{Pcfact}_{dr,i}\right) \tag{B.120}$$

$$\begin{aligned} \frac{d}{dt}\left(\mathbf{Pfact}_{h=CAP,dr,i}\right) &= \frac{1}{\tau_{prices}}\left(actualpcapital_{dr,i} - \mathbf{Pfact}_{h=CAP,dr,i}\right) \\ \frac{d}{dt}\left(\mathbf{Pfact}_{h=LAB,dr,i}\right) &= \left(\left(\frac{1}{labratio_{dr,i}}\right)^{\alpha^{plab}} - 1\right)\mathbf{Pfact}_{h=LAB,dr,i} \end{aligned} \tag{B.121}$$

B.6.2 Auxiliaries

$$actualpcfact_{dr,i} = \frac{\sum_h \left(factorsd_{h,dr,i}\,\mathbf{Pfact}_{h,dr,i}\right)}{qcompfactd_{dr,i}} \tag{B.122}$$

$$qcompfactd_{dr,i} = \gamma^{fact}_{dr,i}\left[\sum_h \left(\delta^{fact}_{h,dr,i}\left(factorsd_{h,dr,i}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}} \tag{B.123}$$

$$\eta^{fact}_{dr,i} = \frac{\epsilon^{fact}_{dr,i} - 1}{\epsilon^{fact}_{dr,i}} \tag{B.124}$$

$$actualpcapital_{dr,i} = \frac{capitaltyped_{cap=BuilC,dr,i}\,\mathbf{Pbuiltcap}_{dr,i}}{qcapitald_{dr,i}} + \frac{capitaltyped_{cap=NatC,dr,i}\,\mathbf{Pcompnaturalcapd}_{dr,i}}{qcapitald_{dr,i}} \tag{B.125}$$

$$qcapitald_{dr,i} = \gamma^{cc}_{dr,i}\left[\sum_{cap}\left(\delta^{cc}_{cap,dr,i}(capitaltyped_{cap,dr,i})^{\eta^{cc}_{dr,i}}\right)\right]^{\frac{1}{\eta^{cc}_{dr,i}}} \tag{B.126}$$

$$labratio_{dr,i} = \frac{\sum_i(factorss_{h=LAB,dr,i})}{\sum_i(factorsd_{h=LAB,dr,i})} \tag{B.127}$$

$$\begin{aligned} factorss_{h=LAB,dr,i} &= indlabours_{dr,i} \\ factorss_{h=CAP,dr,i} &= ccapitals_{dr,i} \end{aligned} \tag{B.128}$$

$$factorsd_{h,dr,i} = \left[(\gamma^{fact}_{dr,i})^{\eta^{fact}_{dr,i}}\,\delta^{fact}_{h,dr,i}\frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} compfactord_{dr,i}\,(1 - RWFACTRT_{h,dr}) \tag{B.129}$$

$$factorsu_{h,dr,i} = \frac{\min(factorsd_{h,dr,i}, factorss_{h,dr,i})}{1 - RWFACTRT_{h,dr}} \tag{B.130}$$

$$compfactoru_{dr,i} = \gamma^{fact}_{dr,i}\left[\sum_h\left(\delta^{fact}_{h,dr,i}\left(\frac{factorsu_{h,dr,i}}{1 - RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}} \tag{B.131}$$

$$effectfactorsd_{h,dr,i} = \left[(\gamma^{fact}_{dr,i})^{\eta^{fact}_{dr,i}}\,\delta^{fact}_{h,dr,i}\frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} effectcompfactord_{dr,i}\,(1 - RWFACTRT_{h,dr}) \tag{B.132}$$

$$effectfactorsu_{h,dr,i} = \frac{\min(effectfactorsd_{h,dr,i}, factorss_{h,dr,i})}{1 - RWFACTRT_{h,dr}} \tag{B.133}$$

$$effectcompfactoru_{dr,i} = \gamma^{fact}_{dr,i}\left[\sum_h\left(\delta^{fact}_{h,dr,i}\left(\frac{effectfactorsu_{h,dr,i}}{1 - RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}} \tag{B.134}$$

B.7 Labour module equations

B.7.1 Stocks

$$\frac{d}{dt}(\mathbf{Labour}_{sr}) = NEWLABOUR_{sr}(t) \tag{B.135}$$

$$\frac{d}{dt}(\mathbf{Pavglabour}_{sr}) = \frac{1}{\tau_{prices}}(actualpavglabour_{sr} - \mathbf{Pavglabour}_{sr}) \tag{B.136}$$

B.7.2 Auxiliaries

$$actualpavglabour_{sr} = \frac{\sum_{dr}\left(reglabourest_{sr,dr}\,preglabour_{dr}\right)}{qlabour_{sr}} \tag{B.137}$$

$$qlabour_{sr} = \theta^{lab}_{sr}\left[\sum_{dr}\left(\xi^{lab}_{sr,dr}\left(reglabourest_{sr,dr}\right)^{\phi^{lab}_{sr}}\right)\right]^{\frac{1}{\phi^{lab}_{sr}}} \tag{B.138}$$

$$\phi^{lab}_{sr} = \frac{\psi^{lab}_{sr}+1}{\psi^{lab}_{sr}} \tag{B.139}$$

$$preglabour_{dr} = \mathbf{Pfact}_{h=LAB,dr,i=Ind1} \tag{B.140}$$

$$reglabourest_{sr,dr} = \left[\left(\theta^{lab}_{sr}\right)^{\phi^{lab}_{sr}}\xi^{lab}_{sr,dr}\frac{\mathbf{Pavglabour}_{sr}}{preglabour_{dr}}\right]^{\frac{1}{1-\phi^{lab}_{sr}}}\mathbf{Labour}_{sr} \tag{B.141}$$

$$indlabours_{dr,i} = \sum_{sr}\left(reglaboursupply_{sr,dr}\right)\frac{factorsd_{h=LAB,dr,i}}{\sum_i\left(factorsd_{h=LAB,dr,i}\right)} \tag{B.142}$$

$$reglaboursupply_{sr,dr} = reglabourest_{sr,dr}\,LSFCONVERT_{dr} \tag{B.143}$$

B.8 Capital module equations

B.8.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Builtcapital}_{dr,i}\right) = newcapital_{dr,i} - depreciation_{dr,i} \tag{B.144}$$

$$\frac{d}{dt}\left(\mathbf{Naturalcapital}_{dr,i}\right) = 0 \tag{B.145}$$

$$\frac{d}{dt}\left(\mathbf{Pbuiltcap}_{dr,i}\right) = \left(\left(\frac{1}{builtratio_{dr,i}}\right)^{\alpha^{pbuiltcap}} - 1\right)\mathbf{Pbuiltcap}_{dr,i} \tag{B.146}$$

$$\frac{d}{dt}\left(\mathbf{Pcompnaturalcapd}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcnaturalcapd_{dr,i} - \mathbf{Pcompnaturalcapd}_{dr,i}\right) \tag{B.147}$$

$$\frac{d}{dt}\left(\mathbf{Pcompnaturalcaps}_{dr,nct}\right) = \frac{1}{\tau_{prices}}\left(actualpcnaturalcaps_{dr,nct} - \mathbf{Pcompnaturalcaps}_{dr,nct}\right) \tag{B.148}$$

$$\frac{d}{dt}\left(\mathbf{Pnaturalcap}_{dr,i,nct}\right) = \left(\left(\frac{1}{naturalcapratio_{dr,i,nct}}\right)^{\alpha^{pnatcap}} - 1\right)\mathbf{Pnaturalcap}_{dr,i,nct} \tag{B.149}$$

$$\frac{d}{dt}\left(\mathbf{Rcapincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(capitalincome_{dr} - \mathbf{Rcapincome}_{dr}\right) \tag{B.150}$$

B.8.2 Auxiliaries

$$actualpcnaturalcaps_{dr,nct} = \frac{\sum_i \left(indnaturalcaps1_{dr,i,nct}\,\mathbf{Pnaturalcap}_{dr,i,nct}\right)}{\mathbf{Naturalcapital}_{dr,nct}} \tag{B.151}$$

$$indnaturalcaps1_{dr,i,nct} = \left[(\theta^{natcap}_{dr,nct})^{\phi^{natcap}_{dr,nct}}\,\xi^{natcap}_{dr,i,nct}\,\frac{\mathbf{Pcompnaturalcaps}_{dr,nct}}{\mathbf{Pnaturalcap}_{dr,i,nct}}\right]^{\frac{1}{1-\phi^{natcap}_{dr,nct}}}\,\mathbf{Naturalcapital}_{dr,nct} \tag{B.152}$$

$$\phi^{natcap}_{dr,nct} = \frac{\psi^{natcap}_{dr,nct}+1}{\psi^{natcap}_{dr,nct}} \tag{B.153}$$

$$actualpcnaturalcapd_{dr,i} = \frac{\sum_{nct}\left(naturalcapd_{dr,i,nct}\,\mathbf{Pnaturalcap}_{dr,i,nct}\right)}{actualindcompnaturalcaps_{dr,i}} \tag{B.154}$$

$$actualindcompnaturalcaps_{dr,i} = \gamma^{nattyp}_{dr,i}\left[\sum_{nct}\left(\delta^{natcap}_{dr,i,nct}\,(naturalcapd_{dr,i,nct})^{\eta^{natcap}_{dr,i}}\right)\right]^{\frac{1}{\eta^{natcap}_{dr,i}}} \tag{B.155}$$

$$\eta^{natcap}_{dr,i} = \frac{\epsilon^{natcap}_{dr,i}-1}{\epsilon^{natcap}_{dr,i}} \tag{B.156}$$

$$naturalcapd_{dr,i,nct} = \left[(\gamma^{natcap}_{dr,i})^{\eta^{natcap}_{dr,i}}\,\delta^{natcap}_{dr,i,nct}\,\frac{\mathbf{Pcompnaturalcapd}_{dr,i}}{\mathbf{Pnaturalcap}_{dr,i,nct}}\right]^{\frac{1}{1-\eta^{natcap}_{dr,i}}}\,\frac{capitaltyped_{cap=NatC,dr,i}}{NATCAPCONVERT_{dr,i}} \tag{B.157}$$

$$\begin{aligned} capitaltyped_{cap=BuilC,dr,i} &= \left[(\gamma^{cc}_{dr,i})^{\eta^{cc}_{dr,i}}\,\delta^{cc}_{cap=BuilC,dr,i}\,\frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pbuiltcap}_{dr,i}}\right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \\ &\quad\times factorsd_{h=CAP,dr,i} \\ capitaltyped_{cap=NatC,dr,i} &= \left[(\gamma^{cc}_{dr,i})^{\eta^{cc}_{dr,i}}\,\delta^{cc}_{cap=NatC,dr,i}\,\frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pcompnaturalcapd}_{cap=NatC,dr,i}}\right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \\ &\quad\times factorsd_{h=CAP,dr,i} \end{aligned} \tag{B.158}$$

$$\eta^{cc}_{dr,i} = \frac{\epsilon^{cc}_{dr,i}-1}{\epsilon^{cc}_{dr,i}} \tag{B.159}$$

$$naturalcapratio_{dr,i,nct} = \frac{indnaturalcaps_{dr,i,nct}}{naturalcaptyped_{dr,i,nct}} \tag{B.160}$$

$$indnaturalcaps_{dr,i,nct} = \frac{indnaturalcaps1_{dr,i,nct}}{\sum_i \left(indnaturalcaps1_{dr,i,nct}\right)}\mathbf{Naturalcapital}_{dr,nct} \tag{B.161}$$

$$builtratio_{dr,i} = \frac{builts_{dr,i}}{capitaltyped_{cap=BuilC,dr,i}} \tag{B.162}$$

$$builts_{dr,i} = \mathbf{Builtcapital}_{dr,i}\, KSFCONVERT_{dr,i} \tag{B.163}$$

$$ccapitals_{dr,i} = \gamma^{cc}_{dr,i}\left[\delta^{cc}_{cap=BuilC,dr,i}\left(builts_{dr,i}\right)^{\eta^{cc}_{dr,i}} + \delta^{cc}_{cap=NatC,dr,i}\left(naturalcapitalsq_{dr,i}\right)^{\eta^{cc}_{dr,i}}\right]^{\frac{1}{\eta^{cc}_{dr,i}}} \tag{B.164}$$

$$naturalcapitalsq_{dr,i} = compnaturalcaps_{dr,i}\, NATCAPCONVERT_{dr,i} \tag{B.165}$$

$$compnaturalcaps_{dr,i} = \gamma^{natcap}_{dr,i}\left[\sum_{nct}\left(\delta^{natcap}_{dr,i,nct}(indnaturalcaps_{dr,i,nct})^{\eta^{natcap}_{dr,i}}\right)\right]^{\frac{1}{\eta^{natcap}_{dr,i}}} \tag{B.166}$$

$$\begin{aligned} capitalincome_{dr} =& \frac{\sum_i \left(factorsu_{h=CAP,dr,i}\,\mathbf{Pfact}_{h=CAP,dr,i}\right)}{1 - RWFACTRT_{h=CAP,dr}} + capregtransout_{DReg1\rightarrow SReg2,DReg2\rightarrow SReg1} \\ &+ \sum_i(\mathbf{Industrybalance}_{dr,i}) \end{aligned} \tag{B.167}$$

$$capregtransout_{dr} = \mathbf{Rcapincome}_{dr} \times CREGTRANSRT_{dr} \tag{B.168}$$

$$capentertrans_{dr} = \mathbf{Rcapincome}_{dr} \times CENTTRANSRT_{dr} \tag{B.169}$$

$$capgovttrans_{g,dr} = \mathbf{Rcapincome}_{dr} \times CGOVTTRANSRT_{g,dr} \tag{B.170}$$

$$caplocalhhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CHHLDTRANSRT_{dr} \tag{B.171}$$

$$capreghhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CRHTRANSRT_{dr} \tag{B.172}$$

$$newcapital_{dr,i} = mobileinvest_{dr,i} + immobileinvest_{dr,i} \tag{B.173}$$

$$\begin{aligned} mobileinvest_{dr,i} =& \left[\frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}} + \sum_c(SETINVESTCQ_{dr,c})\right] \\ &\times MOBILESH_{dr,i}\, mobileinvestsh_{dr,i} \end{aligned} \tag{B.174}$$

$$\begin{aligned} immobileinvest_{dr,i} =& \left[\frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}} + \sum_c(SETINVESTCQ_{dr,c})\right] \\ &\times (1 - MOBILESH_{dr,i})\, capincomesh_{dr,i} \end{aligned} \tag{B.175}$$

$$mobileinvestsh_{dr,i} = \frac{mobileinvest1_{dr,i}}{\sum_i(mobileinvest1_{dr,i})}\, ALLOCATESH_{dr} + INVESTCONSTSH_{dr,i} \tag{B.176}$$

$$mobileinvest1_{dr,i} = (INVESTPARAM_{dr,i}\, netreturn_{dr,i})^{EINVEST_{dr,i}} \times capincomesh_{dr,i} \tag{B.177}$$

$$netreturn_{dr,i} = grossreturn_{dr,i} - RDEP_{dr,i} \tag{B.178}$$

$$grossreturn_{dr,i} = \frac{\mathbf{Pbuiltcap}_{dr,i}\, KSFCONVERT_{dr,i}}{\mathbf{Pinvestcc}_{dr}} \tag{B.179}$$

$$capincomesh_{dr,i} = \frac{indcapincome_{dr,i}}{\sum_i (indcapincome_{dr,i})} \tag{B.180}$$

$$indcapincome_{dr,i} = builtuse_{dr,i}\, \mathbf{Pbuiltcap}_{dr,i} \tag{B.181}$$

$$builtuse_{dr,i} = min(\mathbf{Builtcapital}_{dr,i}\, KSFCONVERT_{dr,i}\,,\, capitaltyped_{cap=BuilC,dr,i}) \tag{B.182}$$

$$depreciation_{dr,i} = \mathbf{Builtcapital}_{dr,i} \left[RDEP_{dr,i}\, (1 + DEPSHFT)\right] \tag{B.183}$$

B.9 Investment and savings module equations

B.9.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Casurplus}\right) = \frac{1}{\tau_{casurplus}}(actualcasurplus - \mathbf{Casurplus}) \tag{B.184}$$

$$\frac{d}{dt}\left(\mathbf{Interestrt}\right) = \frac{1}{\tau_{interest}}\,(targetinterestrt - \mathbf{Interestrt}) \tag{B.185}$$

$$\frac{d}{dt}\left(\mathbf{Pinvestcc}_{dr}\right) = \frac{1}{\tau_{prices}}\,(actualpinvestcc_{dr} - \mathbf{Pinvestcc}_{dr}) \tag{B.186}$$

B.9.2 Auxiliaries

$$actualcasurplus = rwexpenditure - rwincome \tag{B.187}$$

$$targetinterestrt = \begin{cases} ACTUALINTERESTRT(t) & \text{for} \quad t \leq 6 \\ desiredinterestrt & \text{for} \quad t > 6 \end{cases} \tag{B.188}$$

$$\begin{aligned} desiredinterestrt =\ & \mathbf{Inflationrt} + INTERESTCONST + INTERESTINFLW\, (\mathbf{Inflationrt} - 0.02) \\ & - INTERESTGDPW\, gdpgap \end{aligned} \tag{B.189}$$

$$gdpgap = \frac{NATURALGDP(t)}{realgdp} - 1 \tag{B.190}$$

$$realinterestrt = \mathbf{Interestrt} - \mathbf{Inflationrt} \tag{B.191}$$

$$savingstotal_{dr} = rwsavings_{dr} + regsavings_{dr} - savregtransout_{dr} \tag{B.192}$$

$$rwsavings_{dr} = rwsavingstotal\, RWREGSAVSH_{dr} \tag{B.193}$$

$$rwsavingstotal = \begin{cases} ACRWSAVINGS(t) & \text{for} \quad t \leq 6 \\ [100\, \mathbf{Interestrt} - WORLDINTERESTRT(t)] & \\ \times INTERESTWEIGHT & \text{for} \quad t > 6 \\ + WORLDGDPINDX(t)\, GDPWEIGHT + RWSAVCONST & \end{cases} \tag{B.194}$$

$$regsavings_{dr} = entsavtrans_{dr} + hhldsavings_{dr} + \sum_{g} (govtsavings_{g,dr}) + savregtransout_{DReg1 \leftrightarrow DReg2} \tag{B.195}$$

$$savregtransout_{dr} = SAVREGTRANSBS_{dr} \frac{\mathbf{Rhhldincome}_{dr}}{BASEHHLDACCOUNT_{dr}} \tag{B.196}$$

$$qinvestcc_{dr} = \gamma_{dr}^{investc} \left[\sum_{c} \left(\delta_{dr,c}^{investc} \, (disinvestconsump_{dr,c})^{\eta_{dr}^{investc}} \right) \right]^{\frac{1}{\eta_{dr}^{investc}}} \tag{B.197}$$

$$\eta_{dr}^{investc} = \frac{\epsilon_{dr}^{investc} - 1}{\epsilon_{dr}^{investc}} \tag{B.198}$$

$$disinvestconsump_{dr,c} = \left[(\gamma_{dr}^{investc})^{\eta_{dr}^{investc}} \delta_{dr,c}^{investc} \frac{\mathbf{Pinvestcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}} \right]^{\frac{1}{1-\eta_{dr}^{investc}}} \frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}} \tag{B.199}$$

$$actualpinvestcc_{dr} = \frac{\sum_{c} \left(disinvestconsump_{dr,c}\, \mathbf{Pcompcommd}_{dr,c} \right)}{qinvestcc_{dr}} \tag{B.200}$$

$$investconsumpq_{dr,c} = disinvestconsump_{dr,c} + SETINVESTCQ_{dr,c} \tag{B.201}$$

$$investindirecttax_{dr} = aggregateinvestv_{dr} \frac{INVESTINDIRECTTAXRT_{dr}}{1 - INVESTINDIRECTTAXRT_{dr}} \tag{B.202}$$

$$\begin{aligned} aggregateinvestv_{dr} = & (realinterestrt\, ALPHA_{dr} + savingstotal_{dr}\, BETA_{dr} + INVESTCONST_{dr}) \\ & \times (1 - INVESTINDIRECTTAXRT_{dr}) \end{aligned} \tag{B.203}$$

B.10 Rest of world module equations

B.10.1 Stocks

$$\frac{d}{dt} (\mathbf{Exchangert}) = \left(\left(\frac{1}{bopratio} \right)^{\alpha^{exchangert}} - 1 \right) \mathbf{Exchangert} \tag{B.204}$$

B.10.2 Auxiliaries

$$bopratio = \frac{rwincome}{rwexpenditure} \tag{B.205}$$

$$rwincome = \sum_{dr} \Bigg(rwlabourincome_{dr} + \sum_{c} \left(nzimportpurchases_{dr,c} \right) + entrwtrans_{dr} \\ + hhldrwtrans_{dr} + \sum_{g} \left(govtrwtrans_{g,dr} \right) - rwdirecttax_{dr} \Bigg) \tag{B.206}$$

$$rwlabourincome_{dr} = \sum_{i} (rwlaboursupply_{dr,i}\, preglabour_{dr}) \tag{B.207}$$

$$rwlaboursupply_{dr,i} = factorsu_{h=LAB,dr,i} \frac{RWFACTRT_{h=LAB,dr}}{1 - RWFACTRT_{h=LAB,dr}} \tag{B.208}$$

$$nzimportpurchases_{dr,c} = importdemand_{dr,c}\, pimpcommnz_{dr,c} \tag{B.209}$$

$$rwdirecttax_{dr} = (entrwtrans_{dr} + rwlabourincome_{dr})\, RWDIRECTTAXRT_{dr} \tag{B.210}$$

$$rwexpenditure = \sum_{dr} \left(rwsavings_{dr} + rwenttrans_{dr} + rwhhldtrans_{dr} + rwindirecttax_{dr} \right) \\ + \sum_{sr} \sum_{c} (nzexportsales_{sr,c}) \tag{B.211}$$

$$rwindirecttax_{dr} = \sum_{c} (rwindirecttaxdisag_{dr,c}) \tag{B.212}$$

$$rwindirecttaxdisag_{dr,c} = nzexportsales_{sr \to dr,c}\, RWINDIRECTTAXRT_{dr} \tag{B.213}$$

$$nzexportsales_{sr,c} = \frac{expcommoditys_{sr,c}\, actualpexports_{sr,c}}{\mathbf{Exchangert}} \tag{B.214}$$

$$actualpexports_{sr,c} = \frac{actualexports_{sr,c}\, \mathbf{Pexpcomm}_{sr,c}}{expcommoditys_{sr,c}} \tag{B.215}$$

$$actualexports_{sr,c} = \min \left(expcommodityd_{sr,c}, expcommoditys_{sr,c} \right) \tag{B.216}$$

B.11 Output variable module equations

B.11.1 Stocks

$$\frac{d}{dt} (\mathbf{Gdpindex}) = \frac{1}{\tau} (actualgdpindex - \mathbf{Gdpindex}) \tag{B.217}$$

$$\frac{d}{dt} (\mathbf{Inflationrt}) = \frac{1}{\tau} (targetinflationrt - \mathbf{Inflationrt}) \tag{B.218}$$

B.11.2 Auxiliaries

$$targetinflationrt = \begin{cases} ACINFLATIONRT(t) & \text{for} \quad t \leq 6 \\ desiredinflationrt & \text{for} \quad t > 6 \end{cases} \tag{B.219}$$

$$desiredinflationrt = 4\left(\frac{cpif(t) - cpif(t-0.25)}{cpif(t-0.25)}\right) \tag{B.220}$$

$$cpif = \sqrt{cpip \times cpil} \tag{B.221}$$

$$cpil = 1000\,\frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, BASEHHLDCONSUMP_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\, BASEHHLDCONSUMP_{dr,c}\right)} \tag{B.222}$$

$$cpip = 1000\frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, hhldconsump_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\, hhldconsump_{dr,c}\right)} \tag{B.223}$$

$$actualgdpindex = 1000\sqrt{gdpindexl \times gdpindexp} \tag{B.224}$$

$$\begin{aligned} gdpindexl = &\left[\sum_{dr}\sum_{c}\left(\left(BASEHHLDCONSUMP_{dr,c} + BASEINVESTCONSUMP_{dr,c}\right.\right.\right. \\ &\left.\left. + \sum_{g}\left(BASEGOVTCONSUMP_{g,dr,c}\right)\right)\mathbf{Pcompcommd}_{dr,c}\right) \\ &\left. + \sum_{dr}\sum_{c}\left(\left(BASEEXPORTS_{dr,c} - BASEIMPORTS_{dr,c}\right)\frac{PCOMMWORLD_{c}(t)}{\mathbf{Exchangert}}\right)\right] \\ &\div \\ &\left[\sum_{dr}\sum_{c}\left(\left(BASEHHLDCONSUMP_{dr,c} + BASEINVESTCONSUMP_{dr,c}\right.\right.\right. \\ &\left.\left. + \sum_{g}\left(BASEGOVTCONSUMP_{g,dr,c}\right)\right)\, BASEPCOMPCOMMD_{dr,c}\right) \\ &\left. + \sum_{dr}\sum_{c}\left(\left(BASEEXPORTS_{dr,c} - BASEIMPORTS_{dr,c}\right)\frac{BASEPCOMMWORLD_{c}}{BASEEXCHANGERT}\right)\right] \end{aligned} \tag{B.225}$$

$$gdpindexp = \Bigg[totalexpenditure - \sum_{dr}\Bigg(hhldindirecttax_{dr} + investindirecttax_{dr}$$
$$+rwindirecttax_{dr} + \sum_{g}\left(govtindirecttax_{dr,g}\right)\Bigg)\Bigg]$$
$$\div$$
$$\Bigg[\sum_{dr}\sum_{c}\Bigg(\Bigg(hhldconsump_{dr,c} + investconsumpq_{dr,c} + stockchangessupply_{sr\rightarrow dr,c}$$
$$+\sum_{g}(govtconsump_{g,dr,c})\Bigg)\ BASEPCOMPCOMMD_{dr,c}\Bigg)$$
$$+\sum_{dr}\sum_{c}\Bigg(\left(expcommoditys_{sr\rightarrow dr,c} - importdemand_{dr,c}\right)\frac{BASEPCOMMWORLD_{c}}{BASEEXCHANGERT}\Bigg)\Bigg] \tag{B.226}$$

$$realgdp = 1000\frac{totalexpenditure}{actualgdpindex} \tag{B.227}$$

$$totalexpenditure = \sum_{c}\Bigg(\sum_{dr}\Bigg(stockchangessupply_{sr\rightarrow dr,c} - nzimportpurchases_{dr,c}$$
$$\Bigg(hhldconsump_{dr,c} + \sum_{g}\left(govtconsump_{g,dr,c}\right) + investconsumpq_{dr,c}\Bigg)\ \mathbf{Pcompcommd}_{dr,c}\Bigg)$$
$$+\sum_{sr}\left(nzexportsales_{sr,c}\right)\Bigg)$$
$$+\sum_{dr}\Bigg(hhldindirecttax_{dr} + investindirecttax_{dr} + rwindirecttax_{dr} + \sum_{g}\left(govtindirecttax_{g,dr}\right)\Bigg) \tag{B.228}$$

$$stockchangessupply_{sr,c} = \sum_{dr}(regcdomcomms_{sr,dr,c} - regdomcommd_{sr,dr,c}) \tag{B.229}$$

B.12 Scenario setting equations

B.12.1 Auxiliaries

$$addtravelcosts_{dr} = \sum_{c}\left(ADDHHLDTRAVEL_{dr,c}(t)\ \mathbf{Pcompcommd}_{dr,c}\right) \tag{B.230}$$

$$maxprod_{dr,i} = \left(qasplannedprod_{dr,i} \times OPERABILITY_{sr\rightarrow dr,i}(t)\right)\ Pcindustrys_{sr\rightarrow dr,i} \tag{B.231}$$

$$qasplannedprod_{dr,i} = \max\left(qdesiredprod_{dr,i}(t = SHOCKINITIATION), qdesiredprod_{dr,i}\right) \tag{B.232}$$

$$qdesiredprod_{dr,i} = \frac{\mathbf{Desiredprod}_{dr,i}}{Pcindustrys_{sr\rightarrow dr,i}} \tag{B.233}$$

$$dommarginq_{dr} = \sum_{sr}\sum_{c}\left(regcdomcomms_{sr,dr,c}\, DMARGINSHOCKCOEF_{sr,dr,c}(t)\right) \tag{B.234}$$

$$exportmargindemand_{sr,m} = \sum_{c}\left(expcommoditys_{sr,c}\, EMARGINSHOCKCOEF_{sr,c,m}(t)\right) \tag{B.235}$$

$$exportmarginsupply_{dr,m} = \sum_{sr}\left(exportmargindemand_{sr,m}\, REGSHEXPMAR_{sr,dr,m}(t)\right) \tag{B.236}$$

$$importmargindemand_{dr,m} = \sum_{c}\left(\mathbf{Estimports}_{dr,c}\, IMARGINSHOCKCOEF_{dr,c,m}(t)\right) \tag{B.237}$$

$$importmarginsupply_{sr,m} = \sum_{dr}\left(importmargindemand_{dr,m}\, REGSHIMPMAR_{sr,dr,m}(t)\right) \tag{B.238}$$

$$\begin{aligned} marginconsumpq_{dr,c} = {} & \big(exportmarginsupply_{dr,m=Road} + importmarginsupply_{sr\rightarrow dr,m=Road} \\ & + dommarginq_{dr}\big) \times ROADMAP_{c} \\ & + \big(exportmarginsupply_{dr,m=Rail} + importmarginsupply_{sr\rightarrow dr,m=Rail}\big) \\ & \times RAILMAP_{c} \end{aligned} \tag{B.239}$$

$$pimprailmargins_{sr,dr} = \sum_{c}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\, RAILMAP_{c}\right) \tag{B.240}$$

$$pimproadmargins_{sr,dr} = \sum_{c}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\, ROADMAP_{c}\right) \tag{B.241}$$

$$\begin{aligned} pimportmargins_{dr,c} = {} & \left(\sum_{sr}\left(REGSHIMPMAR_{sr,dr,m=Road}(t)\, pimproadmargins_{sr,dr}\right)\right) \\ & \times IMARGINSHOCKCOEF_{dr,c,m=Road}(t) \\ & + \left(\sum_{sr}\left(REGSHIMPMAR_{sr,dr,m=Rail}(t)\, pimprailmargins_{sr,dr}\right)\right) \\ & \times IMARGINSHOCKCOEF_{dr,c,m=Rail}(t) \end{aligned} \tag{B.242}$$

$$pexprailmargins_{dr} = \sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, RAILMAP_{c}\right) \tag{B.243}$$

$$pexproadmargins_{dr} = \sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, ROADMAP_{c}\right) \tag{B.244}$$

$$\begin{aligned} pexportmargins_{sr,c} = {} & \left(\sum_{dr}\left(REGSHEXPMAR_{sr,dr,m=Road}(t)\, pexproadmargins_{dr}\right)\right) \\ & \times EMARGINSHOCKCOEF_{sr,c,m=Road}(t) \\ & \left(\sum_{dr}\left(REGSHEXPMAR_{sr,dr,m=Rail}(t)\, pexprailmargins_{dr}\right)\right) \\ & \times EMARGINSHOCKCOEF_{sr,c,m=Rail}(t) \end{aligned} \tag{B.245}$$